A comment on 'The trousers fracture test for viscoelastic elastomers', JAM 90(7), 071010, 2023

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Abstract

The Rivlin-Thomas classical energy balance model for tearing tests suggests that the fracture energy Γ is proportional to the work of the external load in the legs (potential energy), for not too large stretches in the legs, so the increase of Γ with loading rate is observed or measured, but not really explained by the model. Shrimali & Lopez-Pamies (SLP) have recently built a theory on viscoelastic fracture from recent experimental evidence of a critical (stretch rate independent) stretch on nucleation of cracks in pure shear tests. The theory in other words incorporates this evidence in an otherwise energy based criterion. In the paper "The trousers fracture test for viscoelastic elastomers" (J Appl Mech ASME (2023) 90(7), 071010), they seem to obtain the critical (stretch rate independent) stretch condition for the case of trouser tests (applying their theory from a long enough crack) both in *nucleation* and *steady state propagation*. This outcome seems reasonable since for a linear material, looking at the elastic limit cases of very slow and very fast rate it would produce an increase of the load, and hence of the fracture energy, proportional to the increase of the modulus, which is in line with what has been found experimentally, although possibly in contrast with classical rate-independent cohesive models, at least for crack nucleation.

Keywords:
Viscoelasticity, trousers tests, Shrimali & Lopez-Pamies theory

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1. Introduction

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Early investigations on rubber material fracture focused on generalizing Griffith's condition to non-linear behaviour, incapsulating the effect of loading rate into a measured "tearing energy" which takes into account of all the complex processes occurring at the crack tip. For example, Rivlin-Thomas [1], looked at pure shear specimen (see Fig.1).

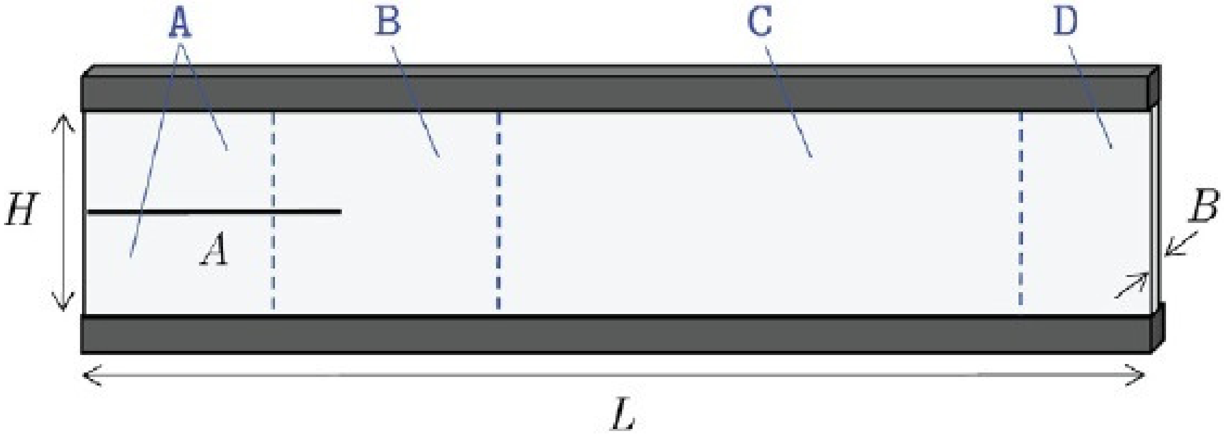

Fig.1 - A pure-shear test specimen

They suggested that the only change of strain energy during an infinitesimal propagation of the crack is the relaxation of the energy of a small piece of material well ahead of the crack (in region C) under uniform state of stress to the unloaded state well behind the crack tip (region A). The area near the crack tip, B, obviously dissipates energy which we enclose into "fracture energy", and therefore Rivlin and Thomas suggested the analysis holds also for non linear elastic materials, provided the fracture energy is no longer the adiabatic value (or a surface energy) Γ_0 but an increased value Γ to be found. Rivlin-Thomas equation gives then

$$\Gamma = S\left(\lambda_c\right) H \tag{1}$$

where $S\left(\lambda_c\right)$ is the area under the stress-stretch curve for the pristine material up to stretch λ_c . The rubber materials considered by Rivlin and Thomas were also clearly viscoelastic, but their loading rates were slow or anyway were not varied systematically, so the equation above was intended for *a* *given loading rate.* Greensmith and Thomas ([2]) later were probably the first to find the very large impact of viscoelasticity in fracture energy at different rates or temperatures.

