

Empirical observations of temperature

Our feeling of hotness comes from everyday experiences. These experiences indicate that many levels of hotness exist, and that all levels of hotness can be mapped to a real variable.

Most of our feelings, however, cannot be mapped to a real variable. Think of happiness, love, anxiety. It is therefore extremely remarkable that the levels of hotness can be mapped to a real variable. What makes hotness so different from happiness?

To appreciate the difference, we will reflect upon how we discover the levels of hotness by using a particular kind of everyday experience: thermal contact.

We make the word hotness—in the precise way used here—synonymous to temperature. An essential step to understand how thermodynamics works is to get to know temperature. How is temperature abstracted from everyday experiences?

Thermal contact. Energy resides in the interactions and motions of all the molecules in a piece of cheese, as well as in a glass of wine. When a piece of cheese touches a glass of wine, the vibration of the molecules in the cheese will interact with the vibration of the molecules in the wine, through the vibration of the molecules in the glass. After some time, energy is redistributed between the cheese and the wine.

Say the wine and the cheese have been kept as two separate isolated systems for such a long time that each by itself is in equilibrium. However, the two isolated systems are not in equilibrium with each other. We now regard the composite of the two systems as an isolated system, and allow the wine and the cheese to interact in a particular way: energy can go from the wine to the cheese, or the other way around, but energy does not leave or enter the composite system. According to the principle of the conservation of energy,

When a constraint internal to an isolated system is lifted, the energy of the system remains unchanged.

When the composite of the wine and cheese is made an isolated system, the amount of energy gained by the wine is the same as that lost by the cheese, or vice versa.

Will energy go from the wine to the cheese, or the other way around? This question is *not* answered by the principle of the conservation of energy, but is central to our understanding of thermodynamics.

This mode of interaction, which re-allocates energy between the two systems, is called *thermal contact*. The amount of energy that goes from one system to the other is called *heat*.

To focus on thermal contact, we make heat transfer the only mode of interaction between the wine and the cheese. We block all other modes of interaction. The wine and the cheese do not exchange molecules. The separation between them—the glass—is rigid, so that the pressure in the wine does no work to the cheese.

In reality, any contact of two systems does more than just re-allocating energy. For example, when two bodies touch each other, they may form atomic bonds. As another example, the presence of two bodies in space modifies the electromagnetic field. For the time being, we assume that such effects are negligible, and focus exclusively on a single mode of interaction: the two systems can only exchange energy.

Our everyday experience with thermal contact may be distilled in terms of several salient observations. From these empirical observations we will abstract the idea of temperature.

Observation 1: After being in thermal contact for a long time, two systems cease to exchange energy. When all other modes of interaction are blocked, two systems in thermal contact for a long time will cease to exchange energy, a condition known as *thermal equilibrium*. This observation shows that, when the two systems are in thermal contact, their approach to thermal equilibrium is *irreversible*.

Observation 2: If two systems are separately in thermal equilibrium with a third system, the two systems are in thermal equilibrium with each other. This observation is known as *the zeroth law of thermodynamics*.

Following this observation, all systems in thermal equilibrium are said to have the same *level of hotness*. We define a level of hotness by an experiment. We bring two systems into thermal contact, and check if they exchange energy. If the two systems in thermal contact do not exchange energy, we say that they have the same level of hotness. If the two systems in thermal contact exchange energy, we say that they have different levels of hotness. Our everyday experience indicates that many levels of hotness exist.

In thermodynamics, the word “hot” is used strictly within the context of thermal contact. It makes no thermodynamic sense to say that one movie is hotter than the other, because the two movies cannot exchange energy.

The word hotness is synonymous to temperature. The phrase “a level of hotness” is synonymous to “a degree of temperature”.

Name levels of hotness any way you like. So many levels of hotness exist in our world. We are interested in all of them. To talk about them individually, we need to give each level of hotness a name. For example, ice melts at a particular level of hotness. This level of hotness has acquired a long list of names: melting point of ice, freezing point of water, 0C, 32F, 273.15K, etc.

A scheme to name all levels of hotness is called *a scale of hotness*. We will explain how various naming schemes work shortly, but all naming schemes are arbitrary decisions made by human beings. The situation is analogous to naming streets in a city. The streets are real: they exist regardless how we name them. We can name the streets by using names of presidents, or names of universities. We can use numbers. We can be inconsistent, naming streets in one part of the city by numbers, and those in another part of the city by names of presidents. We can even give the same street several names, using English, Chinese, and Spanish. Some of us are superstitious, believing that the reality of a street is affected by how we name it.

As shown by the above empirical observations, levels of hotness are real: they exist in the experiment of thermal contact. How to name the levels of hotness is our choice. The names exist between our lips, and in our ears and books. Some naming schemes might be more convenient than others. For now, consider one specific naming scheme.

