One, no one, and one hundred thousand crack propagation criteria: have we learned the lesson by G.I. Barenblatt?

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Some preliminaries about scaling, analogies and engineering solutions

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From Scaling phenomena in fatigue and fracture G.I. Barenblatt: Lawrence Berkeley National Laboratory (University of California, 2004 Paper LBNL-56765

Scaling laws

$$y = C x_1^{\alpha_1} \dots x_n^{\alpha_n} \tag{1}$$

which often appear in modeling phenomena in nature, engineering and society, seem to be trivial. Very often they are considered only as a first, simplest attempt to represent a set of data. Indeed, laws are trivial, but non-trivial is why they are trivial! The reason of their importance is that the scaling laws always reveal a deep feature of processes under consideration: *self-similarity*. For processes developing in time self-similarity means that the phenomenon is reproducing itself in scales, which vary in time:

$$\mathbf{U}(\mathbf{r}, t) = \mathbf{U}_0(t) f\left(\frac{\mathbf{r}}{\ell(t)}\right). \tag{2}$$

Establishing scaling laws and self-similarity was always considered as an important, sometimes crucially important step in construction of engineering and physical theories. In the pre-computer era they were considered as special, "exact" solutions illuminating complicated models, elegant, sometimes useful, but nevertheless restricted in their value, elements of theories. Later, when computers entered into play, the role of such solutions did not diminish, just the contrary. However, the general attitude to them changed: they started to attract attention mainly as *"intermediate asymptotics"* — an important element of physical or engineering theories describing the behavior of systems when the

2 Scaling laws obtained by dimensional analysis

I will present here the state of art of these matters with some instructive examples of scaling laws and scaling phenomena. Naturally all except one of the examples will be related to fracture and fatigue, the examples from other fields in particular turbulence can be found in my books [16,17]. However, I want to make a short *intermezzo*: to demonstrate one of the milestones not only of fluid mechanics, but of engineering science as a whole achieved by similarity methods. I mean the famous scaling law obtained first by Sir Geoffrey Taylor [18] for the radius r_f of the shock wave formed after an atomic explosion (Figure 1):

$$r_f = \left(\frac{Et^2}{\rho_0}\right)^{1/5} . \tag{3}$$

Here E is the energy of the explosion, t — the time after the explosion, and ρ_0 — the air density before the explosion. Scaling law (3) was obtained by G.I. using *dimensional analysis* (I will demonstrate this technique on examples from fracture and fatigue later, the explanation of it with details and examples can be found in [16,17].) I emphasize specially: of crucial importance was not the formal application of rather simple rules of dimensional analysis, but the preliminary idealization of the problem which was invented by G.I. Taylor: He made two basic assumptions — the explosion is instantaneous and concentrated at a point. Furthermore, the initial atmospheric pressure p_0 is negligible in comparison with the pressure behind the shock wave. This basic idealization allowed G.I. Taylor to establish the self-similarity of the phenomenon and to assume that the radius of the shock wave r_f can depend on E, t, and ρ_0 only. After this assumption

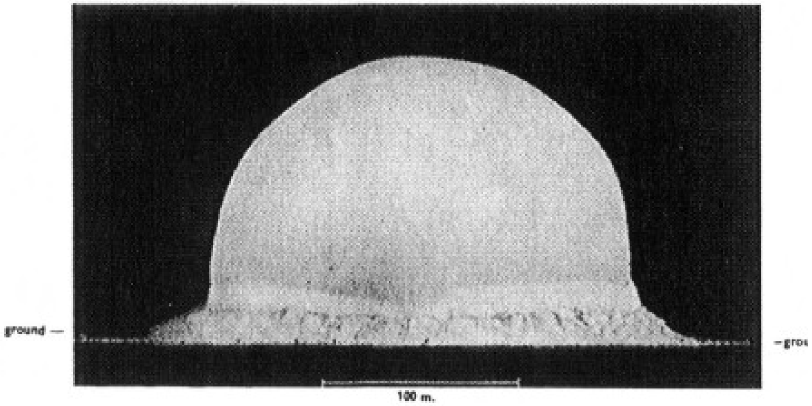


Figure 1: Photograph of the fireball of the atomic explosion.

