● **Nominal (First Piola-Kirchhoff) stress** $\mathbf{S} = J\mathbf{F}^{-1} \cdot \boldsymbol{\sigma}$ $\quad S_{ij} = JF_{ik}^{-1}\sigma_{kj}$

● **Material (Second Piola-Kirchhoff) stress** $\boldsymbol{\Sigma} = J\mathbf{F}^{-1} \cdot \boldsymbol{\sigma} \cdot \mathbf{F}^{-T}$ $\quad \Sigma_{ij} = JF_{ik}^{-1}\sigma_{kl}F_{jl}^{-1}$

The inverse relations are also useful – the one for Kirchhoff stress is obvious – the others are

$$\boldsymbol{\sigma} = \frac{1}{J}\mathbf{F} \cdot \mathbf{S} \quad \sigma_{ij} = \frac{1}{J}F_{ik}S_{kj} \qquad \boldsymbol{\sigma} = \frac{1}{J}\mathbf{F} \cdot \boldsymbol{\Sigma} \cdot \mathbf{F}^{T} \quad \sigma_{ij} = \frac{1}{J}F_{ik}\Sigma_{kl}F_{jl}$$

The **Kirchoff stress** has no obvious physical significance.

The **nominal stress tensor** can be regarded as the internal force per unit undeformed area acting within a solid, as follows

1. Visualize an element of area dA in the deformed solid, with normal $\mathbf{n}$, which is subjected to a force $d\mathbf{P}^{(n)}$ by the internal traction in the solid;
2. Suppose that the element of area dA has started out as an element of area dA_0 with normal $\mathbf{n}_0$ in the undeformed solid, as shown in the figure;
3. Then, the force $d\mathbf{P}^{(n)}$ is related to the nominal stress by $dP_j^{(\mathbf{n})} = dA_0 n_i^0 S_{ij}$

To see this, note that one can show (see Appendix D) that

$$dA\mathbf{n} = J\mathbf{F}^{-T} \cdot dA_0\mathbf{n}_0 \qquad dAn_i = JF_{ki}^{-1}n_k^0 dA_0$$

Recall that the Cauchy stress is defined so that

$$dP_i^{(\mathbf{n})} = dAn_j\sigma_{ji}$$

Substituting for dAn_j and rearranging shows that

$$dP_i^{(\mathbf{n})} = JdA_0 n_k^0\left(F_{kj}^{-1}\sigma_{ji}\right) = dA_0 n_k^0 S_{ki}$$

NOW, CONSIDERING:

THE EQUATION,

$$dAn_i = JF_{ki}^{-1}n_k^0 dA_0$$

HERE, IN FACT **dAni** IS WRITTEN IN TERMS OF dA0 nk0, Right?

Now considering the equations below:

$$dP_i^{(\mathbf{n})} = dAn_j\sigma_{ji}$$

$$dP_i^{(\mathbf{n})} = JdA_0 n_k^0\left(F_{kj}^{-1}\sigma_{ji}\right) = dA_0 n_k^0 S_{ki}$$

The above equations are just the same – only dA is written in terms of dAo- right?

So, even by using Cauchy stress we get essentially same equation.Then why use Piolla Kirchhoff stress at all?