Name a level of hotness after a physical event. For example, we can name a level of hotness after a pure substance that melts at this hotness. This practice is easily carried out because of the following empirical observation: at the melting point, a pure substance is a mixture of solid and liquid, and the level of hotness remains unchanged as the proportion of the solid and liquid changes.

Thus, a system is said to be at the level of hotness named “WATER” when the system is in thermal equilibrium with a mixture of ice and water at the melting point. Here are four distinct levels of hotness: WATER, LEAD, ALUMINUM, GOLD. We can similarly name other levels of hotness by using a large number of pure substances.

How does the above naming scheme differ from words used in everyday life, such as cold, cool, warm, and hot? When we say a system is at the level of hotness WATER, we mean a specific experimental observation: the system is in

thermal equilibrium with an ice-water mixture. By contrast, when we use the word cold, we may not have a specific level of hotness in mind.

Observation 3: Levels of hotness are ordered. When two systems of different levels of hotness are brought into thermal contact, energy goes only in one direction from one system to the other, but not in the opposite direction. This observation is a version of *the second law of thermodynamics*.

This observation does not violate the principle of the conservation of energy, but is not implied by the conservation of energy. Energy going either way is conserved.

This observation of the unidirectional flow of energy allows us to order any two levels of hotness. When two systems are brought into thermal contact, the system losing energy is said to have a higher level of hotness than the system gaining energy. For example, we say that hotness “LEAD” is lower than hotness “ALUMINUM” because, upon brining melting lead and melting aluminum into thermal contact, we observe that the amount of solid lead decreases, while the amount solid aluminum increases. Similar experiments show us the order of the four levels of hotness as follows:

WATER, LEAD, ALUMINUM, GOLD.

Observation 4: Levels of hotness are continuous. Between any two levels of hotness A and B there exists another level of hotness. The experimental demonstration goes like this. We have two systems at hotness A and B, respectively, where hotness A is lower than hotness B. We can always find another system, which loses energy when in thermal contact with A, but gains energy when in thermal contact with B.

Name all levels of hotness by a real variable. All levels of hotness are ordered, so that we can name them by using a set of numbers. Levels of hotness are continuous, so that we cannot name them by using integers, but we can name them by using a real variable. Around 1720, Fahrenheit assigned the number 32 to the melting point of water, and the number 212 to the boiling point of water. What would he do for other levels of hotness? Mercury is a liquid within this range of hotness and beyond, sufficient for most purposes for everyday life. When energy is added to mercury, mercury expands. The various volumes of mercury could be used to name the levels of hotness.

What would Fahrenheit do for a high level of hotness when mercury is a vapor, or a low level of hotness when mercury is a solid? He could switch to

materials other than mercury; for example, you can use a flask of gas. You can also use phenomena other than thermal expansion.

As illustrated by the melting-point scale of hotness, a non-numerical naming scheme of hotness perfectly captures all we care about everyday experience of hotness. Naming levels of hotness by using numbers makes it easier to memorize that hotness 80 is hotter than hotness 60. Our preference to a numerical scale reveals more about the prejudice of our brains than the nature of hotness.

Using numbers to name levels of hotness does not authorize us to apply arithmetic rules: adding two levels of hotness has no more empirical significance than adding the addresses of two houses on a street. House number 2 and house number 7 do not add up to become house number 9. Also, raising the temperature from 0C to 50C is a different process from raising temperature from 50C to 100C.

Map one scale of hotness to another. Thus, mercury performs the duty of a *one-to-one function*. The domain of the function consists of various levels of hotness, and the range of the function consists of various volumes of mercury. The function maps levels of hotness to volumes of mercury. The latter can be represented by a real variable.

Any such choice of a function sets up a *scale of hotness*. Once a scale of hotness is set up, any monotonically increasing function maps this scale to another scale of hotness. For example, in the Celsius scale, the freezing point of water is set to be 0C, and the boiling point of water 100C. We further set the Celsius (C) scale to be linear in the Fahrenheit (F) scale. These prescriptions give the transformation between the two scales of hotness:

$$C = \frac{5}{9}(F - 32) .$$

In general, the transformation from one scale of hotness to another need not be linear. Any increasing function will preserve the order of the levels of hotness. Any smooth function will preserve the continuity of the levels of hotness.

Observation 5: All levels of hotness are hotter than a certain level of hotness. This certain level of hotness can be approached, but not attained. This empirical observation is known as a version of the *third law of thermodynamics*. So far as we know, all levels hotness hotter than this certain level can be attained. That is, there is no upper limit to the levels of hotness.

Following this observation, we can name this certain level of hotness zero, and name all other levels of hotness by a positive real variable. Such a scale of hotness is called an *absolute scale*.

