Introduction to Fracture

Ramadas Chennamsetti

rd_mech@yahoo.co.in

Introduction

- Design of a component
  - Yielding – Strength
  - Deflection – Stiffness
  - Buckling – critical load
  - Fatigue – Stress and Strain based
  - Vibration – Resonance
  - Impact – High strain rates
  - Fracture - ???

rd_mech@yahoo.co.in

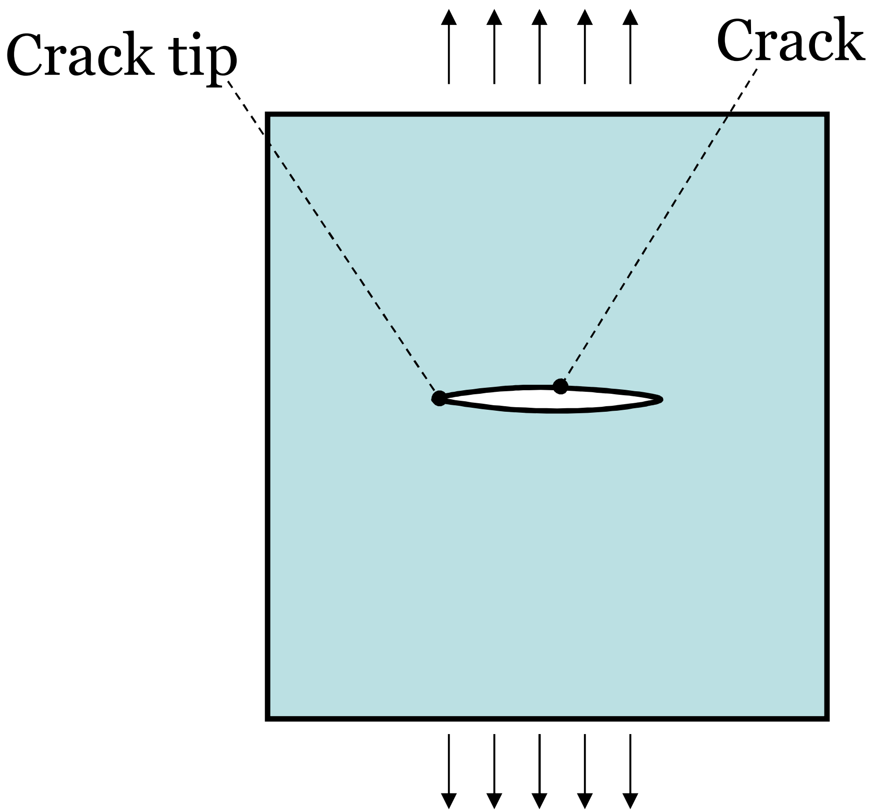
Introduction

- Design based on Strength of Materials / Mechanics of Solids
- Strength based design – Check for allowable stress
- Stiffness based design – Check for allowable deformation / deflection
- Presence of defects in the material – ideal
- Imperfections – Higher factor of safety

rd_mech@yahoo.co.in

Introduction

- Fracture – separation of a body in response to applied load



Fracture – Relieving stress and shed excess energy

Main focus – whether a known crack is likely to grow under a certain given loading condition

Introduction

- Fracture mechanics approach – Implicit assumption – Crack exists
- Severity of the existing crack when loads applied on the structure / component
- Application of Fracture Mechanics (FM) to Crack growth – Fatigue failure
- 1920s – Griffith developed right ideas for growth of a crack – Working parameter
- Modern FM – 1948 – George Irwin – devised a working parameter

Ramadas Chennamsetti
rd_mech@yahoo.co.in

Introduction

- Irwin's work – Mainly for brittle materials – Introduced Stress Intensity Factor (SIF) and Energy Release Rate (G) – Linear Elastic Fracture Mechanics (LEFM) – Plastic deformation negligible
- Irwin's theory – application of FM to design problems
- Focus was on crack tip – not on the crack

rd_mech@yahoo.co.in

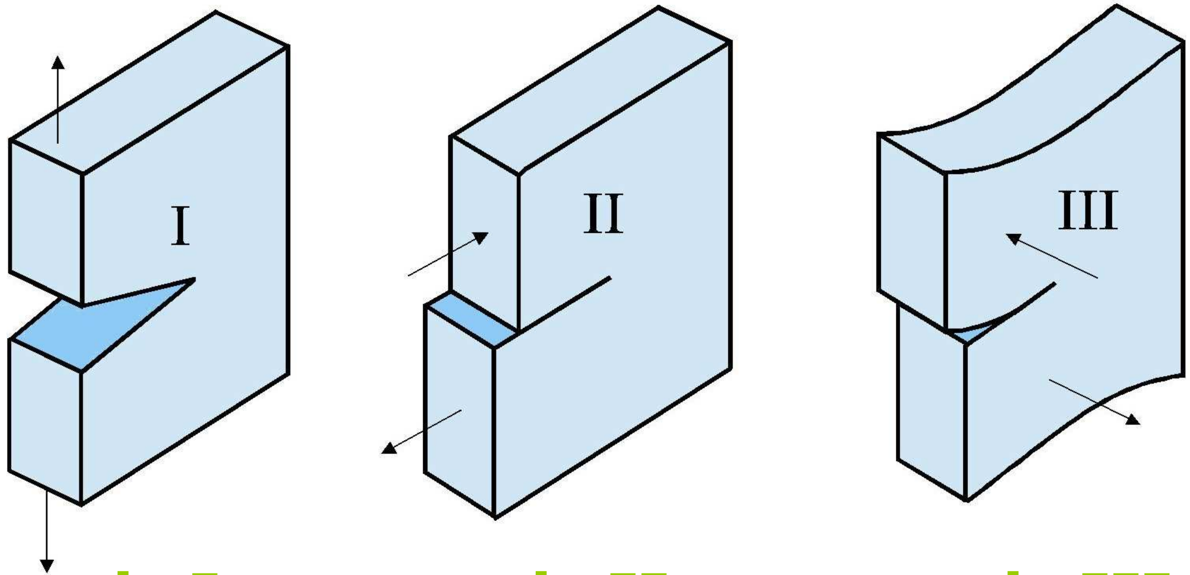
Introduction

- Crack Tip Opening Displacement (CTOD) – 1961 – Wells
- J – Integral – 1968 – Rice
- CTOD and J – integral – Ductile materials – large plastic zone at tip

rd_mech@yahoo.co.in

Modes of fracture

Modes of Fracture –

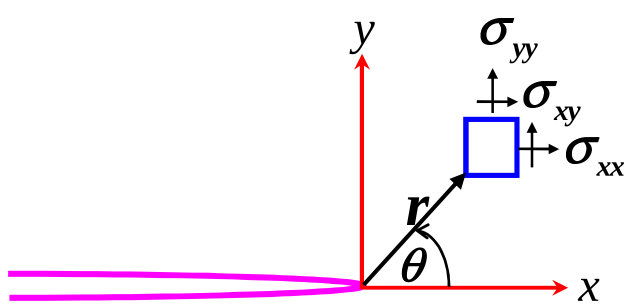


mode I	mode II	mode III
K_I	K_{II}	K_{III}
Opening	Sliding	Tearing

Stresses at crack tip

- From Linear Elastic Fracture Mechanics (LEFM) the stresses near the crack tip in Mode I –

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \begin{bmatrix} 1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2} \\ \sin\frac{\theta}{2}\sin\frac{3\theta}{2} \\ 1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2} \end{bmatrix}$$



- The displacements near the crack tip –

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{K_I\sqrt{r}}{2G\sqrt{2\pi}} \begin{bmatrix} \cos\frac{\theta}{2}(\kappa-1+2\sin^2\frac{\theta}{2}) \\ \sin\frac{\theta}{2}(\kappa+1-2\cos^2\frac{\theta}{2}) \end{bmatrix}$$

