Method to analyze programmable deformation of dielectric elastomer layers

Xuanhe Zhao and Zhigang Suo [a]
School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138

Abstract

A layer of a dielectric elastomer expands its area when a voltage is applied across its thickness. The layer can be programmed to deform in three dimensions by using patterned prestretches, electrodes, and stiffeners. To aid the design of such programmable deformation, this paper describes a computational method, exploiting the fact that the prescribed voltage sets the nominal electric field. The method is demonstrated with examples including formation of wrinkles, movements of a hand-like actuator, and nonlinear oscillation of a balloon.

Key words: dielectric elastomer, programmable deformation, nominal electric field

[a] suo@seas.harvard.edu

Intense effort is being made to develop dielectric elastomers for applications such as sensors, actuators, and generators. [1-3] The essential part of such a device is a layer of a dielectric elastomer sandwiched between two compliant electrodes. When a voltage is applied between the electrodes, the layer reduces its thickness and expands its area. For the layer to deform appreciably, the electric field needed is high, on the order of 10^8V/m. To avoid excessively high voltage in use, the layer must be thin, typically below 1 mm.

Programmable deformation in three dimensions can be achieved with patterned prestretches, electrodes, and stiffeners. [1-3] Existing computational methods, however, have only been demonstrated for relatively simple configurations. [4-11] This paper describes a method that can be embedded in commercial finite element software. Consequently, the method can be widely used to explore dielectric elastomer devices of new configurations and functions.

Our method is based on a general theory of elastic dielectrics. [12-17] The synopsis here follows closely Ref [17]. When an elastic dielectric deforms from a reference state to a current state at time t, a material particle of coordinate $\mathbf{X}$ in the reference state moves to a place of coordinate $\mathbf{x}(\mathbf{X},t)$ in the current state. The deformation gradient is

$$F_{iK} = \frac{\partial x_i(\mathbf{X},t)}{\partial X_K},$$

(1)

The nominal electric field is

$$\tilde{E}_K = -\frac{\partial \Phi(\mathbf{X},t)}{\partial X_K},$$

(2)

where $\Phi(\mathbf{X},t)$ is the electrical potential of the material particle $\mathbf{X}$ in the current state.

Let $\mathbf{B}(\mathbf{X},t)dV(\mathbf{X})$ be the applied force on an element of volume, $\mathbf{T}(\mathbf{X},t)dA(\mathbf{X})$ the

applied force on an element of surface, $\rho(\mathbf{X})dV(\mathbf{X})$ the mass of the element of volume, $Q(\mathbf{X},t)dV(\mathbf{X})$ the charge on the element of volume, and $\Omega(\mathbf{X},t)dA(\mathbf{X})$ the charge on the element of surface. The elastic dielectric is held at a fixed temperature, so that the thermodynamic state of an element of volume is characterized by a free-energy function, $\hat{W}(\mathbf{F},\tilde{\mathbf{E}})$. Every element of the body is in a state of local thermodynamic equilibrium if

$$\int \frac{\partial \hat{W}}{\partial F_{iK}} \frac{\partial \xi_i}{\partial X_K} dV = \int \left(B_i - \rho \frac{\partial^2 x_i}{\partial t^2} \right) \xi_i dV + \int T_i \xi_i dA \qquad (3)$$

holds for arbitrary function $\xi_i(\mathbf{X})$, and

$$\int \frac{\partial \hat{W}}{\partial \tilde{E}_K} \frac{\partial \zeta}{\partial X_K} dV = \int Q \zeta dV + \int \Omega \zeta dA \qquad (4)$$

holds for arbitrary function $\zeta(\mathbf{X})$. The integrals extend over the volume and surface of the body, respectively.

Equations (1)-(4) define an initial value problem that evolves the fields $\mathbf{x}(\mathbf{X},t)$ and $\Phi(\mathbf{X},t)$. The following data need be prescribed: the fields of displacement, velocity and electric potential at an initial time, the applied forces $\mathbf{B}(\mathbf{X},t)$ and $\mathbf{T}(\mathbf{X},t)$, the electric charges $Q(\mathbf{X},t)$ and $\Omega(\mathbf{X},t)$, the mass density $\rho(\mathbf{X})$, and the free-energy function $\hat{W}(\mathbf{F},\tilde{\mathbf{E}})$. Also prescribed are displacement and voltage on parts of the boundary.