Observation 6: Ideal gas. Historically, gases played significant roles in formulating thermodynamics. Let p be the pressure, V the volume, and N the number of molecules. When the gas is thin, i.e., N/V is small, experiments indicate that flasks of gases with the same value of pV/N do not exchange energy when brought into thermal contact. This observation holds regardless the identity of the gas. Consequently, the values of pV/N can be used to set up a scale of hotness, known as the ideal-gas scale of temperature. Write

$$\theta = pV/N.$$

This equation relates a scale of temperature θ to measurable quantities p , V and N . This relation, known as the *law of ideal gases*, was discovered empirically before the fundamental postulate was formulated. The ideal-gas scale of temperature is an absolute scale of temperature.

Because N is a pure number, and pV has the unit of energy, the ideal gas scale of temperature defined above has the same unit as energy.

Kelvin scale of temperature. An international convention, however, specifies the unit of energy by Joule (J), and the unit of temperature by Kelvin (K). The Kelvin scale of temperature is defined as follows:

1. The Kelvin scale of temperature, T , is proportional to the ideal-gas scale of temperature, θ .
2. The unit of the Kelvin scale, K, is defined by marking the triple point of pure water as $T = 273.15 \text{ K}$.

Thus, we write

$$\theta = kT,$$

where k is the conversion factor between the two scales of temperature. The conversion factor k is known as the Boltzmann constant.

The conversion factor between the two units of temperature can be determined experimentally. For example, you can bring pure water in the state of triple point into thermal contact with a flask of an ideal gas. When they reach thermal equilibrium, a measurement of the pressure, volume and the number of molecules in the flask gives a reading of the temperature of the triple point of

water in the unit of energy. Such experiments or more elaborate ones give the conversion factor between the two units of temperature:

$$k \approx 1.38 \times 10^{-23} \text{ J/degree}.$$

Other scales of temperature. Any scale of temperature can be mapped to any other scale of temperature. For example, the Celsius scale C relates to the Kelvin scale T by

$$C = T - 273.15 \text{ K} \quad (T \text{ in the Kelvin unit}).$$

This modern definition of the Celsius scale differs from the historical definition. Specifically, the melting point and the boiling point of water are not used to define the modern Celsius scale. Rather, these two levels of hotness are determined by experimental measurements. The experimental values are as follows: water melts at 0°C and boils at 99.975°C , according to the International Temperature Scale of 1990 (ITS-90).

The merit of using Celsius in everyday life is evident. It feels more pleasant to hear that today's temperature is 20°C than 0.0253eV or 293K . It would be mouthful to say today's temperature is $4.0434 \times 10^{-23} \text{ J}$. The hell would sound less hellish if we were told it burns at the temperature GOLD. Nevertheless, our melting-point scale of hotness can be mapped to the absolute temperature scale in either the Kelvin unit or the energy unit:

Melting-point scale	Kelvin unit	Energy unit
WATER	273.15 K	0.0236 eV
LEAD	600.65 K	0.0518 eV
ALUMINUM	933.60 K	0.0806 eV
GOLD	1337.78 K	0.1155 eV

Experimental determination of temperature (thermometry).

The art of measuring temperature is called *thermometry*. How does a doctor determine the temperature of a patient? She uses a thermometer. Let us say that she brought a mercury thermometer into thermal contact with the patient. Upon reaching thermal equilibrium with the patient, the mercury expands a certain amount, giving a reading of the temperature of the mercury, which is also the temperature of the patient.

The manufacturer of the thermometer must assign an amount of expansion of the mercury to a value of temperature. This he does by bringing the

thermometer into thermal contact with a flask of an ideal gas. He determines the temperature of the gas by measuring its volume, pressure, and number of molecules. Also, by heating or cooling the gas, he varies the temperature and gives the thermometer a series of markings.

Experimental determination of heat (calorimetry). The art of measuring heat is called *calorimetry*. By definition, heat is the energy transferred from one system to the other during thermal contact, when all other modes of interactions between the two systems are blocked. Heat and temperature can be determined by independent experimental measurements. For example, we can pass an electric current through a resistor immersed in a pot of water. The Joule heating of the resistor adds energy to water. The amount energy added to water is the electrical power, IV , over some time.

As another example, the energy absorbed by a mixture of ice and water is proportional to the volume of the ice melted. The proportionality constant is measured, e.g., by using the method of Joule heating. We can subsequently measure energy gained or lost by a system by placing it in thermal contact with a mixture of ice and water. The volume of the ice remaining in the mixture can be used to determine the energy transferred to the system.

Why is hotness so different from happiness? Most our feelings cannot be mapped to a real variable. Think of happiness, love, anxiety. It is therefore extremely remarkable that this particular type of feeling—hotness—can be mapped to a real variable.

What makes this difference between hotness and happiness? One way to answer this question would be to see if we can point to empirical observations that enable us to associate happiness with a real number. This we do not know how to do. Instead, we will *understand* the above empirical observations of thermal contact in terms of two great principles of our world: the fundamental postulate and the conservation of energy. This I hope to do at a later point of the course.

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