ENERGY

Energy. The world has many parts: stars, planets, animals, molecules, electrons, protons, photons, etc. The parts move relative to one another, and interact with one another. The motion and interaction carry *energy*. Energy is a fundamental concept. We do not know how to define energy in terms of more fundamental concepts. We do know, however, how to keep track of energy by measurement and calculation.

We have been learning bookkeeping tricks in many sciences. Mechanics tells us how to calculate the kinetic energy of a flying bullet, the gravitational energy of the bullet, and the elastic energy in a spring. Electrodynamics tells us how to measure the electrostatic energy in a capacitor, the electrical energy flowing in transmission lines, and the electromagnetic energy in a beam of light. Chemistry tells us how to measure energy in all kinds of food, as well as in all kinds of fuels—coal, crude oil, and natural gas. Quantum mechanics tells us how to calculate energy stored in the molecules.

We will rely on your knowledge of energy in the same way as we rely on your knowledge of space, matter, and charge. We will not make the strange demand that you forget the ceaseless and rapid movements of molecules. With this picture in mind, you can imagine how energy goes from place to place, and converts from one form to another.

For example, when your hand warms a glass of wine, it is because the vibration of the molecules of your hand couples the vibration of the molecules in the wine, through the vibration of the molecules of glass. The coupling causes energy of your hand to go into the wine. The molecules of your hand do not go into the wine.

The principle of the conservation of energy. Of our world we know an empirical fact: energy is conserved. Energy can be converted from one form to another, but cannot be created or destroyed. This empirical fact is known as *the principle of the conservation of energy*.

Because energy is conserved, we speak as if energy were a fluid. Energy *flows*—or *transfers*—from one place to another. Energy *stores* somewhere. A moving object *carries* energy. Talking about energy like talking about a fluid is harmless, so long as you are not tricked to believe that energy is a fluid. A fluid is a substance. Energy is a property that the substance has.

Thermodynamics will help us do many things, but not everything. Notably, thermodynamics will not tell us what energy is. Nor will thermodynamics tell us why energy is conserved. Indeed, we do not know why energy is conserved. It just is. The conservation of energy is an empirical fact—a law of nature, like the conservation of space, the conservation of mass, and the conservation of charge.

Forms of Energy

We catalog energy into various forms. These forms of energy appear in many sciences. The arts of calculating and measuring energy have become sophisticated. Here we list a few examples.

Kinetic energy. A flying object carries kinetic energy $mv^2/2$, where m is the mass of the object, and v is the magnitude of the velocity.

Gravitational energy. An object near the surface of the Earth carries gravitational energy mgz, where m is the mass of the object, z is the height of the object above the surface of the Earth, and $g \approx 9.8\mathrm{m/s^2}$ is the gravitational acceleration. The gravitational energy is also known as the potential energy due to gravity.

Elastic energy. Pull a spring, and the spring elongates. Upon releasing the pull, the spring returns to its initial length. Pulling changes the energy of the spring. If we maintain the elongation, the spring is a device that stores energy.

To visualize how pulling of the spring changes the energy, it is helpful to think about the atomic picture of the spring. Consider a spring made of a metal. The metal consists many grains, and each grain is a crystalline lattice of atoms. The atoms are held together by a sea of electrons, known as metallic bonds. When the force is small enough, the spring is perfectly elastic. The pulling does not change the neighbors of individual atoms, but distort bonds between the atoms slightly.

Electrostatic energy. Glass is a good insulator of electricity. An electrical insulator is also known as a dielectric. Start with a thin sheet of glass, and coat its two surfaces with two layers of a metal. The two metallic layers are called electrodes, and the metal/dielectric/metal sandwich is called a capacitor.

Apply a voltage V between the two electrodes using an external circuit. One electrode gains a positive charge $+Q$, and the other electrode gains a negative charge $-Q$. The electric charges on the two electrodes are of the opposite polarity and of equal amount.

The glass is an insulator of electricity. The applied voltage does not cause electrons to move cross the glass, but does distort electron cloud. This distortion—known as polarization—changes energy in glass. When the capacitor is disconnected from the external circuit, the charges remain on the electrodes. Thus, capacitor is a device that stores electrical energy.