Plane strain $\kappa = 3-4\upsilon$

Plane stress $\kappa = \frac{3-\upsilon}{1+\upsilon}$

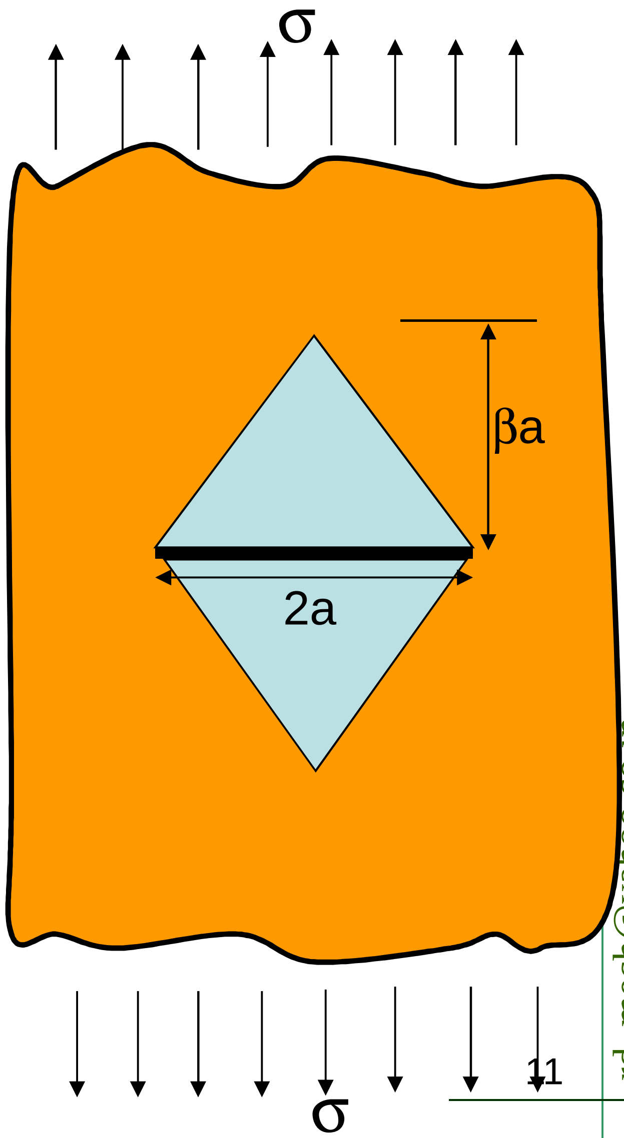
Griffith's theory

- Theory of unstable crack growth
- Theory establishes a relation – unstable crack growth
- Basic underlying principle – Energy balance
- Introducing a crack in a stressed / loaded component – release of strain energy
- Reduction in stiffness
- What happens to change in strain energy ??? or released strain energy

Griffith's theory

- An infinite body
- Linear elastic – subjected to stress σ
- Initially there was no crack
- Strain energy stored

$$U_o = \frac{\sigma^2}{2E} Vol$$



rd_mech@yahoo.co.in

Griffith's theory

- Crack size = 2a – the crack surfaces – traction free – no traction loads acting
- Material above and below – stress free to some extent
- Stress relived portion – assuming triangular distribution
- Height of triangle = β
- Griffith carried out extensive calculations and experiments => $\beta = \pi$

Ramadas Chennamsetti
rd_mech@yahoo.co.in

Griffith's theory

- Strain energy after introducing crack = Strain energy before introducing crack – Strain energy loss (relived)

$$=> U = U_o - \frac{1}{2}\frac{\sigma^2}{E}(\text{Volume of } \Delta\text{le})$$

$$=> \Delta U = U_o - U = \frac{1}{2}\frac{\sigma^2}{E}\left(2 \times \frac{1}{2} 2a\beta at\right)$$

$$=> \Delta U = \frac{\sigma^2 \beta a^2 t}{E} = E_R$$

Energy released because of presence of crack

Griffith's theory

- When a body is cracked – breaking of material bonds – Energy required for breaking bonds – Source of energy ???
- Loss / Deficiency in strain energy => caters for formation of crack
- Formation of crack => Generation of new traction free surfaces
- Energy used for breaking bonds stored as surface energy on newly formed surfaces

rd_mech@yahoo.co.in

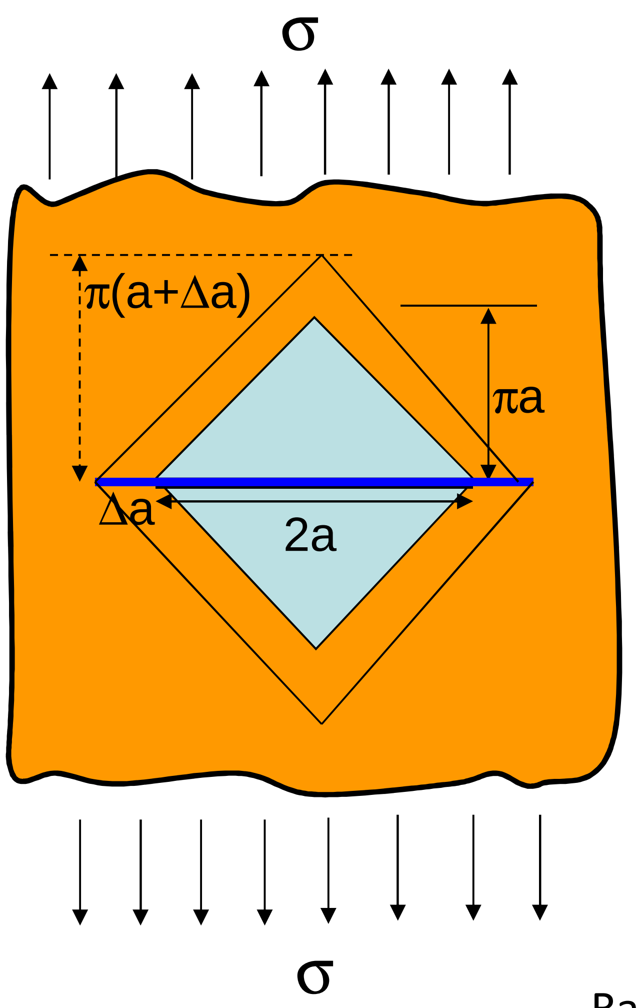
Griffith's theory

- Two key aspects in crack growth
  - How much energy is released – strain energy – when crack advances
  - Minimum energy required for the crack advance in forming two new surfaces
- Some external work is done
  - Increase in strain energy
  - Surface energy – crack growth

rd_mech@yahoo.co.in

Griffith's theory

- Assume a crack of '2a' size exists



Initial stress free area,

$A_1 = 2 * ½ (2a*\pi a) = 2\pi a^2$

Crack grows by Δa on both sides

Stress free area after crack growth,

$A_2 = 2*1/2 (2(a+\Delta a)*\pi(a+ \Delta a)$

$=> A_2 = 2\pi(a+\Delta a)^2$

Griffith's theory

- Change in stress free area

$$=> \Delta A = A_2 - A_1 = 2\pi\left[(a+\Delta a)^2 - a^2\right]$$

$$=> \Delta A = 2\pi(2a\Delta a) = 4\pi a\Delta a$$

$$\Delta V = 4\pi a\Delta a t$$

- Change in strain energy

$$=> \Delta U = \frac{1}{2}\frac{\sigma^2}{E}\Delta V$$

$$=> \Delta U = \frac{1}{2}\frac{\sigma^2}{E}(4\pi a\Delta a t) = \Delta E_R$$

$$=> \frac{\Delta E_R}{\Delta a} = \frac{2\pi\sigma^2 a t}{E} \quad ---(1)$$

Griffith's theory

- Surface energy required to create new area

$$=> \Delta W_s = \gamma_s (4\Delta at)$$

$$=> \frac{\Delta W_s}{\Delta a} = 4\gamma_s t \quad ---(2)$$

Unstable crack growth takes place, if

$$\frac{dE_R}{da} = \frac{dW_s}{da}$$

$$=> \frac{2\pi\sigma^2 at}{E} = 4\gamma_s t$$

$$=> 2E\gamma_s = \sigma^2 \pi a$$

Griffith's theory

- Expression

$$2E\gamma_s = \sigma^2 \pi a$$

'a' existing crack length - Stress required

to grow crack

$$=> \sigma_c^2 = \frac{2E\gamma_s}{\pi a}$$

Given loading - Stress - maximum allowable

crack size - Damage tolerant design

$$a_c = \frac{2E\gamma_s}{\pi\sigma^2}$$

rd_mech@yahoo.co.in

Griffith's theory

- γ_s = Specific surface energy – Surface energy per unit area of crack surface
- Area of crack = 2(2a*t) = 4at
- Surface energy stored => 4at γ_s
- Energy balance

