ISOGEOMETRIC ANALYSIS: TOWARD INTEGRATION OF CAD AND FEA

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BOOK REVIEW

Contents: Preface

When I took this book into my hands for the first time, quickly glancing through its pages, my first impression was that – from a purely external perspective – this is the most beautiful scientific book that I have ever seen. (I am excluding popular science books from this statement; this book matches some of them in its beauty.) The authors, editors and publishers should be congratulated for giving so much attention not just to the content but also to the way the book looks. It is extremely inviting to read.

On top of this, the book is very well-written. This comes with no surprise, since it follows the tradition of the crystal-clear writing of T.J.R. Hughes. The combination of excellent writing, beautiful production and the importance of the subject (as discussed below) makes this book a real gem.

Isogeometric analysis (IGA) is a body of theory and techniques invented originally by T.J.R. Hughes and further developed by his group, including the co-authors J.A. Cottrell and Y. Bazilevs, and others. IGA seeks to unify the fields of Computer-Aided Design (CAD) and Finite Element Analysis (FEA). Although early concepts of CAD-FEA unification have appeared in the pioneering work of Mark S. Shephard and his RPI group, IGA is the first instance in which the unification goes as deep as
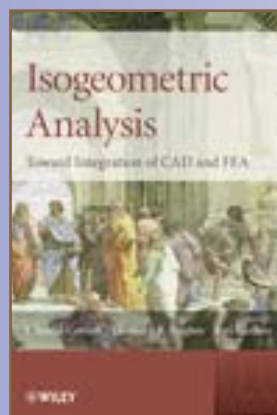
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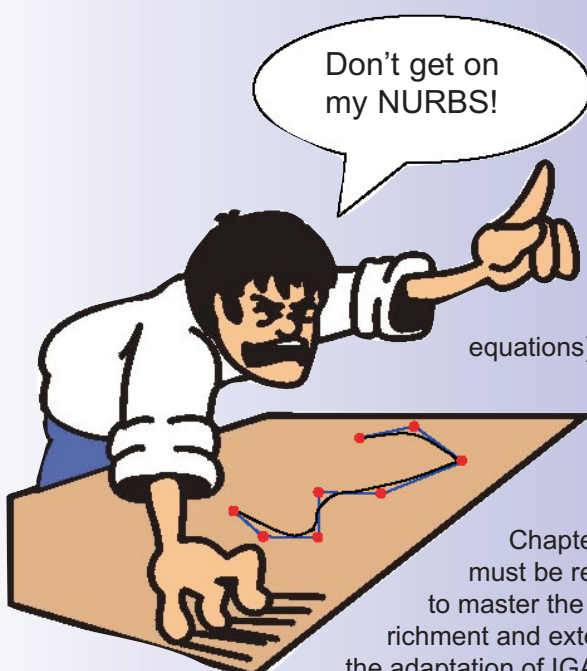givolid@aerodyne.technion.ac.il

T.J.R. Hughes

the approximation space. The book assumes no knowledge in CAD, but it does assume basic knowledge in FEA. As written in the preface, "a background in FEA at the level of Hughes, 2000 is ideal preparation for understanding this book." It is ideal preparation also because the notation and general approach employed in the two books are quite similar.

The key idea behind IGA is that the functional basis used to model the geometry (as traditionally employed in CAD) also serves as the basis for the solution space of the numerical scheme. The 'building blocks' of this basis are B-splines, which are explained very nicely in the first half of Chapter 2. Many concepts which are new to most FEA community members are introduced here (B-spline functions, knot, knot vector, knot multiplicity, index space, control point, control net, B-spline curve, k-refinement, etc.). A common misconception related to B-splines is explained on pp. 22 and 24: despite the fact that B-splines have a larger support than standard finite element functions for polynomial degree larger than 1, the bandwidth associated with the two types of functions is exactly the same. Section 2.1.4 discusses various ways of refinement (without altering the geometry and parameterization), using (1) knot insertion, (2) order elevation, or (3) k-refinement (which very importantly controls the smoothness), as well as their combination.

Non-Uniform Rational B-Splines (NURBS) are the standard tool in CAD, and under IGA become also integral part of analysis. NURBS are explained in the second half of Chapter 2 using two points of view (geometric and algebraic). The chapter ends with a long illustrative tutorial to generating a NURBS mesh. See *Fig. 1*.