Electrical energy. Metals are good conductors of electrons. Water is a good conductor of ions. Both electrons and ions are carriers of electric charge. The amount of electric charge crossing a plane per unit time is called electric current. An electric current carries power (energy per unit time) VI, where V is the voltage, and I is the current. A conductor transmits electrical energy.

Electromagnetic energy. A beam of light is composed of particles known as photons. A photon carries energy hf, where f is the frequency of the photon, and $h \approx 6.6 \times 10^{-34}\,\mathrm{Js}$ is the Planck constant.

Chemical energy. The energy stored in chemical bonds can be calculated using quantum mechanics, and can also be determined by experimental measurements. The values of chemical energy of many molecules are available online. We will describe them later in the course. Energy of molecules can also be calculated using quantum mechanics.

Internal energy. A bullet is made of a metal. The metal is an aggregate of a large number of atoms. Even when the bullet appears to be stationary to us, atoms of the bullet move relative to one other rapidly and ceaselessly. The energy associated with the microscopic movements is known as the internal energy, U. When we fire the bullet into the air, the bullet also acquires kinetic energy and potential energy. The total energy E of the flying bullet consists of the kinetic energy, the potential energy, and the internal energy:

$$E = \frac{1}{2}mv^2 + mgz + U .$$

Conversion of Energy

Conversion between kinetic and potential energy. In mechanics you have learned that a flying object conserves the sum of its kinetic energy and potential energy. Sometimes we call the sum

$$\frac{1}{2}mv^2 + mgz$$

the mechanical energy of a flying object. The sum of the kinetic energy and the potential energy is independent of time. That is, the mechanical energy is conserved. When the object goes high, its speed decreases. When the object falls, its speed increases. This conservation of mechanical energy can be deduced from Newton's second law; see Supplement.

Converting gravitational energy into internal energy. As another example of the conversion of energy from one form to another, imagine that we put a steal ball on top of a jar of honey. The gravitational force will pull the ball down. The descending ball loses the gravitational energy, and the ball and the honey gains in internal energy. This conversion is facilitated by the viscous flow of the honey.

More examples of conversion. Here are some more examples of converting energy from one form to another. If any one of them looks unfamiliar, look it up online. You can certainly add more examples.

	kinetic	gravi-tational	electro-magnetic	electrical	chemical	nuclear	thermal
kinetic	turbine	falling object	solar sail	accelerator	gun explosion	atomic bomb	steam engine
gravi-tational	rising object	seesaw		electric pump		atomic bomb	
electro-magnetic						atomic bomb	
electrical	generator	hydro-electric			fuel cell battery		
chemical			photo-synthesis	charging battery	chemical reaction	atomic bomb	
nuclear							
thermal	friction	falling object	radiator	electric stove	chemical reaction	nuclear reaction	heat exchanger

Transfer of Energy

Isolated system and open system. We can regard any part of the world as a *system*. We classify a system by how it interacts with the rest of the world. A system that does not interact with the rest of the world is called an *isolated system*. A system that exchanges both energy and matter with the rest of the world is called an *open system*.

Closed system. A system that does not exchange matter with the rest of the world is called a *closed system*. A closed system interacts with the rest of the world by exchanging energy. For the time being, we focus on a closed system. As an example, we can make a cylinder-piston setup a closed system. The setup encloses some matter, e.g., water molecules. We can make the matter inside the setup as a closed system by carefully sealing the piston, so that no water molecules will leak out, and no air molecules will sneak in. The cylinder-piston setup is a closed system: it does not exchange matter, but does exchange energy, with the rest of the world.

The system exchanges energy with the rest of the world in two ways. First, when weights are added on top of the piston, the piston moves down and reduces the volume inside the cylinder-piston setup. The weights transfer energy to the system by *work*. Second, when the cylinder-piston setup is brought over a flame, energy from the flame is transferred into the cylinder through the wall of the cylinder. The flame transfers energy to the system by *heat*.