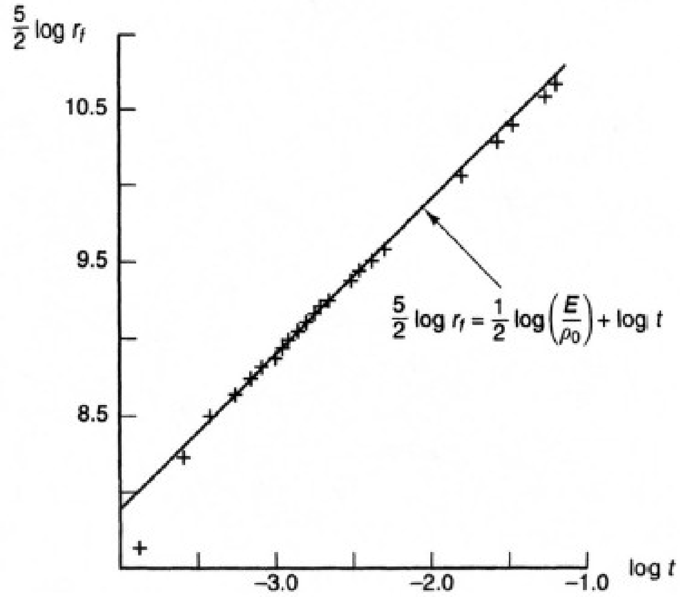


Figure 2: The experimental results confirms scaling law (3) (Taylor [

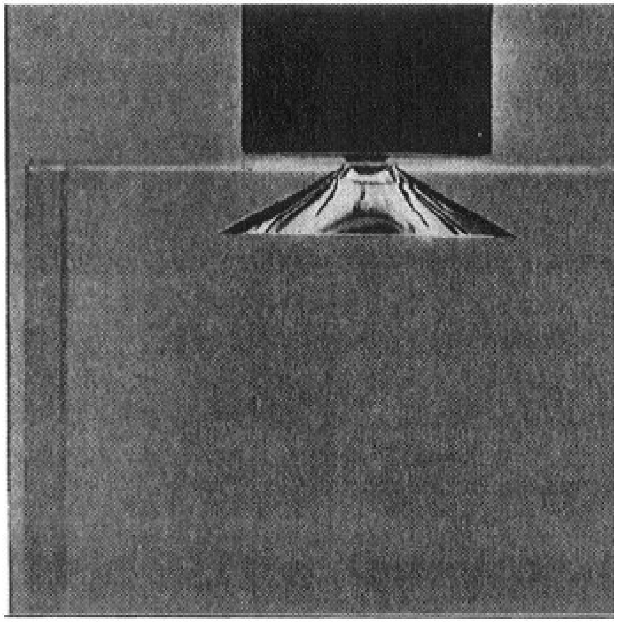


Figure 4: Conical crack in a fused silica block (Benbow [25]).

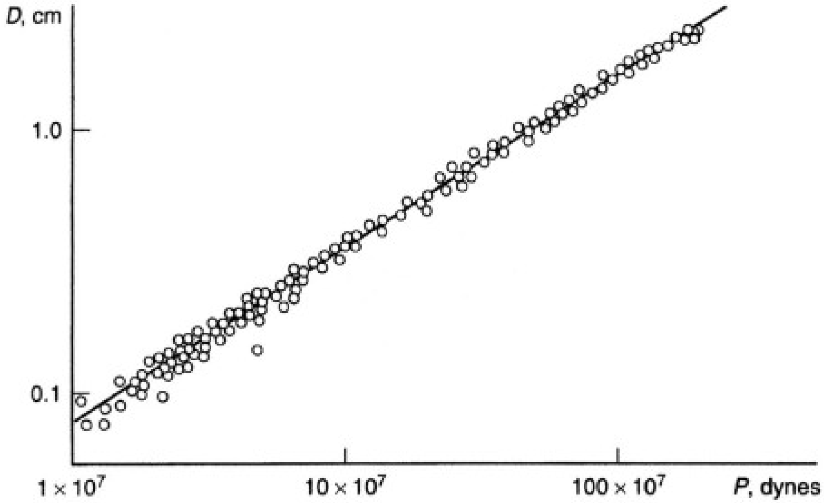


Figure 5: The experimental data confirm scaling law (7) (Benbow [25]).