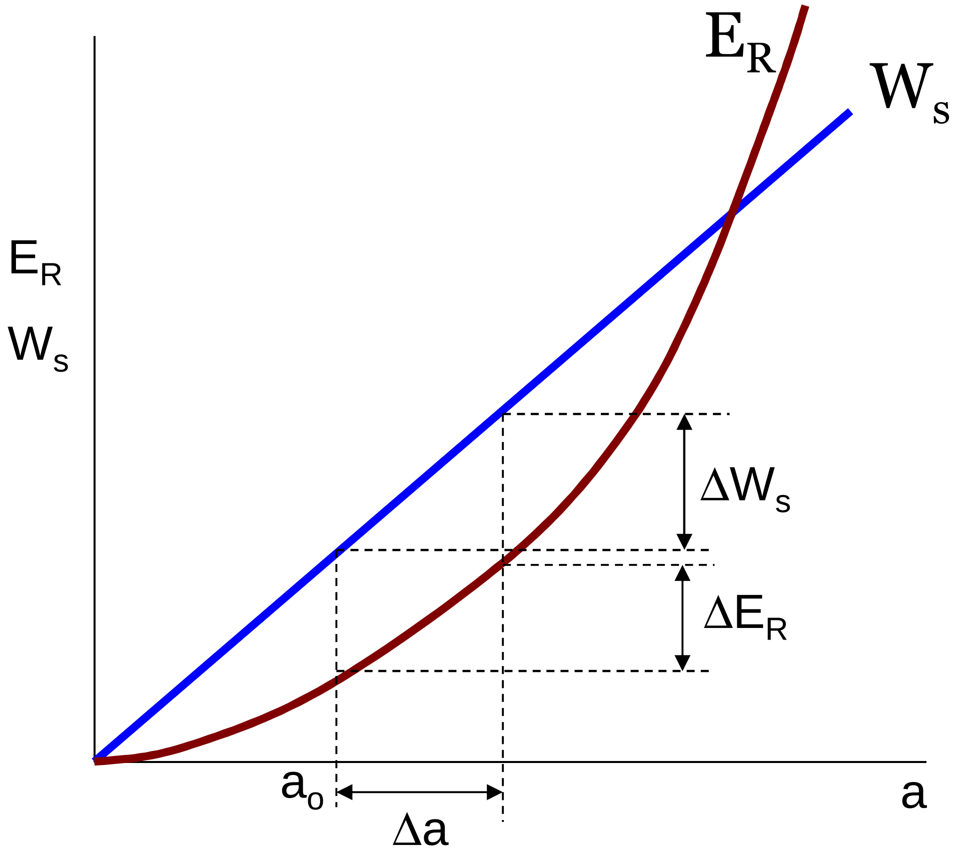
$$=> E_R = W_s$$

$$=> \frac{\pi\sigma^2 a^2 t}{E} = 4at\gamma_s$$

rd_mech@yahoo.co.in

Griffith's theory

- Plot E_R and W_s wrt crack length 'a'



$$E_R = \frac{\pi \sigma^2 a^2 t}{E}$$

$$W_s = 4at\gamma_s$$

Initial crack size = a_o

Crack grows by Δa

Strain energy released = ΔE_R

Surface energy required = ΔW_s

Griffith's theory

- Crack growth takes place, if

$$\Delta E_R \geq \Delta W_s$$

- Not satisfying above inequality – crack remains dormant
- Body acts as a strain energy reservoir
- Surface energy required can be obtained from external sources as well – increase in applied stress – No change in ΔW_s

Ramadas Chennamsetti

rd_mech@yahoo.co.in

Griffith’s theory

- Strain energy used to break the bonds – stored as surface energy
- Strain energy – source & surface energy – sink – Irreversible thermodynamic process
- Crack growth – energy conversion process
- When a crack propagates – strain energy gets reduced and surface energy increases for a constant displacement case

rd_mech@yahoo.co.in

Griffith's theory

- In the limiting case –

$$=> \Delta E_R = \Delta W_s$$

$$=> \lim_{\Delta a \to 0} \frac{\Delta E_R}{\Delta a} = \lim_{\Delta a \to 0} \frac{\Delta W_s}{\Delta a}$$

$$=> \frac{dE_R}{da} = \frac{dW_s}{da}$$

Check the slope of E_R and W_S – Satisfying above condition – onset of crack growth

rd_mech@yahoo.co.in

Griffith's theory

- If strain energy release rate is higher than required rate of surface energy – unstable crack growth

$$\frac{dE_R}{da} > \frac{dW_S}{da}$$

- Difference in energy rates => kinetic energy
- Higher difference => Faster crack growth

rd_mech@yahoo.co.in

Griffith's theory

- Strain energy release rate per unit increase in area during crack growth => Griffiths = G

$$G = \frac{dE_R}{dA}, \quad E_R = \frac{\pi\sigma^2 a^2 t}{E}, \quad A = 2at$$

$$=> dA = 2tda$$

$$=> G = \frac{dE_R}{dA} = \frac{1}{2t}\frac{dE_R}{da} = \frac{\pi\sigma^2 a}{E}$$

Units of G => N.m/m² or J/m²

Griffith's theory

- Crack area (A) and crack surface area (A_s)

Crack area => Simply the area of crack

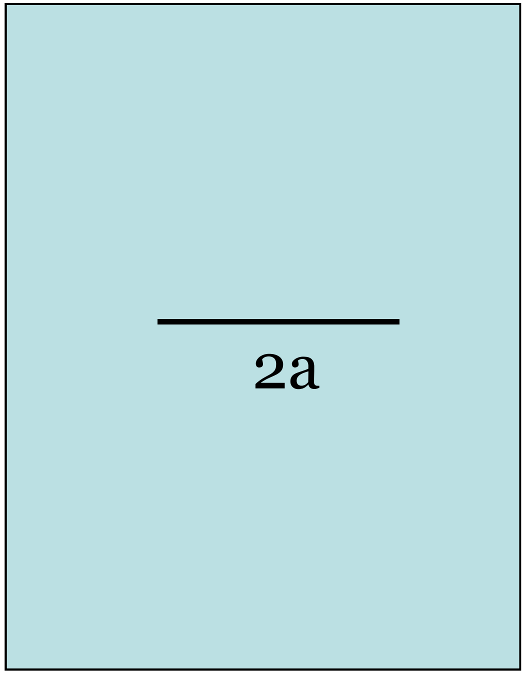
$A = 2a*t = 2at$

Crack surface area => Sum of surface areas of all the crack surfaces

Crack surfaces = Two (top and bottom)

Each => 2at

$A_s = 2(2at) = 4at = 2A$



Griffith's theory

- Surface energy required for crack to grow per unit area of extension – crack resistance – R

$$\Delta W_S = \gamma_S(\text{increase in total surface surface area})$$

$$\text{Crack area} => \Delta A = 2\Delta at$$

$$=> \Delta W_S = \gamma_S(2(2\Delta at)) = 4\Delta at\gamma_S = 2\gamma_S\Delta A$$

$$=> R = \frac{dW_S}{dA} = 2\gamma_S$$

Brittle fracture – no plastic deformation – R = surface energy – Elastic and plastic – R caters for surface energy and energy for plastic deformation at crack tip

rd_mech@yahoo.co.in

Griffith's theory

- Crack start propagating if, G > = R
- Strain energy release rate of a crack must be greater than the crack resistance for the crack to grow
- Crack propagation occurs – G is sufficient to provide all the energy that is required for the crack formation
- Energy release rate is more than crack resistance, crack acquires KE – the growth speed may be faster than speed of a supersonic flight