Work. A weight stores the gravitational energy by its mass and its height. The weight is placed on top of a piston, which seals a cylinder of gas. The lowering of the weight does work to the gas in the cylinder. Transferring energy by organized movements is called work. We can measure work in terms of raising a weight to some height.

Methods of transferring energy by work. You have learned many ways of doing work in mechanics and electrodynamics. The weight exerts a force F on the piston. When the piston moves by distance ds, the weight does work Fds

Let A be the area of the piston. The pressure of the gas is $p = F / A$. When the piston moves by distance ds, the volume of the gas changes by $dV = Ads$, and the pressure of the gas does work by $Fdz = pdV$.

As a second example, a capacitor stores electrical energy using a layer of dielectric to separate two conductors that carry charges of the opposite signs. When the two electrodes connect to a light bulb through a conducting wire, the bulb turns on. As charge flows in the conducting wire, the voltage of the capacitor does work to the charge.

The charges on the electrodes come from the external circuit. Assume that the glass is a perfect insulator, and that no charges leak through it. Thus, the charges on the electrodes can be determined by measuring electric current flowing in the external circuit. At voltage V, when the amount of charge changes from Q to $Q+dQ$, the energy stored in the capacitor changes by VdQ.

Let V be the voltage between the two electrodes of the capacitor. When the charge on the capacitor changes by dQ, the work is VdQ.

As a third example, consider Joule heating. A resister goes through a container of water. When a voltage V is applied to the two ends of te resistor, the work per unit time is IV.

Heat. By contrast, the flame transfers energy into the matter enclosed in the cylinder-piston setup by disorganized movements of molecules. Transferring energy by disorganized movements is called heat.

For the cylinder-piston setup, energy in the matter may leak through the wall of the cylinder, even in the absence of the flame. The leak of energy is through disorganized movements of molecules, and is therefore also heat.

That heat is a way to transfer energy has become evident since about 1840s, through macroscopic experiments and microscopic models. Long before the nature of heat was understood, the quantity of heat had been measured by its macroscopic effects. For example, the heat given off by a flame can be measured by the amount of ice it melts.

Methods of transferring energy by heat. Energy is transferred by heat by several familiar methods:

Conduction. Energy can go through a material. At a macroscopic scale, the material remains stationary. At a microscopic scale, energy is carried by flow of electrons and vibration of atoms.

*Convection***.** Energy can go with the flow of a fluid. This way of energy transfer involves the transfer of matter between systems.

Radiation. Energy can be carried by electromagnetic waves. Because electromagnetic waves can propagate in vacuum, two systems in thermal contact need not be in proximity.

Adiabatic system. When a system and the rest of the word do not exchange matter and do not exchange energy by heat, we call the system an *adiabatic system*. An adiabatic system may still interact with the rest of the world in one mode: exchanging energy by work.

When the rest of the world does work to an adiabatic system, the system gains energy and becomes hotter, and we call the process *adiabatic heating*. When an adiabatic system does work to the rest of the world, the system loses energy and becomes colder, and we call the process *adiabatic cooling*.

For example, we make both the cylinder and the piston using thick materials that block the transfer of matter and heat. But we can move the piston and transfer energy to the system by work. When we add weights, the piston moves done, and the gas gains energy and becomes hotter. When we remove weights, the piston moves up, and the gas loses energy and comes colder.

Joule heating. For example, we can pass an electric current through a resistor immersed in a pot of water. The Joule heating of the resistor adds energy to water. The amount energy added to water is the electrical power, IV, over some time.

Thermal system. When a system and the rest of the world exchange energy by heat, we call the system a *thermal system*. Often we use the phrase "thermal system" in a more restrict sense. A thermal system and the rest of the world do not exchange matter and do not exchange energy by work, but exchange energy *by heat alone*. A thermal system can be obtained by preventing a *closed system* from exchanging energy by work with the rest of the world.

For example, when we block the movement of the piston, but still allow the flame to transfer energy by heat, the cylinder-piston setup becomes a thermal system. A thermal system can also be obtained by opening an *isolated system* in a particular way, allowing the system and the rest of the world to exchange energy by heat only.