We now specialize the general theory to a form suitable for thin layers. The layer is taken to be free of extrinsic charges, so that $Q = 0$ everywhere, and $\Omega = 0$ on the surfaces of the layer uncovered by electrodes. More than one pair of electrodes may be patterned on the layer in order to program complex sequences of deformation. The lateral size of each electrode is large compared to the thickness of the layer, such that the fringe field at the edges of the

3

electrodes can be neglected so far as the overall deformation is concerned. The electric field can also be neglected in part of the layer uncovered by the electrodes. The nominal electric field in the layer between a pair of electrodes is

$$\tilde{\mathbf{E}} = \frac{\Phi}{H}\mathbf{N},$$

(5)

where Φ is the voltage applied between the electrodes in the current state, H the thickness of the layer in the reference state, and $\mathbf{N}$ the unit vector normal to the layer in the reference state. The direction of $\mathbf{N}$ is fixed if the layer in the reference state is planar, but varies if the layer in the reference state is a curved. *In both cases, the prescribed voltage sets the nominal electric field.*

For the layer (2) is replaced by (5) to prescribe the nominal electric field, and (4) is removed from the list of governing equations. Consequently, (1) and (3) define the initial-value problem that evolves the field $\mathbf{x}(\mathbf{X},t)$. The structure of this initial-value problem is identical to that of an elastic body subject to mechanical loads alone. The known nominal electric field (5) enters the problem through the free-energy function $\hat{W}(\mathbf{F},\tilde{\mathbf{E}})$ in a form of an eigenstrain.

We embed this method in the commercial finite element software, ABAQUS, by coding the free-energy function $\hat{W}(\mathbf{F},\tilde{\mathbf{E}})$ and its derivatives in a user-supplied subroutine, UMAT. To illustrate the method we adopt a model material, called the ideal dielectric elastomer, with the free-energy function[18]

$$\hat{W}(\mathbf{F},\tilde{\mathbf{E}}) = \frac{\mu}{2}\left[F_{iK}F_{iK} - 2\log(\det\mathbf{F}) - 3\right] + K(\det\mathbf{F}-1)^2 - \frac{\varepsilon F_{iK}^{-1}\tilde{E}_K F_{iL}^{-1}\tilde{E}_L}{2\det\mathbf{F}},$$

(6)

where μ is the shear modulus, and ε the permittivity. Following a usual practice in computation, we use the second term to approximately enforce incompressibility of the

elastomer, with the constant K set to be much greater than μ.

We verify our method by analyzing several simple configurations (Fig. 1), which have been analyzed by solving algebraic equations or ordinary differential equations (ODE). For a flat layer of an elastomer fully covered by electrodes on both faces, the deformation in the layer is homogenous until the nominal electric field reaches a peak, where the layer undergoes the electromechanical instability. [11] When we use the static option in this calculation, the method reproduces the analytic solution before the instability, but does not go beyond the instability. The situation is similar for the balloon subject to a pressure and a voltage. [14] For a flat layer partially covered by circular electrodes and prestretched, our method reproduces the existing results obtained by the ODE [6], and goes beyond the existing results when the layer loses tension, as discussed below.

When the voltage-induced expansion in area is constrained by surrounding materials, the layer may form wrinkles; Fig 2(a) shows an example[21]. We model a layer patterned with elliptic electrodes, the ratio of the two axes being $A_1 / A_2 = 0.6$. The layer is prestretched by $\lambda_P = 1.1$ and is then subject to a voltage. As the voltage increases, the region with electrodes expands in all directions. The stress in the direction of axis A_2 becomes compressive, while the stress in the direction of A_1 is still tensile. Wrinkles form along the A_2 direction, Fig. (2b). Our method may be used to examine the effects of wrinkles on the operation of the devices.