Recent experiments ([3]) use this equation for different stretch rates, and notice that the critical stretch λ_c for nucleation of the crack is *stretch rate-independent.* In other words, ([3]) measure the effective fracture energy value Γ for a non-linear material, but also as a function also of rate writing an equation

$$\Gamma\left(\dot{\lambda}\right) = S\left(\lambda_c, \dot{\lambda}\right) H \tag{2}$$

where $S\left(\lambda_c, \dot{\lambda}\right)$ is the area under the stress-stretch curve for the pristine material at the stretch rate $\dot{\lambda}$ up to stretch λ_c when nucleation was found to occur for the specimen with precrack longer than the heigth of the specimen H. They found the stretch at nucleation λ_c was approximately stretch rate independent. By doing this, they found an increase of Γ with $\dot{\lambda}$ of a factor about 3.5 (see their Fig.4) at nucleation. Classical models or empirical equations like Gent-Schultz/Schapery/Persson-Brener (see ([4]), ([5])) give the fracture increase as a function of steady state crack speed $\Gamma(V)$, so these approaches seem to give at nucleation by definition $\Gamma(0) = \Gamma_0$ which is contradicted by experiments showing an increase of Γ with $\dot{\lambda}$ of a factor about 3.5. Hence, if we assume the very slow or very fast stretch rates in the experiments are slow and fast enough that we can consider the material elastic, still we cannot use the same fracture energy which is not an intrinsic property of the material, since this is stretch rate dependent $\Gamma\left(\dot{\lambda}\right)$. What we can write is that (simplifying as if the material were linear with relaxed modulus E_R and instantaneous one E_I)

$$\Gamma_{slow}^{nucl} = \Gamma_0 = \frac{1}{2} E_0 \left(\lambda_c^{slow} - 1\right)^2 H \tag{3}$$

and

$$\Gamma_{fast}^{nucl} = 3.5\Gamma_0 = \frac{1}{2} E_I \left(\lambda_c^{fast} - 1\right)^2 H \tag{4}$$

The (non-linear) stress-stretch curves show an increase of modulus (in the linear part) of the order of a factor 3 in ([3]) , so we have explained that two equations (3,4) above work well for a nearly constant

$$\lambda_c^{fast} = \lambda_c^{slow} = \lambda_c \tag{5}$$

which is indeed what appears from a more refined investigation in ([3]). In other words, this is telling us that the increase of fracture energy at nucleation goes proportional to the increase of modulus. This is incidentally similar to the increase observed with the $\Gamma(V)$ Gent-Schultz/Schapery/Persson-Brener models, with a change of speed of propagation by various orders of magnitude. For nucleation, $\Gamma\left(\dot{\lambda}\right)$ seems to follow the same pattern, with a change of stretch rate by various orders of magnitude, which means it will obey similar viscous processes rate-temperature superposition principles. As a consequence, in tearing or peeling experiments, we will not see much difference between nucleation and propagation phases, in terms of fracture energy, as is likely to be the case, since if large discrepancies had been observed, they would have been reported in the extensive investigations like ([5]).

In this pure shear test geometry, however, where tests are conducted at given stretch rate, the stretch will continue to increase immediately after nucleation, and hence we cannot see a steady state propagation process: ([3]) actually report that nucleation corresponds apparently to the maximum force. Shrimali & Lopez-Pamies ([6]) (SLP) then, based on assuming the constant critical stretch λ_c, developed recently an interesting theory for nucleation which tries to explain why the Griffith condition for the strain energy (W) release rate (A is crack area) is seen to increase with stretch rate $\dot{\lambda}$: at nucleation

$$-\left.\frac{\partial W}{\partial A}\right|_{\lambda=\lambda_c} = \Gamma_0\left(1 + f_c\left(\dot{\lambda}\right)\right) \tag{6}$$