block. The scaling law, also obtained by dimensional analysis, for the diameter of the base of the cone crack D under the load P has the form:

$$D = \mathrm{Const}(\nu)\left(\frac{P}{K}\right)^{2/3}. \qquad (7)$$

Fatigue ---- Paris « law »

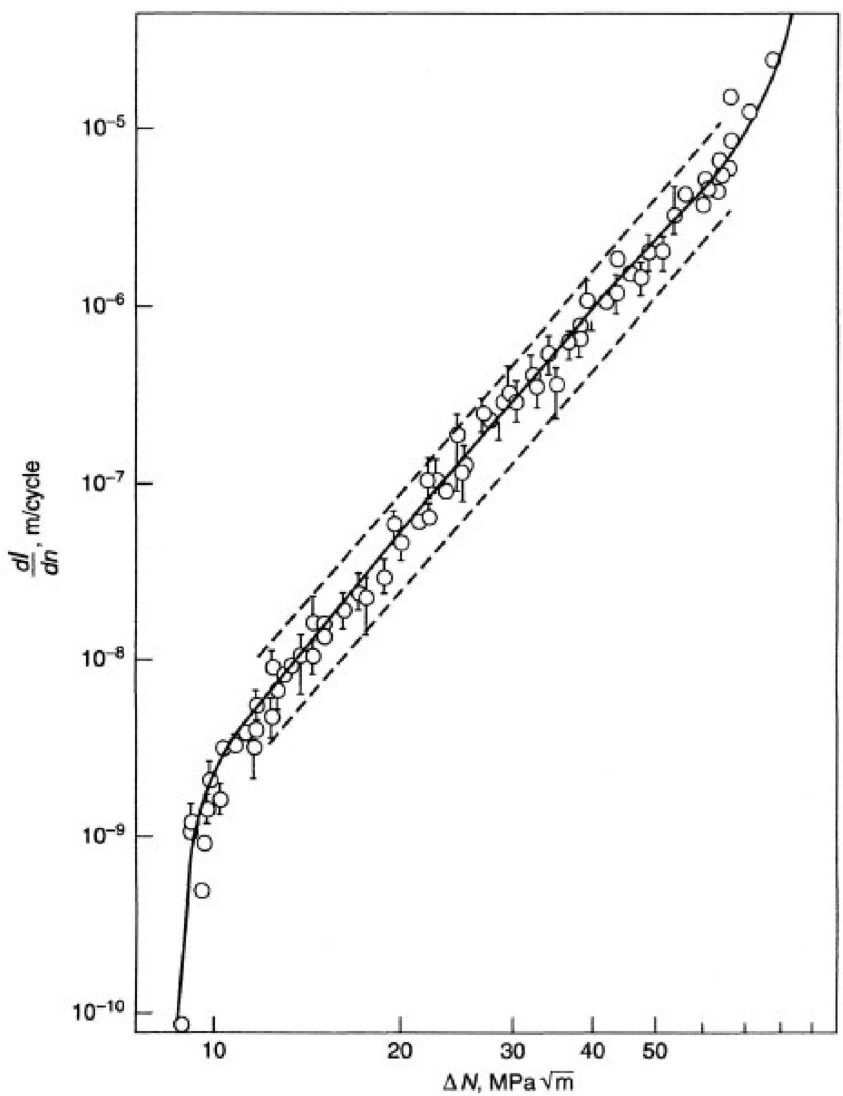


Figure 6: Experimental data for fatigue crack growth in the alluminum alloy confirm Paris law (17) in the major part of the crack velocity range (Botvina [38]).