A hand-like actuator has been demonstrated recently. [9] A prestretched layer of a dielectric elastomer is first attached to a frame cut from a thin foil of a polymer. When the external constrain is removed, this planner structure self-assembles into a three-dimensional

shape.   The structure opens when a voltage is applied across the thickness of the elastomer, and closes when the voltage is withdrawn.   The structure can be used to pick and place objects. Fig 3(a) shows the simulation of the self-assembly from a planar structure to a three-dimensional one.   We also simulate the structure as it opens, closes, and lifts a weight by contact friction, Fig. 3(b).

Once an oscillating voltage is applied across the skin of a pressurized balloon, the balloon oscillates.   The oscillation is nonlinear when the amplitude of the voltage is large enough, as shown in a previous study using an ODE.[22]   We use this phenomenon to illustrate the capability of simulating dynamic deformation.   Fig (4a) shows the applied oscillating voltage, and figure (4b) shows the simulated oscillation of the balloon.   The balloon oscillates with the same frequency as the input voltage. The amplitude of the oscillation, however, varies with another frequency.   The results from our simulation match perfectly with those from solving the ODE.

In summary, we have developed a method to simulate programmable deformation of dielectric elastomer layers.   Our method, embedded in commercial finite element software, can be widely used to explore devices of new configurations and functions.

Figure Captions

FIG. 1.   Several simple configurations: a flat layer fully covered by electrodes on both faces, a spherical balloon subject to a pressure and a voltage, and a prestretched flat layer partially covered by circular electrodes.