Indeed, energy is separated into stored energy at any given instant (further split into “equilibrium” W^{eq} and “non-equlibrium” part W^{Neq} — the latter is the part which should get dissipated ”eventually” if we fix the part of the boundary which is not traction free), and dissipated one:

$$W = W^{eq} + W^{Neq} + W^{v} \tag{7}$$

SLP then argue that the Griffith condition can be written in terms of the equilibrium energy

$$- \left. \frac{\partial W^{eq}}{\partial A} \right|_{\lambda=\lambda_c} = \Gamma_0 \tag{8}$$

which is they call the "true" Griffith law, while

$$\Gamma_0 f_c \left(\dot{\lambda} \right) = - \left. \frac{\partial W^{Neq}}{\partial A} \right|_{\lambda=\lambda_c} - \left. \frac{\partial W^{v}}{\partial A} \right|_{\lambda=\lambda_c} \tag{9}$$

All of the above derivatives need to be taken at constant stretch, i.e. at fixed boundary, so that there is no influence of the contribution of external loads, and moreover for equilibrium at nucleation the stretch has to be the critical one λ_c, which is what gives the fracture energy enhancement. SLP has in other words combined Rivlin-Thomas experimental procedure with (temperature)-rate effects and with the experimental observations of ([3]).

Once again, this result *should not be confused* with the steady state propagation load increase with crack velocity in classical linear theories for semi-infinite cracks like cohesive models of Knauss-Schapery or models based on dissipation (see for a recent comparison Hui et al ([4])). Indeed, for propagation $\Gamma_{fast}^{prop} / \Gamma_{slow}^{prop} = E_I / E_0$ also, but this is obtained with a change of the speed of crack propagation. Also, for nucleation of a linear material under very fast or very slow loading, we can apply the standard Griffith theory, and *if the fracture energy were really independent on rate*, then we would immediately obtain for any specimen shape

$$\frac{\lambda_{c,slow}}{\lambda_{c,fast}} = \sqrt{E_I / E_0} \tag{10}$$

which is in strong contrast to the experimental evidence in ([3]). Therefore, the SLP theory is *not a trivial generalization of the Griffith theory*, and strongly points to the fact that the fracture energy is itself loading *rate-dependent*, namely $\Gamma \left(\dot{\lambda} \right)$.

2. Trouser test

In trouser tests (see Fig.2), one cannot really write clearly that the Stress Intensity Factor K_I is linear with load and the Rivlin-Thomas equation reads instead

$$\Gamma = \frac{2}{B} P \tag{11}$$

where we have considered the usual case that the stretch in the specimen is not large, so that essentially the fracture energy is entirely due to the work of external loads (potential energy).

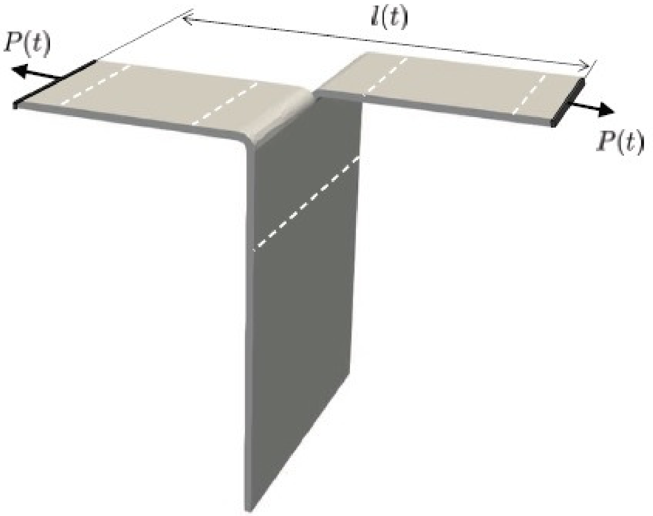

Fig.2 - A trouser test

In a sense, this equation makes it *less* evident than the pure-shear test Rivlin-Thomas equation (1) the role of elastic constants or, more in general, of the constitutive equations. It simply says that there is balance between energy flowing from the external loads into the fracture energy. If one were to assume that fracture energy were an intrinsic property of the material, one would be lead in error. Therefore, this equation is by no means predictive of the effective fracture energy.