Griffith's theory

- Onset of crack growth –

$$\sigma^2 \pi a = 2E\gamma_s$$

- RHS – completely depends on material properties – Constant
- From the above -

$$a \propto \frac{1}{\sigma^2}$$

Maximum allowable crack size depends on loading or

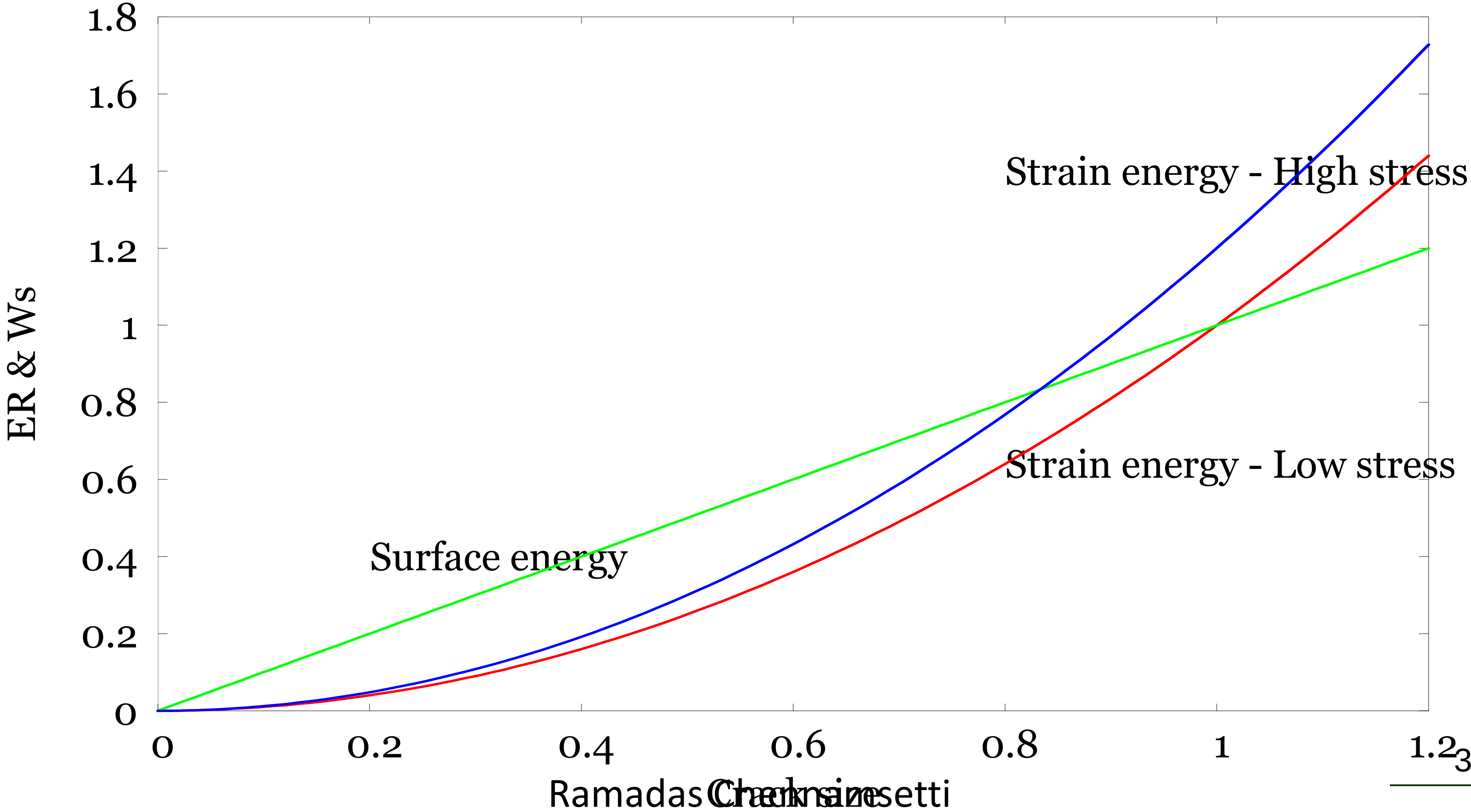
Loading decides the allowable crack size

Bigger cracks – lower loads

rd_mech@yahoo.co.in

Griffith's theory

- Crack growth at different load / stress conditions



Griffith's theory

- Thin and thick brittle plates
  - Thin plate – Plane stress condition
  - Thick plate – Plane strain condition
- Brittle – insignificant plastic zone at crack tip - LEFM
- Ductile materials – considerable plastic deformation at crack tip – EPFM
- Plane stress and planes strain – Poisson ratio
- Allowable crack size of stress different

rd_mech@yahoo.co.in

Griffith's theory

- Thin and thick plates – Griffith's theory

$$\sigma^2 \pi a = 2E\gamma_s \qquad \text{Plane stress}$$

$$\sigma^2 \pi a = 2\left(\frac{E}{1-\upsilon^2}\right)\gamma_s \qquad \text{Plane strain}$$

- RHS higher in plane strain
  - For a given load – higher crack size in thick plate than in thin
  - For a given crack size – higher load in thick plate than in thin plate

Griffith's theory

R&DE (Engineers), DRDO

- Taking square root

$$=> \sigma_c^2 \pi a_c = 2\left(\frac{E}{1-\upsilon^2}\right)\gamma_s$$

$$=> \sigma_c \sqrt{\pi a_c} = \sqrt{2\left(\frac{E}{1-\upsilon^2}\right)\gamma_s}$$

$$=> K_{IC} = \sigma_c \sqrt{\pi a_c} = \sqrt{2\left(\frac{E}{1-\upsilon^2}\right)\gamma_s} = \text{Constant}$$

K_I – Stress Intensity Factor (SIF)

K_c – Stress Concentration Factor (SCF)

Griffith's theory

- Critical strain energy release rate = G_{IC} = R
- Crack growth takes place, if