FIG. 2.   A prestretched layer partially covered by elliptic electrodes.   Wrinkles form when the voltage exceeds a critical value. [21]
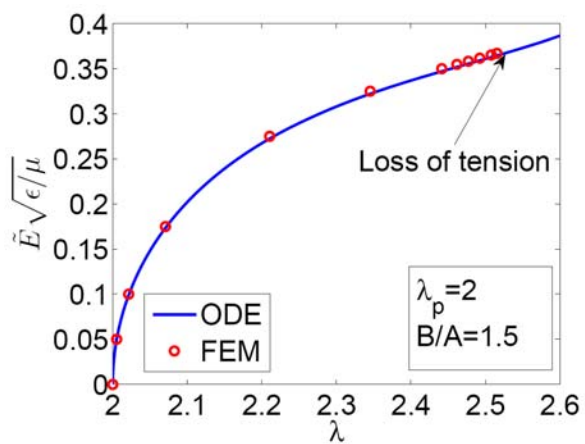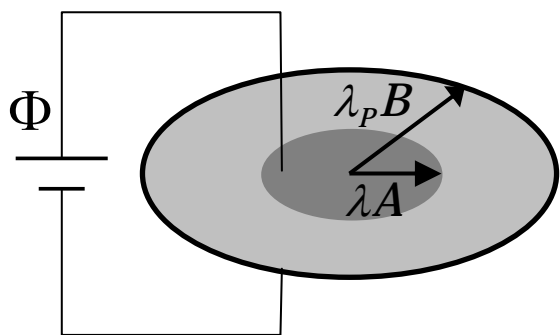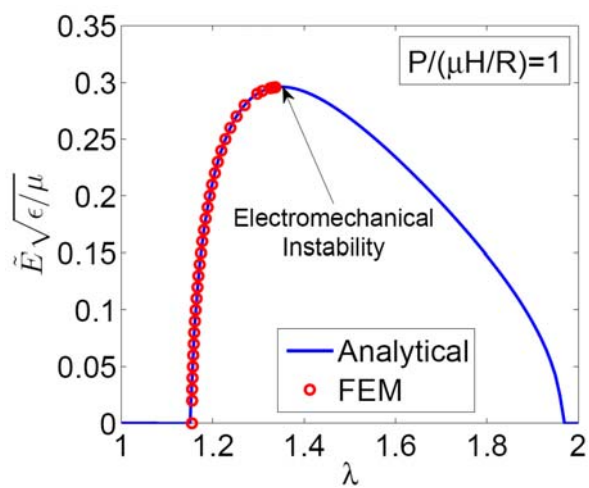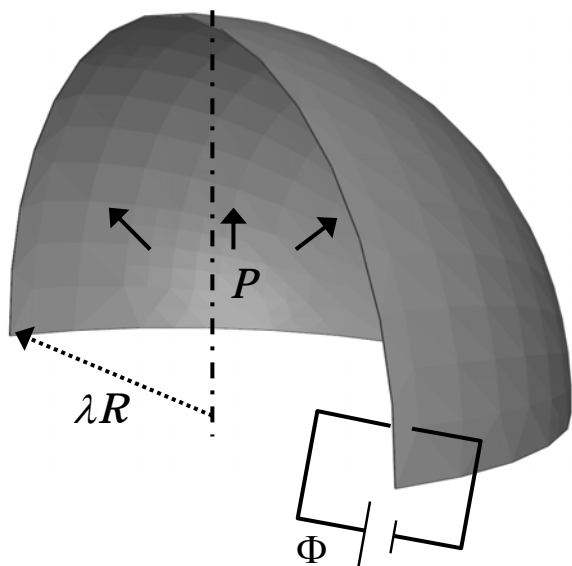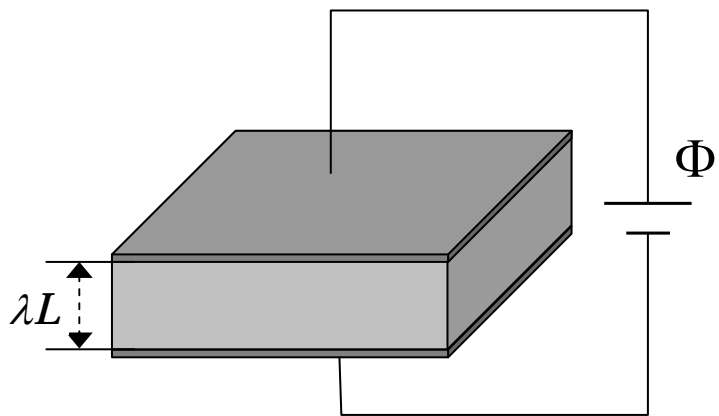
Fig.3. A hand-like actuator. [9] The thickness of the frame is set to be equal to that the dielectric elastomer, and the shear modulus of the frame is set to be twice that of the dielectric elastomer.
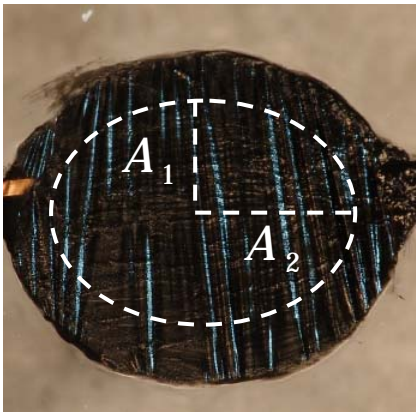
FIG. 4.   Nonlinear oscillation of a balloon subject to a sinusoidal voltage and a constant pressure.
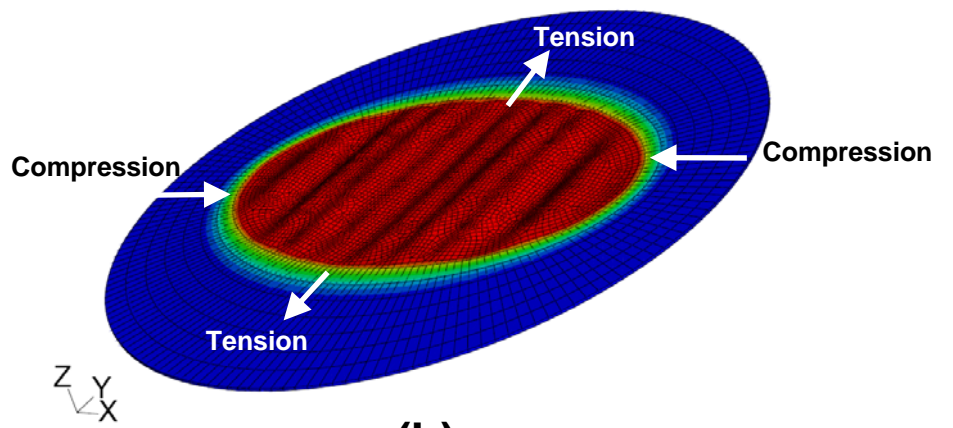
Reference