Shrimali & Lopez-Pamies ([7]) have extended their analysis to the case of tearing (trouser test), which seems to reobtain that nucleation occurs at a rate-independent critical stretch λ_c and propagation continues when the same Griffith condition holding, namely the global stretch remains constant. This shows that, at least when steady state conditions are realized and stick-slip phenomena are not present, one can obtain from the constitutive equations

$$\Gamma_0 f_c\left(V\right) = -\left.\frac{\partial W^{Neq}}{\partial A}\right|_{\lambda=\lambda_c} - \left.\frac{\partial W^{v}}{\partial A}\right|_{\lambda=\lambda_c} \tag{12}$$

where derivatives are taken at constant critical stretch and for the corresponding stretch rate, and where we have stressed now that the function

$f_c(V)$ is a true function of steady state velocity of the crack V, as in classical theories of semi-infinite cracks.

Shrimali & Lopez-Pamies ([7]) results show how that the fracture energy increases with tearing rate, even though viscous dissipation has a maximum at an intermediate speed, which seems to contradict both theories which base the fracture energy purely on dissipation both for infinite size specimen (for which dissipation occurs also at infinite speed although very far from crack tip, and is the cause of the increase of fracture energy, see Hui et al ([4])), and for finite size specimen ([8]) where it was predicted that fracture energy would follow the dissipation and being maximum at intermediate speeds. Moreover, since in the limit of very fast tearing, we have essentially again an elastic material, where the global stretch is not only an average value but is uniform along the length of the entire leg of the specimens, supposing for simplicity that the material is linear, the Rivlin-Thomas equation reads, under the framework of the SLP theory that $\lambda_c = const$

$$\frac{\Gamma_{fast}(V)}{\Gamma_0(V)} = \frac{E_I}{E_0} \tag{13}$$

which is again in general qualitative agreement with classical *experimental* results which span 3 to 4 orders of magnitude in load and in elastic modulus for rypical elastomers (see ([5])). Notice that Knauss-Schapery or deGennes-Persson-Brener theories cannot be directly applied to tearing tests, but have received some indirect verification, which however requires fitting constants and in particular the size of the fracture zone at low speeds is unphysically small in sub-nanometer range (see [4]). In tearing tests, once again, it is the load that has been measured to increase by this large factor, and not the square of the K_I.

The situation of tearing tests is very similar to conditions in steady state peeling where there is an analogous Rivlin equation to (11), but many attempts to use simple rate-independent cohesive models have been largely *unsuccessful* by orders of magnitude ([9], [10]), as increase of fracture energy was observed to be merely a factor of the order of 2 even with $E_I/E_R = 1000$, while Gent [5]) measures Γ typically increased of a factor $10^3 - 10^4$. Gent ([5]) concluded that fracture energy rate dependency must result from a combined effect of both interfacial and bulk dissipation properties. This may be related to what is emerging in recent experiments ([11]) where interfacial fracture energy coming from bond scission seems to reach 80% of the total fracture energy in some conditions. The models for which the increase of fracture energy in steady state propagation comes entirely from bulk dissipation as Knauss-Schapery or deGennes-Persson-Brener (see [4]) seem additionally to predict enourmous increase of temperature near the crack tip ([12]) of the order of 1000°C, while experimentally in a follow-up experimental paper only $1°C$ increase could be found ([13]), and this discrepancy is not really resolved. In the dissipation model, for very fast propagation dissipation occurs in a region 10^8 times larger than the the fracture process zone at very small speed (see [4]). Although his fracture process zone at very small speed is unphysically in the sub-nanometer range, 10^8 of this is still large and should be visible experimentally, and one should see its effect in terms of heating.

3. Discussion

The SLP theory is mainly a theory of nucleation of cracks, and the equivalence of the nucleation and propagation phase in tearing given by Shrimali & Lopez-Pamies ([7]) should not be taken as a firm proof. The SLP theory points clearly to the fact that we have a loading rate-dependent nucleation fracture energy, and not, as often beleived, just a steady state propagation crack speed-dependent fracture energy.