$$G = \frac{\pi \sigma_c^2 a}{E} \geq G_{IC}$$

In the limiting case,

$$G_{IC} = \frac{\pi \sigma_c^2 a}{E} = \frac{K_{IC}^2}{E} = 2\gamma_S = R$$

$$=> K_{IC} = \sqrt{G_{IC} E} = \sqrt{2\gamma_S E}$$

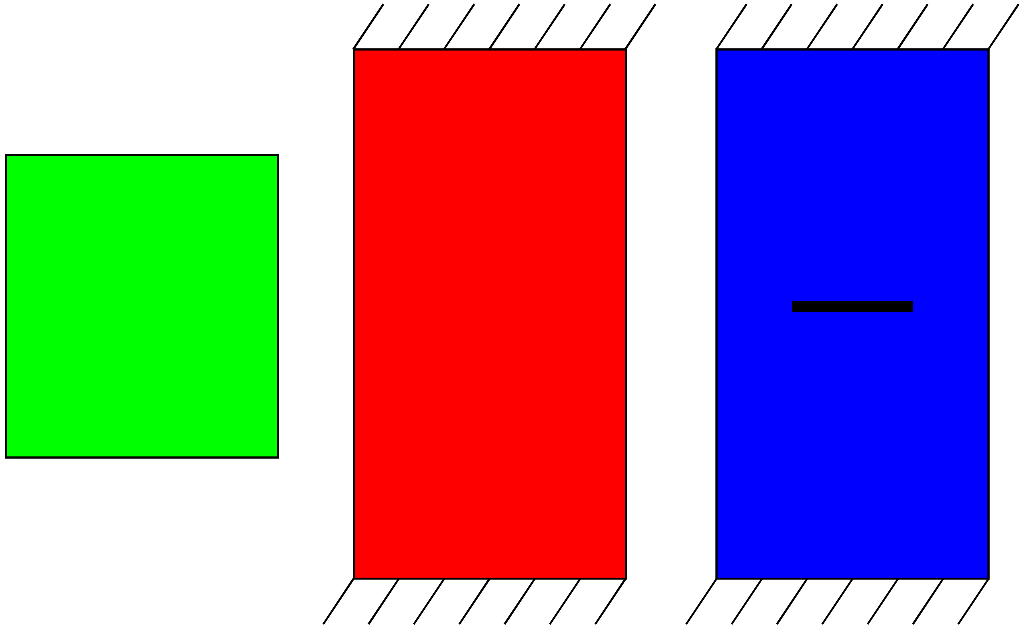
Griffith's theory

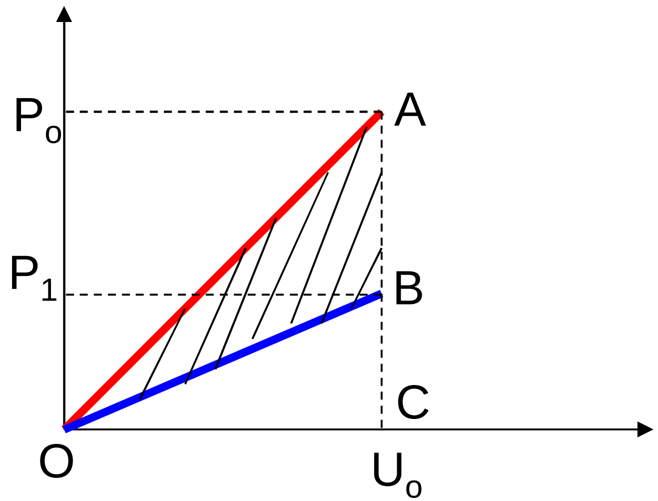
- Typical values of critical stress intensity factor – K_{IC}
  - Glass – 0.5 to 1 MPa $m^{0.5}$
  - Alloy steel – 150 MPa $m^{0.5}$
  - Aluminium alloy – 25 to 40 MPa $m^{0.5}$

rd_mech@yahoo.co.in

Griffith's theory

- Plate under constant displacement





- Plate stiffness reduced
- OAB – Strain energy released

rd_mech@yahoo.co.in

Ramadas Chennamsetti

Griffith's theory

- No crack => More slope – high stiffness – more force required to pull
- Presence of crack => lower slope – reduced stiffness – less force required to pull same length – u_o
- Difference in strain energy – Area OAB used to form new surface / break the bonds
- Major portion of strain energy release takes place at above and below crack

rd_mech@yahoo.co.in

Fracture toughness – total potential

- A body with a crack subjected to external loading – work done on the body
- Utilization of this energy
  - Increase in strain energy
  - Utilization of energy to create two new surfaces
- In this process
  - Point of application of load – may or may not move
  - Force moves => work is done by the force
  - Decrease in stiffness

rd_mech@yahoo.co.in

Conservation of energy

- Work performed per unit time = rate of change of internal energy + plastic energy + kinetic energy + surface energy (crack formation)

$$\dot{W} = \dot{U}_E + \dot{U}_P + \dot{K} + \dot{\Gamma}$$

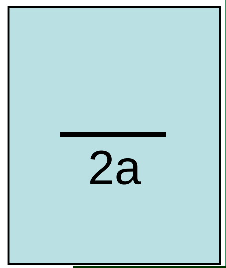
Crack grows slowly – inertia effects negligible => KE = 0

Equation was wrt time. Crack grows with time – change time to crack area

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial A}\frac{\partial A}{\partial t} = \dot{A}\frac{\partial}{\partial A}$$

Chain rule

A = 2at



rd_mech@yahoo.co.in

Conservation of energy

- Change variable to crack

$$\dot{A}\frac{dW}{dA} = \dot{A}\frac{dU_E}{dA} + \dot{A}\frac{dU_P}{dA} + \dot{A}\frac{d\Gamma}{dA}$$

$$\Rightarrow \frac{dW}{dA} = \frac{dU_E}{dA} + \frac{dU_P}{dA} + \frac{d\Gamma}{dA}$$

$$\Rightarrow \frac{dW}{dA} - \frac{dU_E}{dA} = \frac{dU_P}{dA} + \frac{d\Gamma}{dA}$$

$$\Rightarrow -\left(\frac{dU_E}{dA} - \frac{dW}{dA}\right) = \frac{dU_P}{dA} + \frac{d\Gamma}{dA}$$

Conservation of energy

R&DE (Engineers), DRDO

- Ideal brittle material => Negligible plastic deformation => $U_p = 0$

$$\Rightarrow -\left(\frac{dU_E}{dA} - \frac{dW}{dA}\right) = \frac{d\,\Gamma}{dA}$$

- Total potential

$$\pi = U_E + V = U_E - W$$

$$-\frac{d\pi}{dA} = \frac{d\Gamma}{dA}$$

rd_mech@yahoo.co.in

Surface energy

- Surface energy per unit area => γ_s
- Total surface area, $A_s = 2*(2at) = 4at = 2A$
- Change in surface energy to form new crack of length Δa

$$\Delta\Gamma = \gamma_s \Delta A_s = \gamma_s (4\Delta a t) = 2\gamma_s \Delta A$$

$$\Rightarrow \frac{\Delta\Gamma}{\Delta A} = 2\gamma_s \Rightarrow \frac{d\Gamma}{dA} = 2\gamma_s$$

It was shown that, $G_I = 2\gamma_s$

G – total potential

- Surface energy and total potential

$$-\frac{d\pi}{dA} = \frac{d\,\Gamma}{dA}$$

$$\Rightarrow -\frac{d\pi}{dA} = 2\gamma_s = R = G_I$$

Impending of crack growth

$$G_I \geq R$$

G – total potential

- Relation between G_I and π

$$=> G_I = -\frac{d\pi}{dA}$$

$$=> -d\pi = GdA$$

$$d\pi = dU_E - \Delta W_{ext}$$

$$=> \Delta W_{ext} - dU_E = -d\pi = G_I dA$$

$$=> \Delta W_{ext} = dU_E + G_I dA$$

External work done on the body => increase in elastic strain energy and formation of new surfaces

G – total potential

- No movement of external forces, $W_{ext} = 0$

$$G_I = -\frac{dU_E}{dA}$$

When there is no external work done, energy required for crack growth is obtained from the strain energy stored in the body

Conservation of energy => Decrease in strain energy = increase in surface energy

rd_mech@yahoo.co.in

Compliance approach

- Compliance => Inverse of stiffness

$$C = \frac{1}{k}, \qquad P = ku => u = CP$$

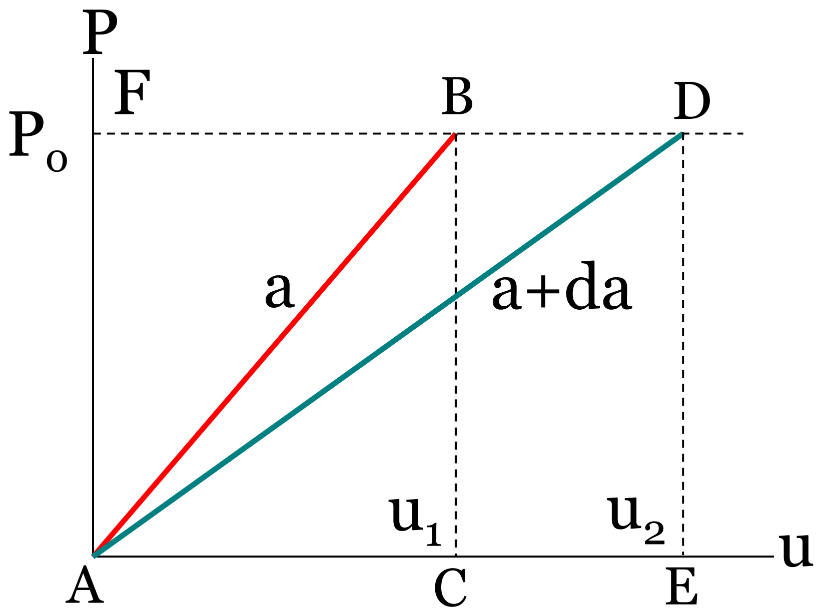
- Fracture toughness – compliance approach
  - Constant load
  - Constant displacement
- Increase in compliance with crack length – loss of stiffness
- Calculation of Fracture toughness, G_I