[1] F. Carpi, D. De Rossi, R. Kornbluh, R. Pelrine, and P. Sommer-Larsen, *Dielectric elastomers as electromechanical transducers* (Elsevier, Oxford, 2008).

[2] A. O'Halloran, F. O'Malley, and P. McHugh, Journal of Applied Physics **104,** 071101 (2008).

[3] Y. Bar-Cohen and Q. M. Zhang, Mrs Bulletin **33,** 173-181 (2008).

[4] F. Carpi and D. De Rossi, Materials Science & Engineering C-Biomimetic and Supramolecular Systems **24,** 555-562 (2004).

[5] M. I. Frecker and W. M. Aguilera, Smart Materials & Structures **13,** 82-91 (2004).

[6] J. S. Plante and S. Dubowsky, International Journal of Solids and Structures **43,** 7727-7751 (2006).

[7] M. Wissler and E. Mazza, Sensors and Actuators a-Physical **120,** 184-192 (2005).

[8] D. K. Vu, P. Steinmann, and G. Possart, International Journal for Numerical Methods in Engineering **70,** 685-704 (2007).

[9] G. Kofod, W. Wirges, M. Paajanen, and S. Bauer, Applied Physics Letters **90,** 081916 (2007).

[10] J. X. Zhou, W. Hong, X. H. Zhao, Z. Q. Zhang, and Z. G. Suo, International Journal of Solids and Structures **45,** 3739-3750 (2008).

[11] X. H. Zhao and Z. G. Suo, Applied Physics Letters **91,** 061921 (2007).

[12] R. A. Toupin, Journal of Rational Mechanics and Analysis **5,** 849-915 (1956).

[13] H. F. Tiersten, J. Acoust.Soc. Am. **57,** 660 (1975).

[14] N. Goulbourne, E. Mockensturm, and M. Frecker, Journal of Applied Mechanics **72,** 899 (2005).

[15] A. Dorfmann and R. W. Ogden, Journal of Elasticity **82,** 99-127 (2006).

[16] R. M. McMeeking and C. M. Landis, Journal of Applied Mechanics-Transactions of the Asme **72,** 581-590 (2005).

[17] Z. G. Suo, X. H. Zhao, and W. H. Greene, Journal of the Mechanics and Physics of Solids **56,** 467-486 (2008).

[18] X. H. Zhao, W. Hong, and Z. G. Suo, Physical Review B **76,** 134113 (2007).

[19] R. Pelrine, R. Kornbluh, Q. B. Pei, and J. Joseph, Science **287,** 836-839 (2000).

[20] J. W. Fox and N. C. Goulbourne, Journal of the Mechanics and Physics of Solids **56,** 2669-2686 (2008).

[21] C. Keplinger, M. Kaltenbrunner, N. Arnold, and S. Bauer, Applied Physics Letters **92** (2008).

