For some reason, I am writing a finite element program using the Timoshenko beam element and Total Lagrangian Formulation for geometric nonlinear analysis.

I adopted formulations in K.J. Bathe's classic book *Finite Element Procedures in Engineering Analysis* (Chapter 6, p358-371).

In the calculation of the strain-displacement transformation matrix, the orthogonal nodal unit vectors at time t are required, which are denoted by ${}^t\boldsymbol{V}_r^k, {}^t\boldsymbol{V}_s^k, {}^t\boldsymbol{V}_r^k$ (see p364,365). This means at each time step these orthogonal unit vectors need to be updated. (Am I right?)

My problem is related to the update of these vectors. In my program, I adopted an Rodrigues formula's for updating, see the book by Crisfied: *Non-linear Finite Element Analysis of Solids and Structures. Volume 2: Advanced Topics*. (Chapter 16.3, equation (16.21). the updating procedure is as follow:

(1) Solve the equations

$${}_0^{t+\Delta t}\boldsymbol{K}^{i-1}\Delta\boldsymbol{U}^i = \boldsymbol{g}^{i-1}$$

and obtain global incremental displacement vector ΔU^i, from which the nodal incremental 'rotational vector' $\Delta\boldsymbol{\theta}^i$ can be extracted.

$$\Delta\boldsymbol{\theta}^i = \begin{bmatrix}\Delta\theta_1^i & \Delta\theta_2^i & \Delta\theta_3^i\end{bmatrix}^T$$

(2) Update the rotation matrix

$${}_0^{t+\Delta t}\boldsymbol{R}^i = (\Delta_0^{t+\Delta t}\boldsymbol{R}^i) * ({}_0^{t+\Delta t}\boldsymbol{R}^{i-1})$$

Where

$$\Delta_0^{t+\Delta t}\boldsymbol{R}^i = \boldsymbol{I}_3 + \frac{\sin\Delta\theta^i}{\Delta\theta^i}\boldsymbol{\Theta}^i + \frac{1}{2}\left[\frac{\sin(\Delta\theta^i/2)}{\Delta\theta^i/2}\right]^2\boldsymbol{\Theta}^i \quad \Delta\theta^i = \left\|\Delta\boldsymbol{\theta}^i\right\| \text{ and}$$

$$\boldsymbol{\Theta}^i = \begin{bmatrix} 0 & -\Delta\theta_3^i & \Delta\theta_2^i \\ \Delta\theta_3^i & 0 & -\Delta\theta_1^i \\ -\Delta\theta_2^i & \Delta\theta_1^i & 0 \end{bmatrix}$$

(3) Update the orthogonal nodal unit vectors

$${}_0^{t+\Delta t}\boldsymbol{V}_r^i = ({}_0^{t+\Delta t}\boldsymbol{R}^i)({}_0^{t+\Delta t}\boldsymbol{V}_r^{i-1}) \qquad {}_0^{t+\Delta t}\boldsymbol{V}_s^i = ({}_0^{t+\Delta t}\boldsymbol{R}^i)({}_0^{t+\Delta t}\boldsymbol{V}_s^{i-1})$$

$${}_0^{t+\Delta t}\boldsymbol{V}_t^i = ({}_0^{t+\Delta t}\boldsymbol{R}^i)(+\Delta t\boldsymbol{V}_t^{i-1})$$

Then the latest vectors are used in the calculation of the strain-displacement transformation matrix for time $t+\Delta t$.

In the real calculation, the strain-displacement matrix is only calculated in the beginning of each load step, then keep unchanged during the iteration while the nodal vectors ${}^t\boldsymbol{V}_r^k, {}^t\boldsymbol{V}_s^k, {}^t\boldsymbol{V}_r^k$ are updated at each iteration.

Can anyone tell me if there are mistakes in the above process? Thanks.