Establishing the scaling laws and self-similarities was of special importance in studies of turbulence and structural strength. Remarkably both these branches of natural and engineering science were started by two great Italians: Leonardo da Vinci, born in a suburb of Florence, and Galileo Galilei, born in Pisa: both these places are not far from Torino where we are assembled these days.

Turbulence is the fluid flow with randomly accumulating and dissipating vortices.

Structural strength is inseparable from *fatigue*. Fatigue in a broader sense is the deformation and flow of solids with accumulating defects, terminating by *fracture*.

Generally in scientific circles turbulence is considered as a major challenge of applied mathematics and classical physics. Among those who made seminal contributions to studies of turbulence were giants — A.N. Kolmogorov and W. Heisenberg, as well as great applied mathematicians of the XX century — L. Prandtl, Th. von Kármán and G.I. Taylor.

At the same time in the circles of physicists and applied mathematicians the problem of structural strength, and in particular the problem of fatigue, definitely remains overshadowed by turbulence. Meanwhile, fatigue and fracture present not less thrilling fundamental problems than turbulence, not to speak about practical importance. In fact, the phenomenon of fatigue is in principle even more complicated! Indeed, stop the turbulent flow of air and/or water, and the fluid becomes indistinguishable from that at the beginning of the motion. This is not the case for fatigue. And there is even something worse. In turbulence we have good experimental reasons to believe that the fluid, like air and water remains Newtonian even in the most complicated flows up to the scales of smallest vortices. At the same time even for quasi-brittle solids the very possibility of using any constitutive equations for the materials near the defects is doubtful!

Parallels between turbulence and fatigue are instructive — also in another aspect. Yes, turbulence is generally considered as a great challenge, and during more than a century an army of engineers and scientists led by geniuses attacked it. But let's ask ourselves with full sincerity: what was achieved during this time as far as the creation of pure self-contained theory of turbulence based on first principles is concerned? The answer is very disappointing — practically nothing! And it is clear now that such a theory will not be constructed in real time — such was in particular the opinion of A.N. Kolmogorov. He claimed that the practical way is to construct models based on special hypotheses relying on the results of experimental studies. Clearly such models could be valid for special classes of flows only. It can be expected that the same path is to be followed in structural strength and fatigue studies.

Here I want to emphasize that practically all significant results in turbulence studies were obtained using similarity considerations and scaling laws. The value of these tools and their technique should be properly understood, and they should be properly used in everyday practice by engineers and researchers

OUTLINE

- Generalized Barenblatt & Botvina dimensional analysis approach to fatigue crack growth.
- Dependence of the Paris' law parameter m on the dimensionless number Z.
- Dependence of the Paris' law parameter C on the dimensionless number Z: complete vs. incomplete (power-law) dependence.
- Correlation between C and m based on the dimensionless number Z.

Classical Paris' "law":

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K^m$$

Generalized representation of fatigue crack growth derived according to dimensional analysis:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \left(\frac{K_{\mathrm{IC}}}{\sigma_y}\right)^2 \Phi\left(\frac{\Delta K}{K_{\mathrm{IC}}}, \frac{\Delta K_{\mathrm{th}}}{K_{\mathrm{IC}}}, \frac{\Delta\sigma_{fl}}{\sigma_y}, \frac{E}{\sigma_y}, \frac{\sigma_y^2}{K_{\mathrm{IC}}^2}h, \frac{\sigma_{fl}^2}{\Delta K_{\mathrm{th}}^2}a; 1-R\right)$$

$$\Pi_1 = \frac{\Delta K}{K_{\mathrm{IC}}}, \quad \Pi_2 = \frac{\Delta K_{\mathrm{th}}}{K_{\mathrm{IC}}}, \quad \Pi_3 = \frac{\Delta\sigma_{fl}}{\sigma_y},$$