A classification of systems. We classify systems according to how they interact with the rest of the world. Different authors may classify systems differently, and may name them differently. It is good to spell out the modes of interaction for each type of systems.

	exchange matter	exchange energy by work	exchange energy by heat
isolated system	no	no	no
closed system	no	yes	yes
thermal system	no	no	yes
adiabatic system	no	yes	no
open system	yes	yes	yes

The first law of thermodynamics. A closed system interacts with the rest of the world in two ways: exchanging energy by work, and exchanging energy by heat. In thermodynamics, the principle of the conservation of energy is stated in a form called the first law of thermodynamics:

> *The increase of the energy of a closed system equals the sum of the energy transferred into the closed system by work and the energy transferred into the closed system by heat.*

Thus, when a closed system receives energy by work W and by heat Q from the rest of world, the internal energy of the closed system increases by

$$\Delta U = W + Q .$$

In writing this expression, we adopt the following sign convention: $\Delta U > 0$ when the internal energy of the system increases, $W > 0$ when the reset of the world transfer energy to the system by work, and $Q > 0$ when the reset of the world transfer energy to the system by heat. Some authors may adopt other sign conventions.

Work vs. heat. A closed system and the reset of the world exchange energy in two ways. Work transfers energy by organized movements. Heat transfers energy by disorganized movements. What do we mean by organized movements and disorganized movements?

If we feel better about the concepts of energy and work than we feel about the concept of heat, we may as well define heat as the change in the energy of a closed system minus the work.

This procedure removes the burden to differentiate precisely what we mean by organized and disorganized movements. We do our best to identify organized movements, and define the change of energy associated with the organized movements as work. Heat is then the change of energy associated with movements that we do not bother to count as organized movements.

Calorimetry

Experimental determination of heat. The art of measuring heat is called *calorimetry*. By definition, heat is the energy transferred from one system to the other during thermal contact, when all other modes of interactions between the two systems are blocked.

As another example, the energy absorbed by a mixture of ice and water is proportional to the volume of the ice melted. The proportionality constant is measured, e.g., by using the method of Joule heating. We can subsequently measure energy gained or lost by a system by placing it in thermal contact with a mixture of ice and water. The volume of the ice remaining in the mixture can be used to determine the energy transferred to the system.

Enthalpy.

$$H = U + pV$$

Supplement A: Kinetic Energy and Gravitational Energy

The sum of the kinetic energy and the potential energy is the mechanical energy. If a process involves no other change of energy, the mechanical energy is conserved. You can view this statement as a consequence of the conservation of energy. You can also view this statement as a consequence of Newton's second law. We now derive the conservation of mechanical energy from Newton's second law.

Conversion between kinetic and gravitational energy. In mechanics you have learned how kinetic energy converts to potential energy. This conversion is a consequence of Newton's second law,

$$ma = f ,$$

where m is the mass of a particle, a the acceleration of the particle, and f he force acting on the particle. For a particle near the surface of the Earth, the Earth exerts on the particle a gravitational force $f = -mg$, where z is the distance of the particle going up, and the negative sign indicates that the force pulls the particle down. By definition, the acceleration a is the rate of change in the velocity v, namely, $a = dv / dt$. The velocity is the rate of change in the distance, $v = dz / dt$.

Newton's second law implies the conservation of mechanical energy. Write Newton's second law as

$$m\frac{dv}{dt} + mg = 0 .$$

Multiply this equation by the velocity v, and we obtain that

$$m\frac{dv}{dt}v + mgv = 0 .$$

Recall an identity in calculus,

$$\frac{dv}{dt}v = \frac{d}{dt}\left(\frac{1}{2}v^2\right).$$

Also recall the definition of the velocity, $v = dz/dt$. The above equation becomes that

$$\frac{d}{dt}\left(\frac{1}{2}mv^2 + mgz\right) = 0 .$$

This equation shows that the sum of the kinetic energy and the potential energy is independent time. That is, the mechanical energy is conserved.

