PLASTICITY PRINCIPLES IN FRICTIONAL PROBLEMS:
Some ideas for future investigations based on recent work

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Abstract: Based on some recent results by Anders Klabring, myself and Jim Barber, showing rigorously that Melan's theorem only works for a very restricted class of frictional problems, we suggest possible avenues of further research, with applications ranging from scientific areas like the sliding of tectonic plates in earthquakes to technological areas such as frictional dampers, contact joints in mechanical and biomechanical assemblies, pipelines, geotechnical systems, and many other components.

Friction is a key factor in many mechanical components, either as an undesired effect, causing energy losses and malfunctioning, or as a desired ingredient of some components such as brakes, clutches or dampers. Components subject to vibratory or oscillatory loading have similarities in behaviour with plastic monolithic bodies, including the possibility, for cyclic loading, of shakedown after the first few loading cycles, or of permanent ratchetting, i.e. incremental slip (see Antoni et al 2007). Shakedown would be beneficial in some cases (those for which incremental sliding may give rise to concerns over further service of the component, like is found in some conrod applications or in "walking" pipelines), or detrimental, in others, when frictional sliding energy expenditure is a desired effect, although at the possible risk of fretting fatigue, like in frictional dampers of turbine blade attachments.

At least three mechanisms are at play in attempting to reduce the possibility of microslip under oscillatory loading, namely:

(i) possibility of residual stresses, developing "shakedown"
(ii) increase of friction coefficient (friction hardening), sometimes up to 3 or 4 times like for light aluminium alloys and
(iii) wear of the sliding parts.

Which mechanism is stronger?

For simple low dimensional models this analogy friction/plasticity seems indeed very strong, and has raised hope in various authors of using Melan and Koiter's theorems for the mechanism (i) which would be very useful for example to define regions of safe exercise for systems which cannot function below the limit of full adhesion, yet would suffer problems in the case of ratchetting. However, Drucker (1954) already proved that the analogy is far from complete for limit loads (i.e. static loading) because of the non-associative nature of Coulomb friction (in that case for simple monotonic loading). Melan's theorem for frictional systems might be enunciated as *"If a set of time-independent tangential displacements at the interface can be identified such that the corresponding residual stresses when superposed on the time-varying stresses due to the applied loads cause the interface tractions to satisfy the conditions for frictional stick throughout the contact area at all times, then the system will eventually shake down to a state involving no slip, though not necessarily to the state so identified."*

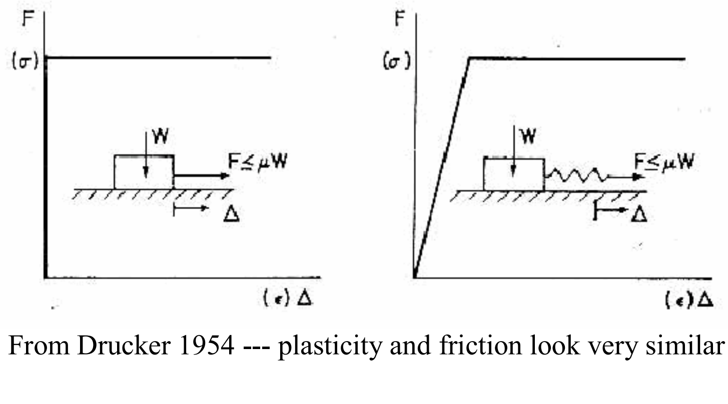

Fig.1 --- From Drucker 1954 --- plasticity and friction look very similar in a 0D model

Jim Barber, Anders Klarbring and I are in the process of publishing a paper showing that only for a limited class of problems involving Coulomb friction the original Melan theorem works for shakedown, and in practise only those for which the contact pressure is fixed, or more in general there is no coupling between tangential displacements and normal pressure at the interface. This is a rather restricted academic class of problems, and for all other systems, including all 3D systems, all incomplete contacts (for which at most residual stresses reduce the size of the slip zone, see Dini and Hills, 2004), and all coupled problems. This suggests that shakedown due to residual stresses (i) is in general not a very efficient mechanisms as it is in plasticity. Indeed, an additional complication occurs when considering the dynamic formulation, for which Adams (1995) has shown that Coulomb friction is known to be unstable in most cases, and would therefore lead to local separation and removal of residual stresses anyway. It is suggested finally that wear and significant increase of friction coefficient (i.e. mechanisms ii and iii) are perhaps more efficient mechanisms for reducing the amount of microslip as it is well known in fretting applications of light alloys. The application of plasticity theorems viceversa is not justified and may be largely underconservative in general. However, more research is needed to look at this problem from this perspective. Clearly, wear takes a long time to develop, although in fretting problems, for example, it would be an efficient mechanism to remove microslip at least under constant amplitude loading (see Ciavarella and Hills, 1998).