For a person that has worked in FEA for many years, grasping these new concepts may pose a challenge. One may attempt to find analogies between these concepts and the more familiar ones, but as the authors write on p. 27, "we should resist this urge, and understand the B-spline technology for what it is, not merely by analogy with things that are already familiar."

Chapter 3 explains how NURBS are brought into use in analysis. This chapter discusses a number of classes of methods (Galerkin, collocation, least-squares and meshless methods) where NURBS can be incorporated. Of course, complete books can be written on each of these methods, but the approach here is to provide the reader with the minimum amount of material needed to understand how these methods work with NURBS. Different ways to enforce boundary conditions are also discussed here, as well as work with multiple patches in general geometries.

Figure 1:
The control lattice for a NURBS-based pipe model. This is Figure 2.44 taken from the book, p. 64, and is part of the tutorial on NURBS modeling.

Don't get on my NURBS!

Chapter 3 ends with a nice and clear comparison of IGA and classical FEA. At this point, the reader already understands the significant benefits entailed in IGA as compared to standard FEA. An additional and very important benefit was pointed out to me by Michel Bercovier: IGA allows one to construct vector basis spaces (like H(div) or H(curl), as done, e.g., by Buffa et al., CMAME, 199, 1143-1152, 2010 for Maxwell's equations) directly and easily, a feature which was unavailable before. In Michel's words: "Approximations that were once regarded as exotic have become, owing to IGA, simple and natural."

Chapters 2 and 3 are the heart of this book. They must be read and understood completely if the reader is to master the material. The other chapters constitute enrichment and extension to this material. Chapters 4-13 discuss the adaptation of IGA to various types of problems, and include
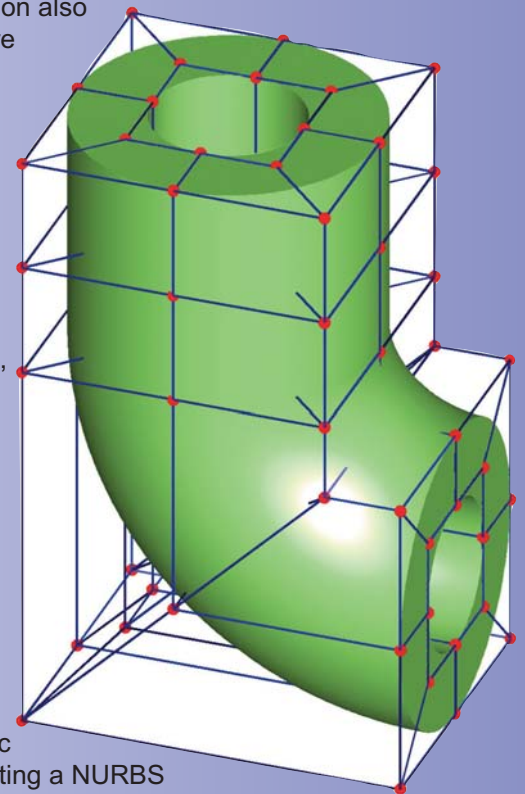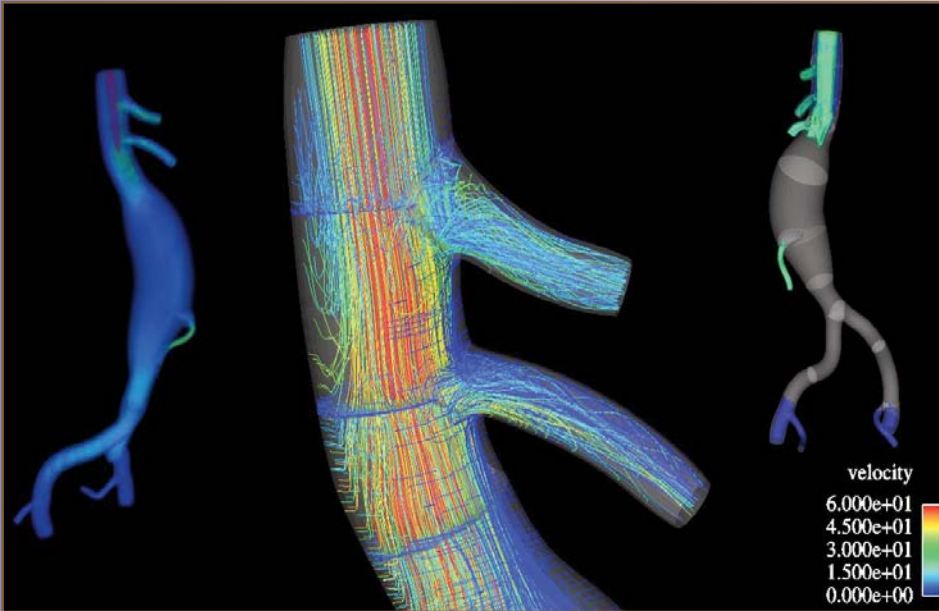
Figure 2:
Blood flow in a vessel as the heart contracts during systole. This is Figure 19.9(a) taken from the book, Chapter 10 (fluid-structure interaction and fluids on moving domains), p. 263