Supplement B: A Function of Many Variables

The theory of a function of many variables is a subject where geometry, algebra and analysis meet. The three branches of mathematics use distinct dialects, which can be a source of confusion, as well as inspiration. We need to translate concepts from one dialect to another. Here we review several simple but very useful ideas.

A function of many variables. Let V be an n-dimensional vector space and S be a scalar set. Let Q be a function that maps a vector $\mathbf{v}$ in V to a scalar s in S:

$$s = Q(\mathbf{v}) .$$

We often write the vector $\mathbf{v}$ in terms of its components, $(v_1,...,v_n)$, and write the function as $Q(v_1,...,v_n)$.

Level set. For a given scalar a in S, the equation

$$Q(\mathbf{v}) = a$$

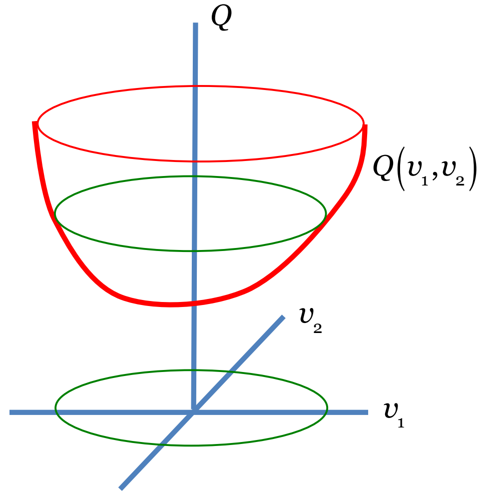
defines a subset of vectors in V, called a *level set*.

We can represent a level set graphically when $n = 2$. Let V be a two-dimensional vector space. We use two vectors in V as a basis. The two vectors serve as the axes of the plane. Each point in the plane corresponds to a vector in the V. For a vector $\mathbf{v}$, let its components relative to the basis be v_1 and v_2.

Write the function as $Q(v_1,v_2)$. We erect Q as the third axes. In the three-dimensional space, the function $Q(v_1,v_2)$ is a surface.

Given a scalar a in S, the equation $Q = a$ represents a plane in the three dimensional space at the height a. The equation $Q(v_1,v_2) = a$ represents the intersection of the plane $Q = a$ and the surface $Q = Q(v_1,v_2)$. The intersection is a curve parallel to the plane (v_1,v_2). We can move this curve vertically to the plane (v_1,v_2), and label this curve by the value a. This curve is a level set.

We can repeat this procedure for another value b, and add the curve $Q(v_1,v_2)=b$ to the plane (v_1,v_2). This procedure leads to a contour plot of the function $Q(v_1,v_2)$.



Gradient. Once again consider a function that maps a vector to a scalar:

$$s = Q(\mathbf{v}).$$

When the vector changes from $\mathbf{v}$ to $\mathbf{v}+d\mathbf{v}$, the scalar changes from $Q(\mathbf{v})$ to $Q(\mathbf{v}+d\mathbf{v})$. According to calculus, for any small change $d\mathbf{v}$,

$$Q(\mathbf{v}+d\mathbf{v})-Q(\mathbf{v}) = \frac{\partial Q(\mathbf{v})}{\partial v_1}dv_1 + ... + \frac{\partial Q(\mathbf{v})}{\partial v_n}dv_n .$$

The n partial derivatives

$$\frac{\partial Q(\mathbf{v})}{\partial v_1},...,\frac{\partial Q(\mathbf{v})}{\partial v_n}$$

are components of another vector, called the *gradient*. Denote the gradient by

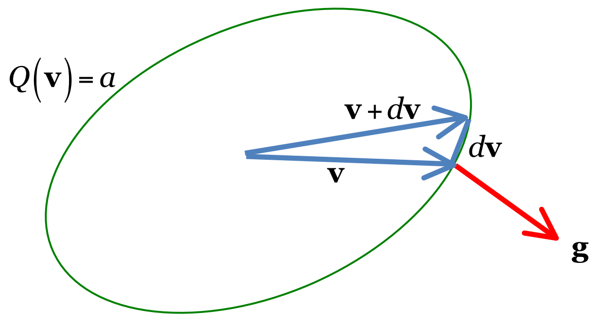
$$g_i = \frac{\partial Q(v_1,...,v_n)}{\partial v_i}.$$

Given a vector $\mathbf{v}$ and a function $Q(\mathbf{v})$, the gradient $\mathbf{g}$ is known.

