Consider the function

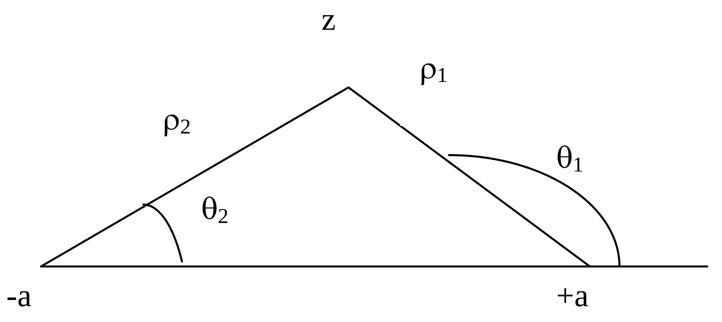
$$\varphi(z) = \frac{1}{\sqrt{z^2 - a^2}}$$

With a brunch cut at –a < x < a

Let

$$z - a = \rho_1 e^{i\theta_1}, \quad -\pi < \theta_1 < \pi$$

$$z + a = \rho_2 e^{i\theta_2}, \;\; 0 < \theta_2 < 2\pi$$



So that

$$\varphi(z) = \frac{1}{\sqrt{\rho_1 \rho_2}} e^{-\frac{i}{2}(\theta_1 + \theta_2)}$$

When z approaches the cut from above

$$\rho_1 = a - x, \theta_1 = \pi, \quad \rho_1 = a + x, \theta_2 = 0$$

So that

$$\varphi_+(x) = \frac{1}{\sqrt{a^2 - x^2}} e^{-\frac{i}{2}(\pi)}$$

When z approaches the cut from below

$$\rho_1 = a - x, \theta_1 = -\pi, \quad \rho_1 = a + x, \theta_2 = 2\pi$$

So that

$$\varphi_+(x) = \frac{1}{\sqrt{a^2 - x^2}} e^{-\frac{i}{2}(\pi)}$$

Consequently

$$\varphi_+(x) + \varphi_-(x) = \frac{2}{\sqrt{a^2 - x^2}} e^{-\frac{i}{2}(\pi)}$$

In your document you found

$$\varphi_-(x) = \frac{1}{\sqrt{a^2 - x^2}} e^{+\frac{i}{2}(\pi)}$$

And

$$\varphi_+(x) + \varphi_-(x) = 0$$

I haven’t understood how did you get this result