$$\Pi_4 = \frac{E}{\sigma_y}, \quad \Pi_5 = \frac{\sigma_y^2}{K_{\mathrm{IC}}^2}h, \quad \Pi_6 = \frac{\sigma_{fl}^2}{\Delta K_{\mathrm{th}}^2}a,$$

$$\Pi_7 = r_1 = 1 - R.$$

$$Z = \Pi_5^2$$

Incomplete self-similarity in Π_1, Π_5, Π_6 and Π_7 gives:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \left(\frac{K_{\mathrm{IC}}^{2-\beta_1}}{\sigma_y^2}\right) \Delta K^{\beta_1} \left(\pi \frac{h}{r_p}\right)^{\beta_2} \left(\pi \frac{a}{a_0}\right)^{\beta_3} (1-R)^{\beta_4} \Phi_2(\Pi_2, \Pi_3, \Pi_4)$$

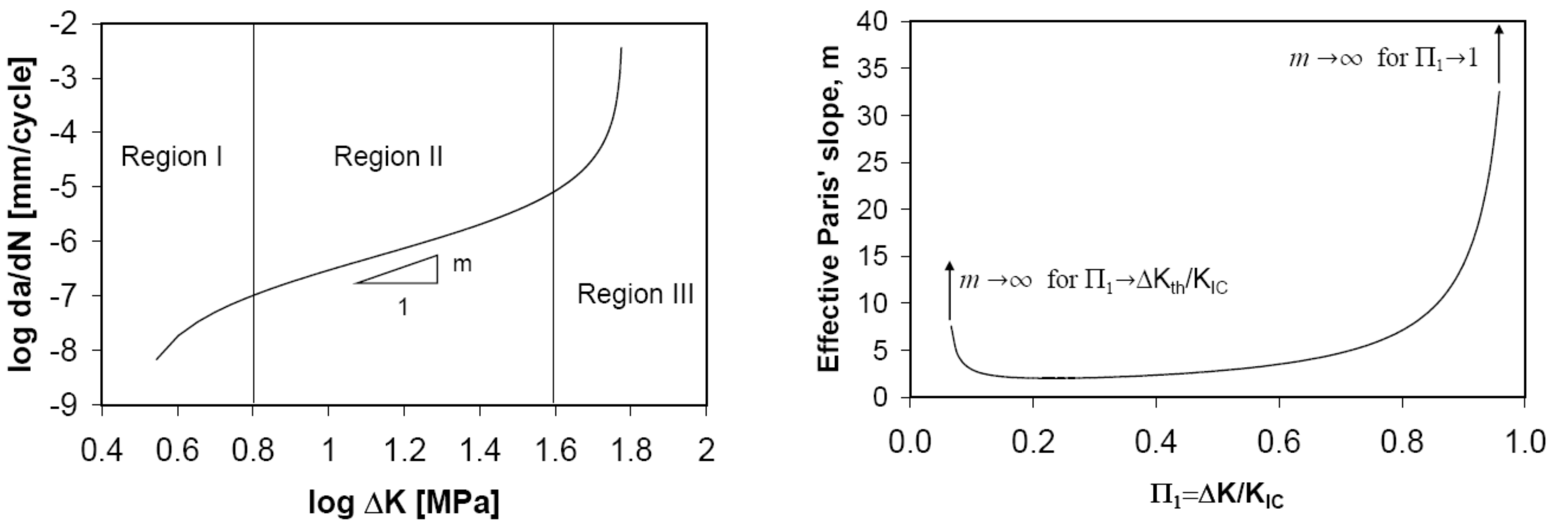
Therefore, the experimentally observed deviations from the simplest power-law regime suggested by Paris is the result of incomplete self-similarity in the various dimensionless numbers:

$$m = \beta_1,$$

$$C = \left(\frac{K_{\mathrm{IC}}^{2-m}}{\sigma_y^2}\right) \left(\pi \frac{h}{r_p}\right)^{\beta_2} \left(\pi \frac{a}{a_0}\right)^{\beta_3} (1-R)^{\beta_4} \Phi_2\left(\Pi_2, \Pi_3, \Pi_4\right).$$

where the exponents β_i can be functions of Π_i.