Figure 3:
Two propellers spinning in opposite directions. This is Figure 10.16(b) taken from the book, chapter 10, p. 274

demonstrations of very impressive applications of IGA. Among them are those shown in *Figs. 2 and 3*.

Finally, a bit more on the fine touches which make this book such a delight to read. The book contains many colorful figures, and is extremely pleasing visually. On pp. 9, 10, 13 and 14, a 'picture gallery' of some prominent researchers in FEA and CAD is shown. Along the entire book there are pictures of additional famous scientists who have contributed to what FEA and CAD are today. Many attempts are done throughout the book to help the reader digest the new material easily; for example, on p. 98, two very helpful tables appear which summarize the differences and similarities between IGA and FEA. At the end of each chapter there are numbered Notes that are referred to from the text. The captions of the figures are very detailed and clear, and the illustrations are well crafted. The reader can learn a great deal by merely looking at the figures and reading their captions.

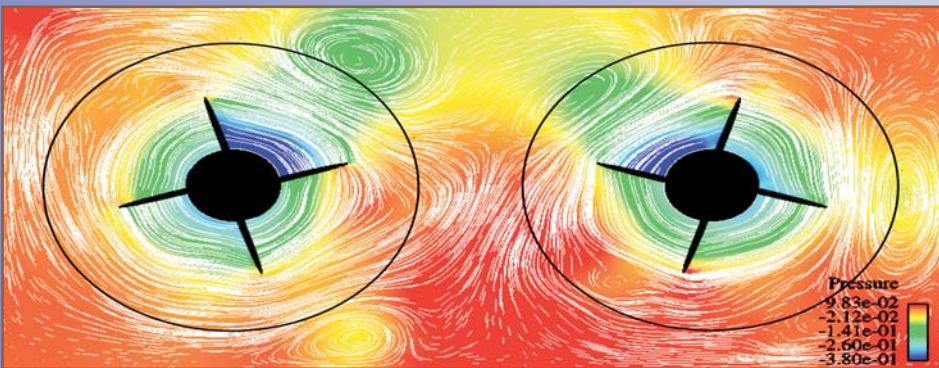The only thing in this book which I found to be slightly less than perfect is the index. For some reason in many excellent books the importance of a detailed index seems to be underestimated. (An extreme example is the classical book by Strang and Fix on the finite element method; this book is a masterpiece but its index "leaves much to be desired" as the British understatement goes.) Most of the entries in the index are cited only once, even though many of them appear in different places in the text, sometimes in an important way. Some entries are altogether missing. Here are a few examples: 'NURBS coordinates' appears in the index, but 'NURBS' itself (defined in pp. 47-51) and 'NURBS curve' (p. 50) do not appear; 'B-spline curve' appears but 'B-spline basis functions' (p. 21) does not appear; 'multiple patches' are discussed on pp. 87-92 but do not appear under 'patches' in the index, and 'multiple' or 'repeated' do not appear either; 'error estimates' (pp. 103-106) and Bézier element (pp. 91-94) do not appear in the index. Fortunately, the book is so well-structured and the table of contents is so revealing that the reader can easily find her way even without the aid of an index.

For decades, geometric design and mesh generation on one hand and analysis on the other hand have been regarded as quite separate from each other. Many researchers and practitioners may feel that this separation is convenient. For example, many researchers who investigate subjects in FEA consider geometry-related techniques a 'black box'. This book shows us the benefits of the integrative geometry-analysis approach, and establishes a language common to both constituencies. Indeed, as written in the explanation to the painting of Raphael which is on the front cover of this book, legend has it that over the door to Plato's Academy in Athens there was an inscription:
Let no man ignorant of geometry enter here!