Ramadas Chennamsetti
rd_mech@yahoo.co.in

Constant load

R&DE (Engineers), DRDO

- Consider a body with some initial crack of length 'a'- Load applied => P_o



Write total potentials when crack sizes are 'a' and 'a+da'

$$\pi_1 = U_1 + V_1$$

$$=> \frac{1}{2} P_o u_1 - P_o u_1 = -\frac{1}{2} P_o u_1$$

$$\pi_2 = U_2 + V_2$$

$$=> \frac{1}{2} P_o u_2 - P_o u_2 = -\frac{1}{2} P_o u_2$$

π_1 = - Area ABF, π_2 = - Area ADF

$$\Delta\pi = \pi_2 - \pi_1$$

rd_mech@yahoo.co.in

Constant load

R&DE (Engineers), DRDO

- Potentials,

$$\pi_1 = -\frac{1}{2} P_o u_1, \quad \pi_2 = -\frac{1}{2} P_o u_2$$

$$=> \Delta\pi = \pi_2 - \pi_1 = -\frac{1}{2} P_o u_2 + \frac{1}{2} P_o u_1$$

$$=> \Delta\pi = \frac{1}{2} P_o (u_1 - u_2) = -\frac{1}{2} P_o \Delta u$$

Strain energy release rate, G_I

$$G_I = -\frac{d\pi}{dA} = \frac{1}{2} \frac{d}{dA} (P_o u) = \frac{P_o}{2} \frac{du}{dA}$$

rd_mech@yahoo.co.in

Constant load

R&DE (Engineers), DRDO

- Use compliance relation

$$u = CP_o$$

$$=> \frac{du}{dA} = P_o \frac{dC}{dA}$$

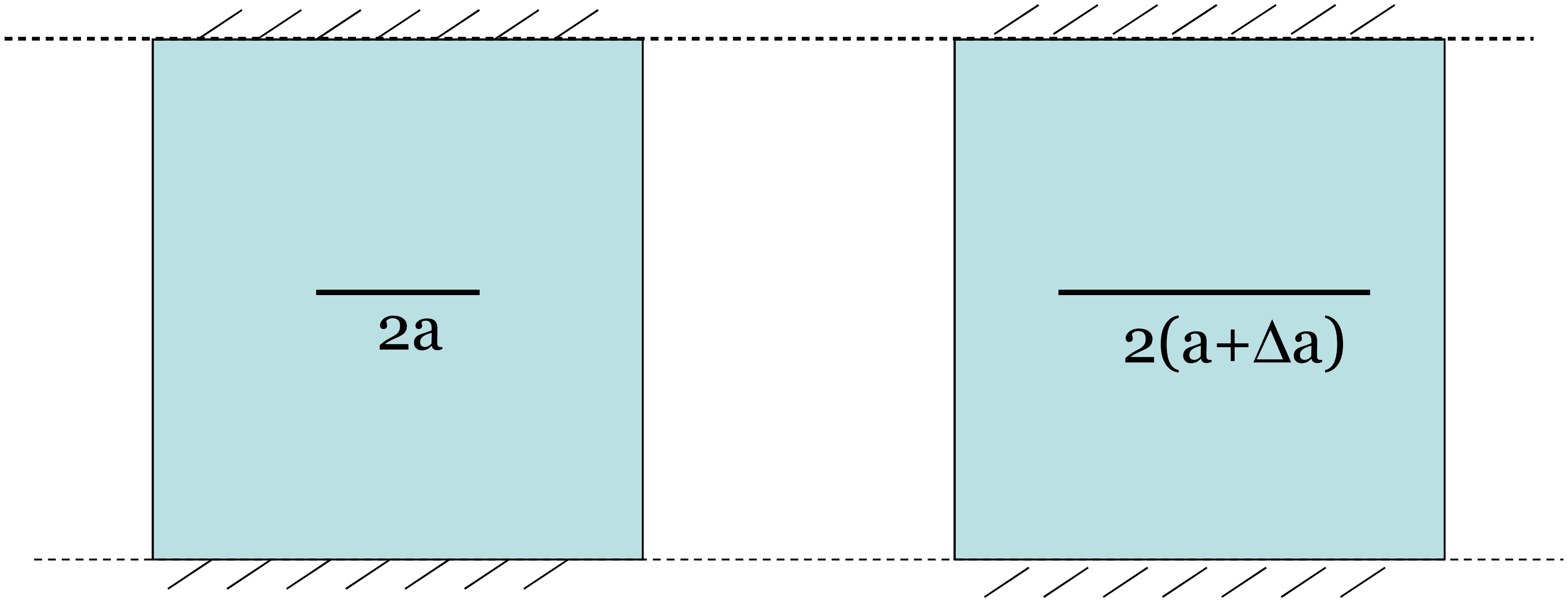
$$G_I = \frac{1}{2} P_o \frac{du}{dA} = \frac{P_o^2}{2} \frac{dC}{dA}$$

$$G_I = \frac{P_o^2}{2} \frac{dC}{dA}$$

Experimental measurements required to estimate change in compliance with crack area

Constant displacement

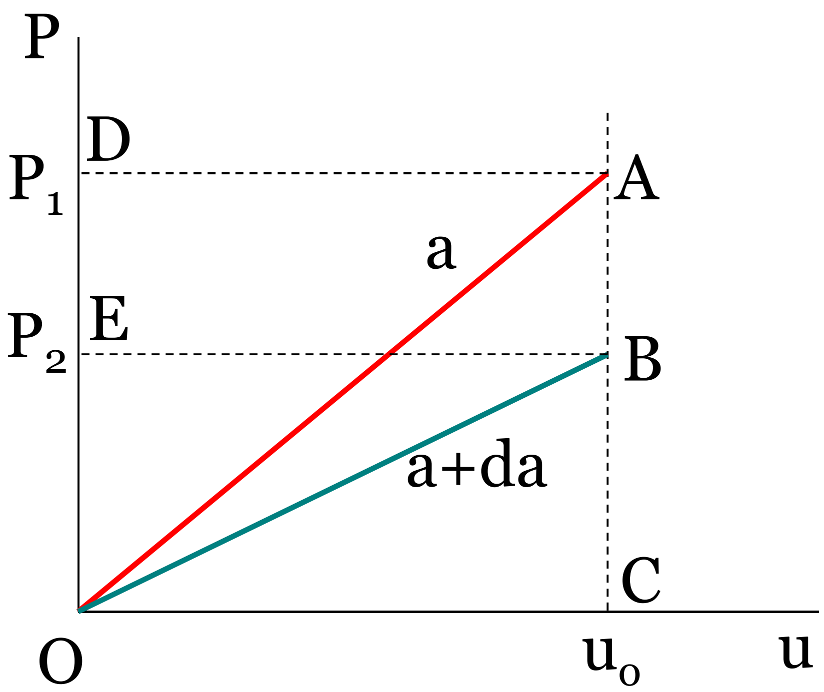
- Constant displacement – Fixed grip condition



Since the grip is fixed no external work is done by the forces, $W_{ext} = 0$

Constant displacement

- Load-displacement curve



Change in potential,

$$\Delta\pi = \frac{1}{2}\left(P_2 - P_1\right)u_o \Rightarrow -\text{ve}$$

$$\Rightarrow \Delta\pi = \frac{1}{2}\Delta P u_o$$

$$\Rightarrow G_I = -\frac{d\pi}{dA} = -\frac{1}{2}u_o\frac{dP}{dA}$$

$$u_o = CP \Rightarrow P\frac{dC}{dA} = -C\frac{dP}{dA}$$

rd_mech@yahoo.co.in

Constant displacement

- This gives,

$$\frac{dP}{dA} = -\frac{P}{C}\frac{dC}{dA}$$

$$G_I = -\frac{1}{2}u_o\frac{dP}{dA} = -\frac{1}{2}u_o\left(-\frac{P}{C}\frac{dC}{dA}\right)$$