Write the change in the value of the function as $dQ = Q(\mathbf{v}+d\mathbf{v})-Q(\mathbf{v})$. We translate the above result in analysis into a statement using words in geometry and algebra: the change in the scalar, dQ, is the inner product of the gradient $\mathbf{g}$ and the vector $d\mathbf{v}$, namely,

$$dQ = g_1 dv_1 + ... g_n dv_n .$$

Level set and gradient. Consider a special change $d\mathbf{v}$ such that the function has the same value at the two vectors, that keeps the value of Q unchanged—that is, $Q(\mathbf{v}+d\mathbf{v}) = Q(\mathbf{v})$, or dQ = 0. This vector $d\mathbf{v}$ is tangent to a level set $Q(\mathbf{v}) = a$. The expression $dQ = g_1 dv_1 + ... g_n dv_n$ says that the inner product of the gradient $\mathbf{g}$ and the vector tangent to the level set $d\mathbf{v}$ vanishes. Consequently the gradient $\mathbf{g}$ is a vector normal to the level set passing through the vector $\mathbf{v}$.



Supplement C: Kinetic Energy and Potential Energy

Newton's second law in the form of vectors. A particle, mass m, moves in a three-dimensional space. Set a point in the space as the origin. Describe any other point in the space by a radial vector $\mathbf{r}$ from the origin to the point. At time t, the particle locates at a point $\mathbf{r}$. The function $\mathbf{r}(t)$ describes the trajectory of the particle.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} .$$

In the three-dimensional space, the radial vector has three components, and the velocity is also a vector of three components. This definition of the velocity represents three independent equations for the three components of the vectors.

At time t, the particle is subject to a force $\mathbf{f}(t)$. Newton's second law takes the form

$$m \frac{d\mathbf{v}(t)}{dt} = \mathbf{f}(t) .$$

The mass m of the particle is conserved, and is independent of time. Both the velocity and the force may vary with time. In the three-dimensional space, the force is a vector of three components. Newton's second law consists of three independent equations for the three components.

Given the mass m and the force $\mathbf{f}(t)$, the definition of the velocity and Newton's second law constitute six ordinary differential equations that evolve six functions of time: the three components of the radial vector $\mathbf{r}(t)$ and the three components of the velocity $\mathbf{v}(t)$. The ordinary differential equation can be integrated in time once the radial vector and the velocity are prescribed at a particular time.

Gradient of a scalar field. $\phi(\mathbf{r})$. $d\phi = \phi(\mathbf{r} + d\mathbf{r}) - \phi(\mathbf{r})$

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial x}dz$$

$$d\mathbf{r} = \begin{bmatrix} dx & dy & dz \end{bmatrix}$$

Similarly, we can regard the three partial derivatives as three components

$$\nabla \phi = \begin{bmatrix} \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{bmatrix}$$

$$d\phi = (\nabla \phi) \cdot (d\mathbf{r})$$

Potential energy. $\mathbf{f} = -\nabla \phi$

Conservation of kinetic energy and potential energy.

$$m\frac{d\mathbf{v}}{dt} + \nabla \phi = 0$$

$$m\mathbf{v} \cdot \frac{d\mathbf{v}}{dt} + \mathbf{v} \cdot \nabla \phi = 0$$

$$(d\mathbf{r}) \cdot (\nabla \phi) = d\phi$$

$$\frac{d}{dt}\left(\frac{1}{2}m\mathbf{v} \cdot \mathbf{v} + \phi\right) = 0$$

$$\frac{1}{2}m\mathbf{v}\cdot\mathbf{v}+\phi=\text{constant}$$

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