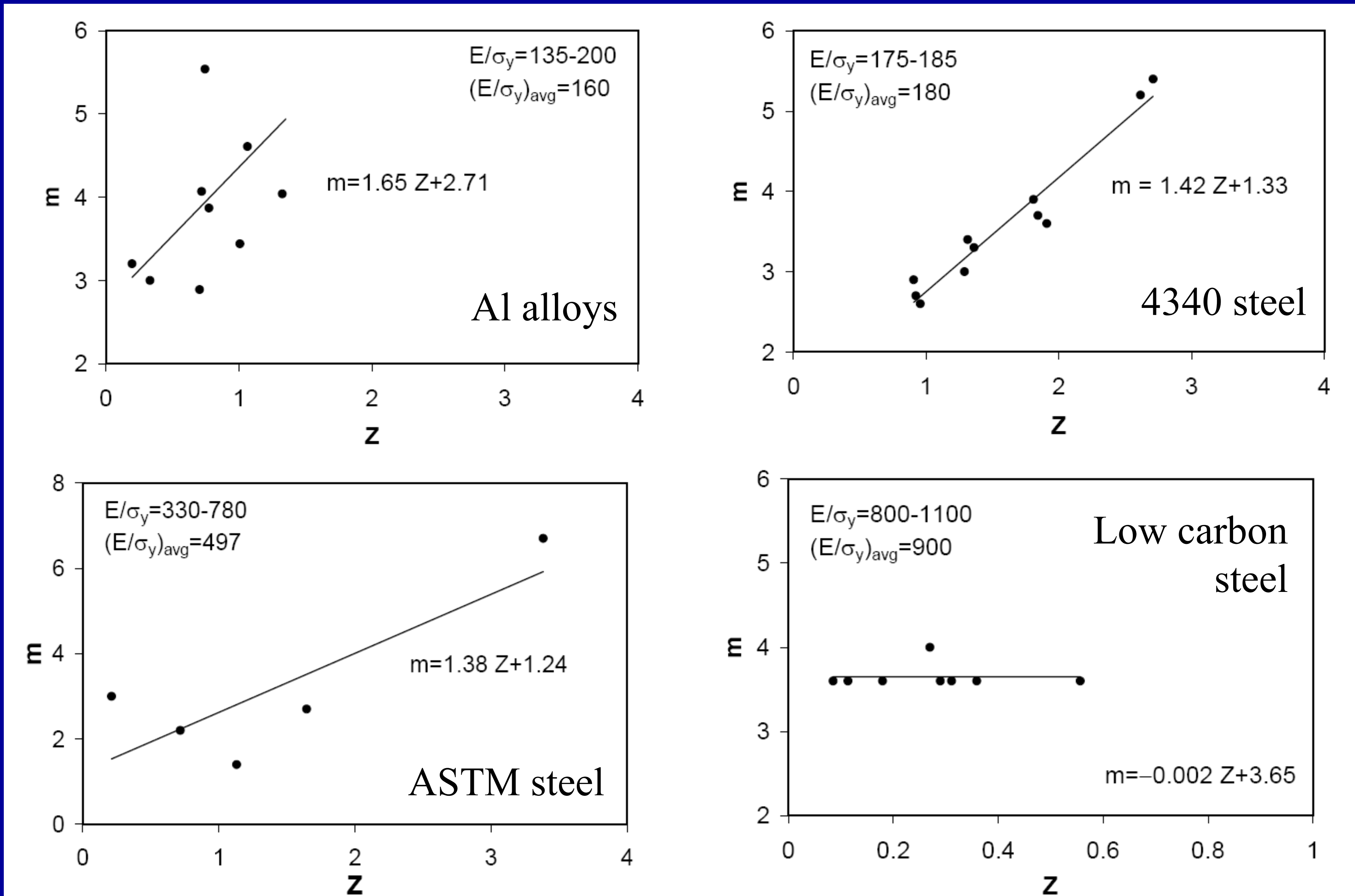
Π_1-dependence of the Paris' law parameter *m*