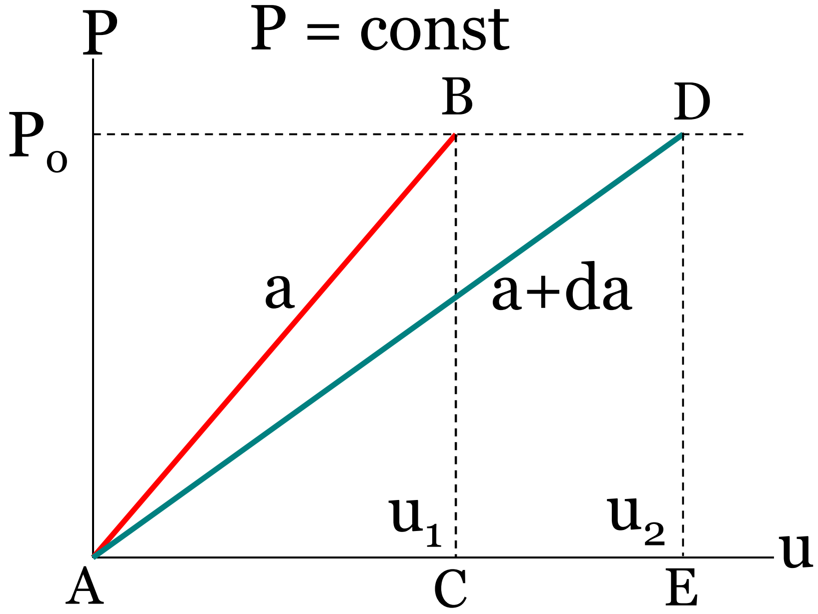
$$G_I = \frac{P^2}{2}\frac{dC}{dA}$$

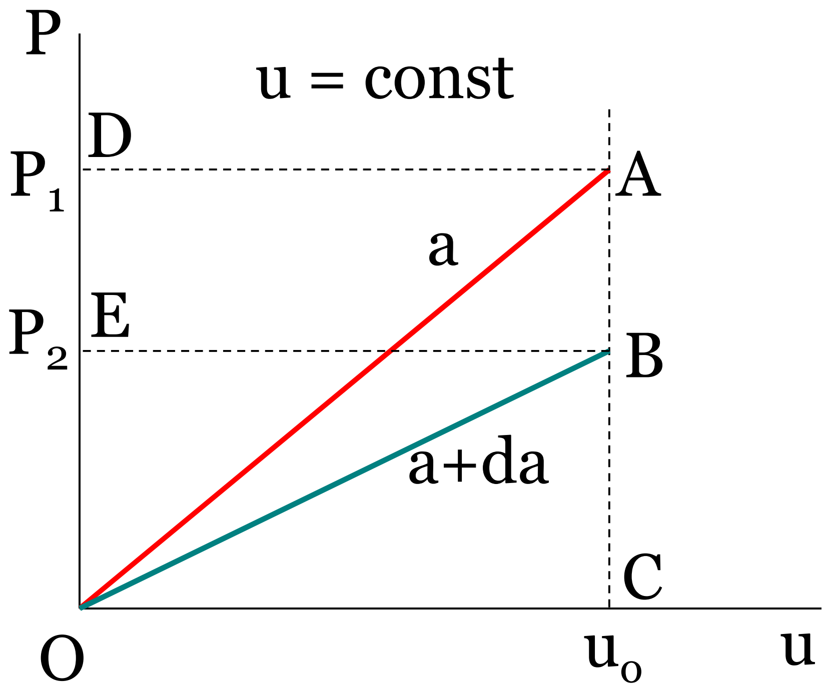
G_I same in both constant load and displacement conditions

rd_mech@yahoo.co.in

Constant load & displacement

- Comparison





$U_2 > U_1$ Increase in strain energy

Caters for crack growth

No external work done

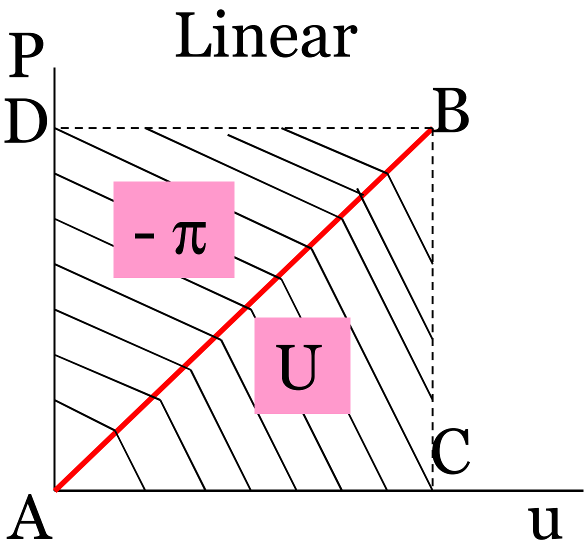
$U_2 < U_1$ Decrease in strain energy – used for crack growth

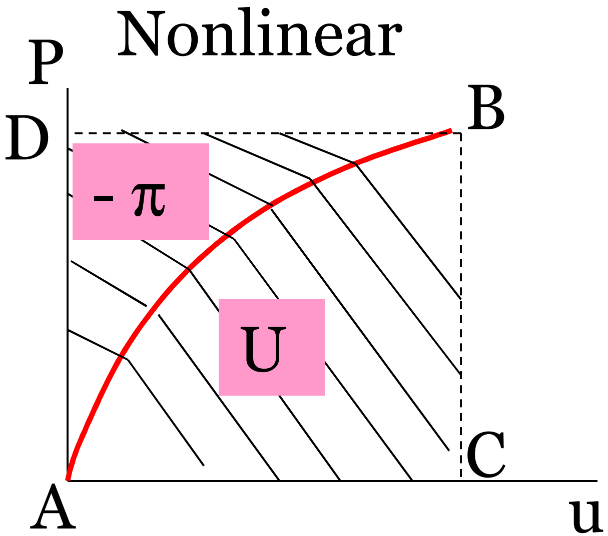
rd_mech@yahoo.co.in

Energy release rate

- Energy release rate referred as rate of strain energy flux flowing towards a crack tip as the crack extends
- Linear and Nonlinear

'J' integral – contour integral





$$J = -\frac{d\pi}{dA}$$

Area 'ABD'

For linear elastic, G = J

Ramadas Chennamsetti

rd_mech@yahoo.co.in

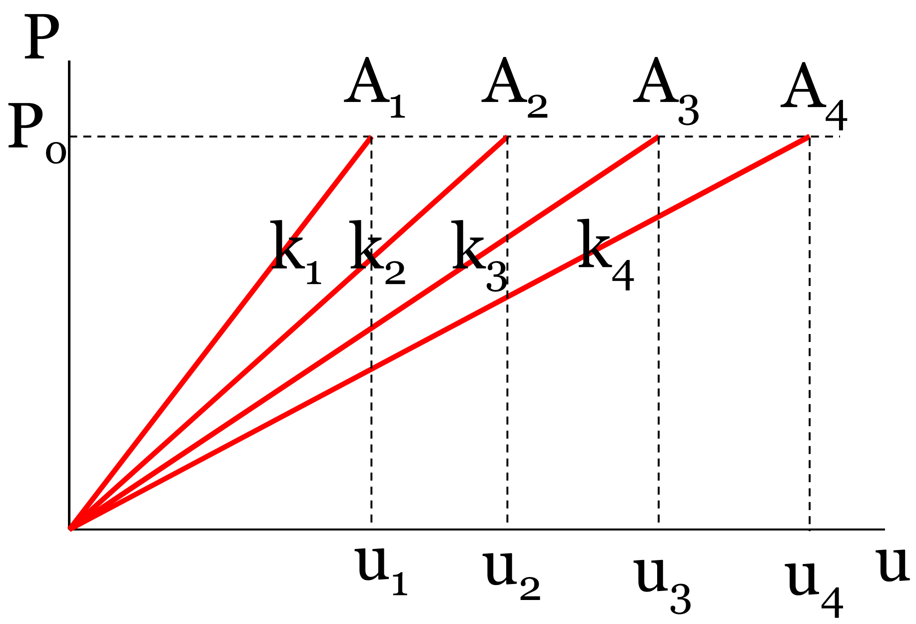
Measurement of ‘G_{IC}’