Moreover, SLP is built on somehow limited experimental evidence of two sets of data on rubber ([3], [14]), tested on three decades of loading rates, not making advantage of temperature-rate superposition principle to extend the loading rate by changing the testing temperature, so the doubt remains that extending the range of loading rates, elongation stretch at nucleation could vary. In the literature, data on elongation or stretch at nucleation are scarce, but we did find a couple of references of interest, on plates with edge cracks. In particular, ([11]) test a cracked PMA-DA-0.4 (poly(methylacrylate) elastomer) at constant displacement rate and their Fig.1b shows stress-strain curves for range of temperatures from 25 to 80 °C. The critical stretch at rupture seems to be increasing with (effective) loading rate: we don't know precisely the nucleation strain, but if we follow the suggestion in ([3]) that it should be indicated by the time where the force reaches the maximum value, then even nucleation strain should be an increasing function of rate, in contrast with the Shrimali & Lopez-Pamies theory, but even more in contrast with the predicted decrease of the simplistic "Griffith theory" with a constant fracture energy (10).

Perhaps even more appropriate is another recent study ([15]) on PolyTetraFluoroEthylene (PTFE). Their material seems to have a ratio between relaxed modulus and instantaneous modulus of about 0.2 and hence we expect from the simplistic "Griffith theory" with a constant fracture energy (10) a decrease of the elongation of a factor $\sqrt{5} = 2.2$ with rate. Instead, from their data we obtain the elongation at nucleation plotted as "nucleation strain" in Fig.3 as computed from $\varepsilon = \Delta l / l_0$ where Δl is the elongation between the grips of the testing machine and l_0 the initial length, as a function of the displacement rate between the grips $\dot{\Delta l}[mm/\min]$, whereas the strain at rupture is reported as "rupture strain". The nucleation strain doesn't seem to follow a clear monotonic trend, whereas the rupture strain is found more clearly to decrease with rate of a factor about 2.6. This shows that nucleation and propagation phases should not be confused.

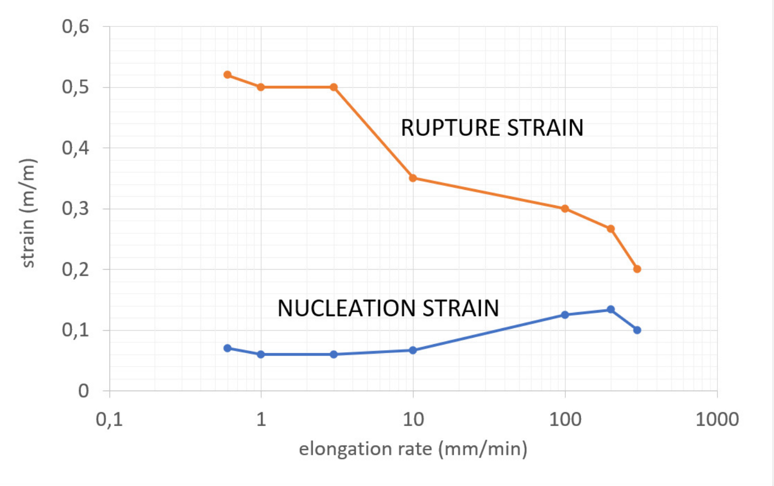

Fig.3- The strain at nucleation and at rupture, as computed from $\varepsilon = \Delta l / l_0$ where Δl is the elongation between the grips of the testing machine and l_0 the initial length. From experiments in a edge cracked specimen ([15]) of PolyTetraFluoroEthylene (PTFE) at constant displacement rate between the grips $\dot{\Delta l}[mm/min]$ on the horizontal axis.

4. Conclusions

We made some comments on the recent Shrimali & Lopez-Pamies model for trouser test of viscoelastic materials, in contrast to classical models both for nucleation and steady-state propagation of cracks, and basic experimental knowledge on this problem. We found that the main assumption of rate-independent critical stretch leads to quite expected results in the limit of very slow and very fast loading compared to the classical experimental literature, whereas there is sharp contrast to simple theoretical linear models for nucleation of cracks, which remains to be clarified. Hence, given the theory is simple and for quite general material, it is a promising proposal. The SLP theory seems to suggest that dissipative processes at the crack tip in a nucleating crack, result in an increased fracture energy similarly to what is observed in steady state propagation. Therefore, nucleation may require rate-dependent cohesive models to be captured.

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