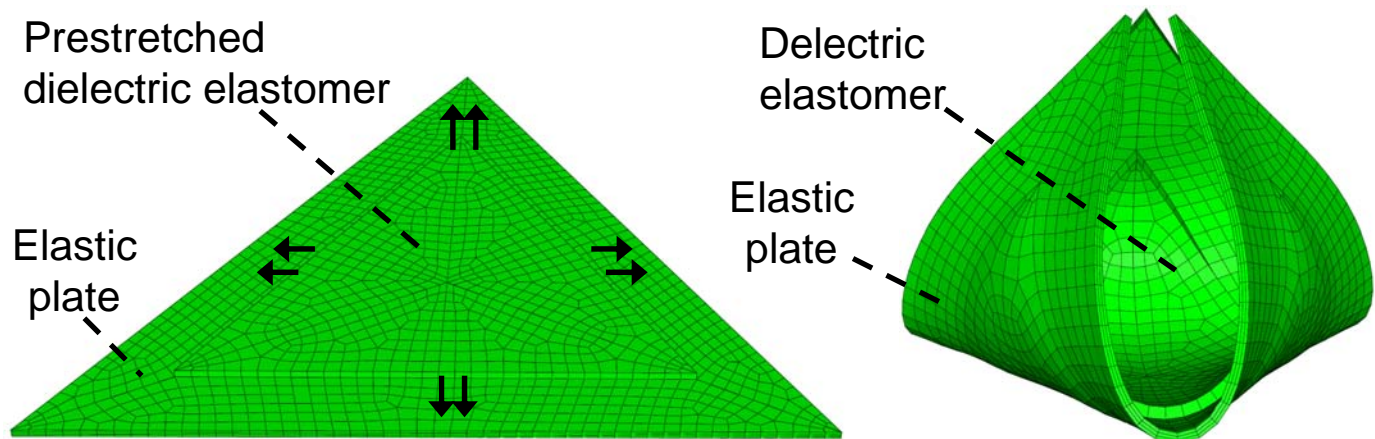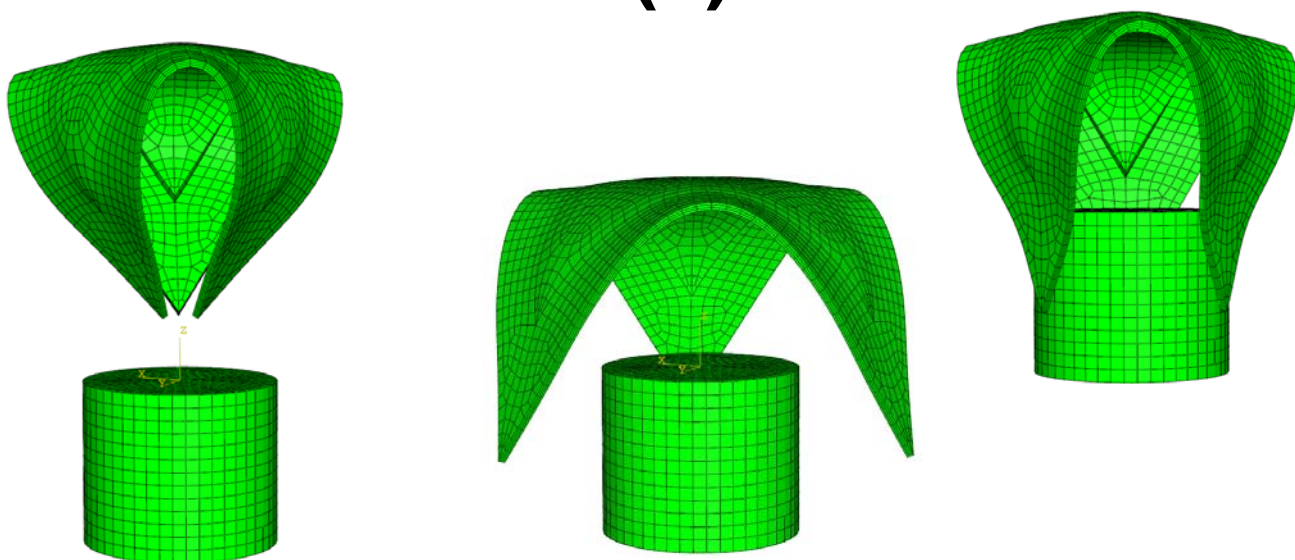[22] J. Zhu, S.Q. Cai, and Z.G. Suo, Unpulished work (2008).

(a)

(b)

Prestretched
dielectric elastomer

Elastic
plate

Delectric
elastomer

Elastic
plate

(a)

(b)

(a)

(b)

(c)