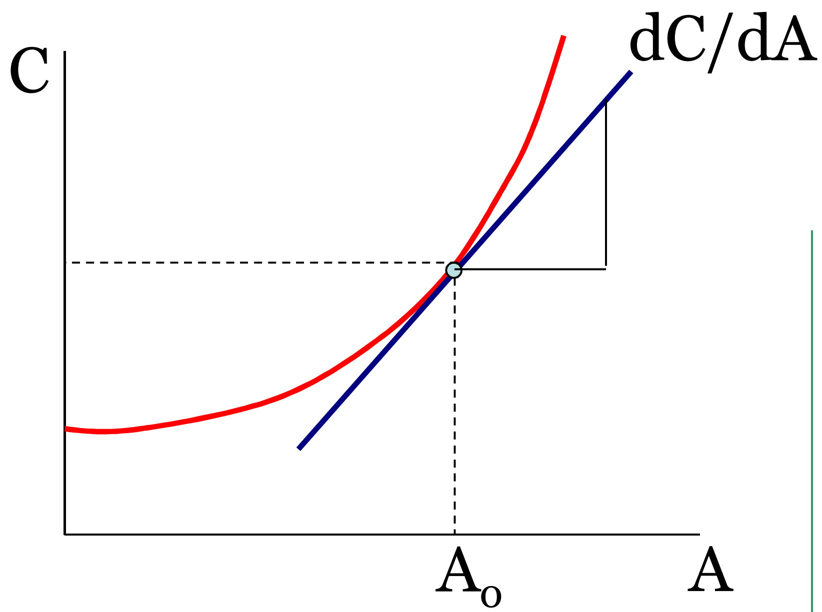
R&DE (Engineers), DRDO

- Expression for G_{IC} – Critical SERR

$$G_{IC} = \frac{P^2}{2}\frac{dC}{dA}$$

Parameter to be measured experimentally => *dC/dA*





rd_mech@yahoo.co.in

Measurement of 'G_{IC}'

- Size of crack => a_o, corresponding area = A_o
- Load required to initiate crack growth, P_c

$$G_{IC} = \frac{P_c^2}{2} \frac{dC}{dA}\bigg|_{A_o}$$

If G_I is expressed in terms of stiffness,

$$G_{IC} = -\frac{P_c^2}{2k_o^2} \frac{dk}{dA}\bigg|_{A_o}$$

dk/dA is -ve

rd_mech@yahoo.co.in

Griffith's theory

- Crack in stiff and flexible components
- Same force applied on stiff and flexible components having same size of crack
- Energy stored

$$U = \frac{1}{2} P^2 C = \frac{1}{2} \frac{P^2}{K}$$

$$=> U \propto \frac{1}{K} (= C)$$

Flexible component stores more energy than stiffer component

More store of energy – availability of energy for crack propagation

Stiff bodies do not store more energy – less release

rd_mech@yahoo.co.in

Surface energy

- Surface energy per unit area – R – material property
- In Fracture Mechanics – R => total energy consumed in propagation of crack
- In ductile materials
  - Plastic deformation takes place – energy required to deform
  - Energy required for a crack to grow is much larger than the surface energy
- Significant plastic deformation

rd_mech@yahoo.co.in

Surface energy

- Crack growth accompanied with plastic deformation

$$G = 2(\gamma_s + \gamma_p) = 2w_f = R$$

γ_p => Plastic work per unit area of surface created,

γ_s => Surface energy per unit area of surface created

- Most of metals – surface energy is much smaller than (1/1000th) of the total energy consumed in crack propagation

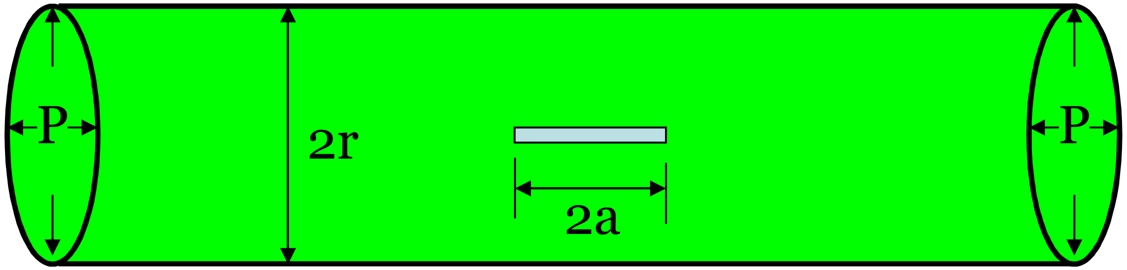
Surface energy

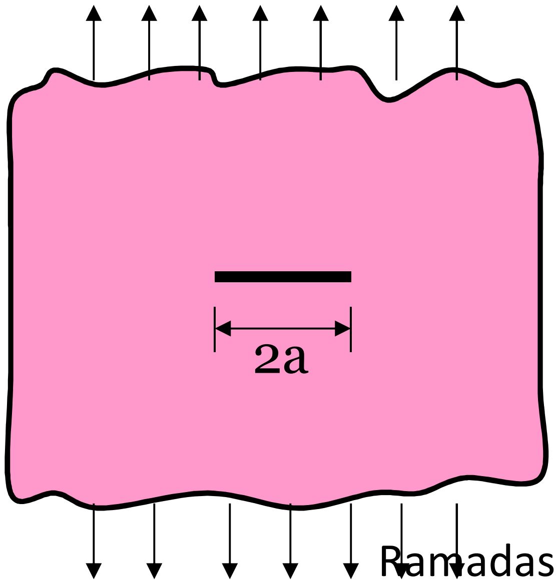
- Crack resistance (R) – depends on material and geometry – thickness
- Plastic zone size at the crack tip depends on thickness
- Bigger plastic zone – more dissipation of energy – Higher crack resistance - Plane stress
- Smaller plastic zone – less dissipation of energy – Lower crack resistance – Plane strain

Ramadas Chennamsetti
rd_mech@yahoo.co.in

Crack in a pipe

- Eg. Longitudinal crack of 10 mm is allowed in a pipe. What is the allowable pressure?





K_{IC} = 93 MPa $m^{0.5}$

Allowable stress = 878 MPa

Crack in a pipe

- Hoop stress causes crack opening – mode – I
- Considering this as an infinite plate with a crack of length 2a

$$\sigma = \frac{p_{\max} r}{t} \quad \Rightarrow \text{Hoop stress}$$

$$K_{IC} = \sigma\sqrt{\pi a_c} \quad \Rightarrow \text{Critical SIF}$$

$$\Rightarrow K_{IC} = \frac{p_{\max} r}{t}\sqrt{\pi a_c}$$

Crack in a pipe

- Maximum pressure

$$K_{IC} = \frac{p_{\max} r}{t}\sqrt{\pi a_c}$$

$$\Rightarrow p_{\max} = \frac{t}{r}\frac{K_{IC}}{\sqrt{\pi a_c}} = \frac{t}{r}\frac{93\times10^{6}}{\sqrt{\pi\times5\times10^{-3}}}$$

$$\Rightarrow p_{\max} = 742.03\times10^{6}\,\frac{t}{r}$$

rd_mech@yahoo.co.in

Crack in a pipe

- Same maximum pressure causes Hoop stress – Check for yield failure

$$\sigma_{allowble} = Y = \frac{p_{\max} r}{t} = 878 \times 10^{6}$$

$$=> p_{\max} = 878 \times 10^{6} \frac{t}{r}$$

If design is based on fracture criteria, it does not fail in yielding

rd_mech@yahoo.co.in


डीआरडीओ स्वर्ण जयंती वर्ष
DRDO Golden Jubilee Year
रक्षा अनुसंधान तथा विकास संगठन
बलस्य मूलं विज्ञानम्
रक्षा मंत्रालय
1958 - 2008
राष्ट्र की सेवा में समर्पित 50 वर्ष
50 Years Of Dedicated Service To The Nation