Limit load problems (static)

We can then make a step back, and consider *static* problems with friction. After all, there is a huge literature starting from Coulomb 1773. Coulomb's method is still very used today and probably is one of the few engineering methods still in use after more than 200 years! It gives the collapse load of a frictional system independently on the concept of stress, let alone the development of plasticity theory. After that, many methods have been devised including kinematic methods, or limit equilibrium, and the slip circle method of Bishop (and others) with application in geotechnical engineering. These essentially assume that the normal pressure on the slip surface (Coulomb yield condition) is determinate from equilibrium conditions. They produce unique solutions (for certain restricted classes of kinematic fields) and have been used for many years quite safely. As far as I know they always give “good” solutions. For a significant classes of problems a unique limit state does occur for friction and each case has to be argued afresh - unless someone can come up with an entirely new theory. However, it would be useful to come up with some more restricted results for limited classes of problems.

Why non-associativity may have *no* strong consequences on some solutions? Take a blocky system (blocks triangular in cross section, plane deformation), and the mechanism is induced by a force, causing friction limit to be reached on all interfaces. To find the magnitude of that force causing "failure" one can assume associativity of relative sliding and apply the kinematic approach of limit analysis. The kinematics will be fictitious as the sliding is not governed by associative rule, but the result will be no different from that for non-associative rule. That is because the problem is statically determinate, and assuming different flow rule cannot affect the solution. In other words: the work rate balance equation written for the incipient failure mechanism is analogous to the principle of virtual work, which is independent of the material properties. This is not true in general; for instance, for continuum the stress state associated with work dissipation terms is not necessarily in equilibrium, and the work rate balance equation written with these terms is not the principle of virtual work. Hence, the reason why the “old” geotechnical methods work well for soil mechanics is probably that slopes, landslides, etc., all are more or less statically determinate. We should investigate this.

Here is another peculiarity: the rate of work dissipation on a frictional interface with sliding governed by normality is zero. More in general, the reason why associativity with traditional Mohr-Coulomb yield condition is used so often in soil mechanics is that the work dissipation becomes independent of the stress state (zero if pure friction), otherwise the kinematic approach cannot be used effectively since the work dissipation cannot be calculated without given stress field. Not less important is the fact that the theorems of limit analysis do not hold for nonassociative materials, although some recent efforts to account for non-associativity in limit analysis appear to be successful. Anyway, we seem to agree that in "statically determinate systems" non-associativity has no strong consequences, so I ask:

1) can we generalize this also to cyclic loading (i.e. the absence of shakedown, incremental collapse or ratchetting?). I am not very familiar with the Koiter's 1960 theorem on that, and I know people in geomechanics do not use that theorem, but I suspect we could do it. We should check if and when the limit state is independent of the initial state of residual stress.

2) if the "fun" starts with statically indeterminate systems (we know that shear bands, chaos, instability are due to friction, i.e. mostly to non-associativity), can we generate some particularly intriguing case, or is this problem intrinsically left to be "case-specific" in all respects? Can we define "weakly" statical indeterminate, and "strongly" statical indeterminate, just as an example? Maybe with critical state theories?

Granular materials

Other applications can include the case of frictional materials, such as powder or granular materials in general. The non-associativity of friction is known to give rise in granular solids to shear bands, the absence of a real elastic regime and rather a phenomenon similar to ratchetting even at modest shear stresses (Alonso-Marroquin et al 2002). However, more recent studies showed a transition between ratcheting and shakedown (Alonso-Marroquin et al. 2004, Garcia-Rojo et al. 2004), depending on the amplitude of the stress variations and the strength of friction. Further recent 3D results on cyclic loading show (Luding et al, 2007) that ratcheting is observed for rather weak friction, while stronger friction seems to work against ratcheting by stabilizing the packing due to the stronger tangential forces. That shakedown here may not be a mechanism due to sliding emerges from the fact that the strain rate shows a sharp transition when friction is increases above a certain value, whereas the fraction of sliding contacts shows a smooth, continuous transition, indicating that both effects are decoupled. Therefore, there are various mechanisms of shakedown, and that due to residual stresses may not be the most efficient. Also, the results of the present paper suggest that the latter mechanism may depend on the load path in general, and hence it cannot be relied on.

Conclusions

There are more open questions in frictional systems than one expects, and some early investigations or engineering solutions, are still remarkable today. This is particularly true for Coulomb's method which is dated 1773, is not based on the concept of stress, let alone plasticity theorems, and yet is still used today! If shakedown theorems have partly lost their appeal today even for pure plasticity applications, since even in plasticity deviations from the purely perfect plastic associative material are significant, and the power of computers has increased since the '50s, the main idea remains that they would permit to define "safe regions", what is not possible for friction, where dependence on initial conditions, as well as separation at the interface (or more in general, reduction of pressure, i.e softening behaviour) are expected to be influenced also by frictional sliding. We still need to solve the issue of what happens and when, depending on loading conditions, but that not necessarily a general method of prediction is possible, other than a transient simulation. Since friction coefficient is not a fixed quantity anyway, and other mechanisms are at play (in particular, wear), friction seems to pose a lot of interesting questions in the years to come. Notice incidentally that we have only considered Coulomb friction, but it is clear that this is not a sufficiently rich model today, particularly in view of its ill-posed nature for a dynamic formulation. It sounds there are more open problems than I can hope to solve alone on my own…

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