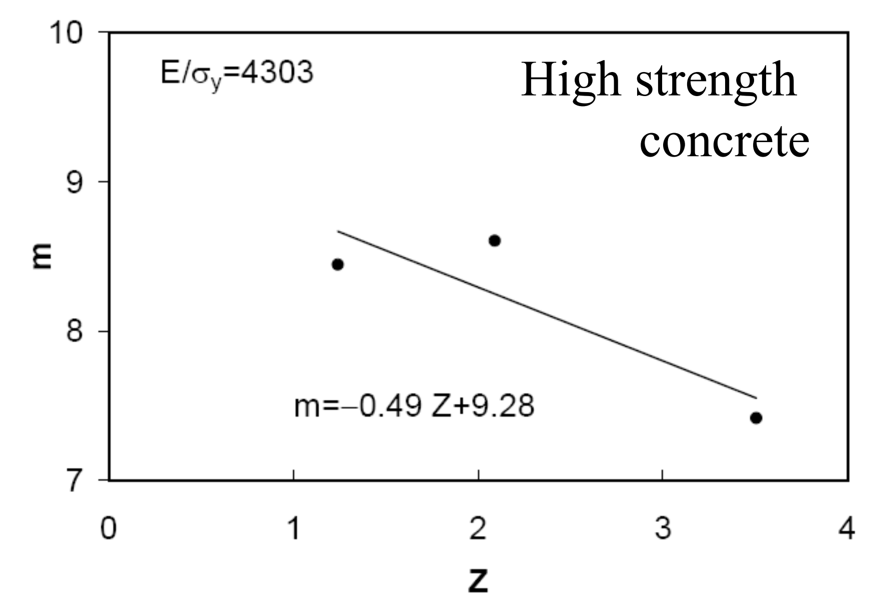
(a) A typical da/dN curve for steel ($R = 0$, $K_{IC} = 60$ MPa$\sqrt{\text{m}}$, $\Delta K_{th} = 3$ MPa$\sqrt{\text{m}}$).

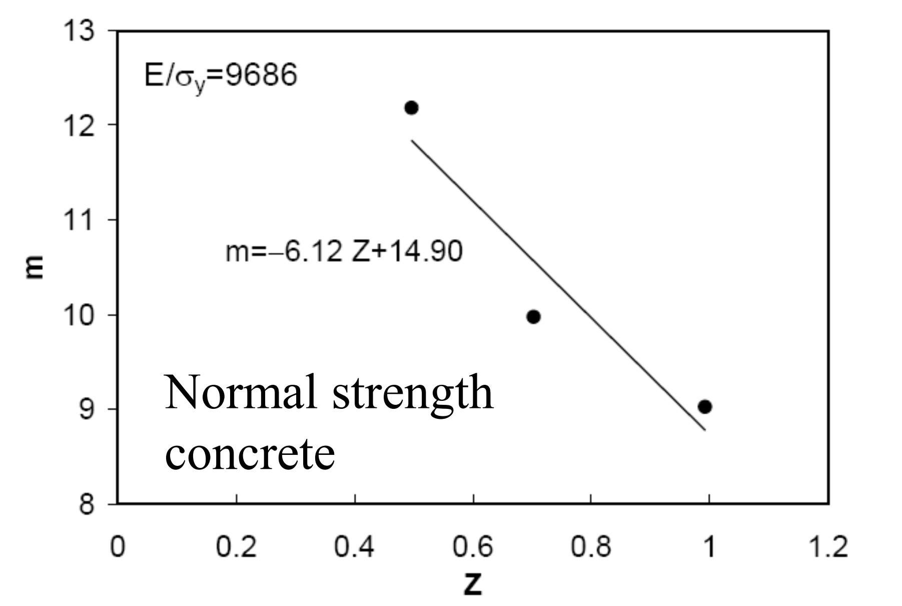
(b) Effective Paris' slope m vs. $\Pi_1 = \Delta K/K_{IC}$ computed from Fig1(a).

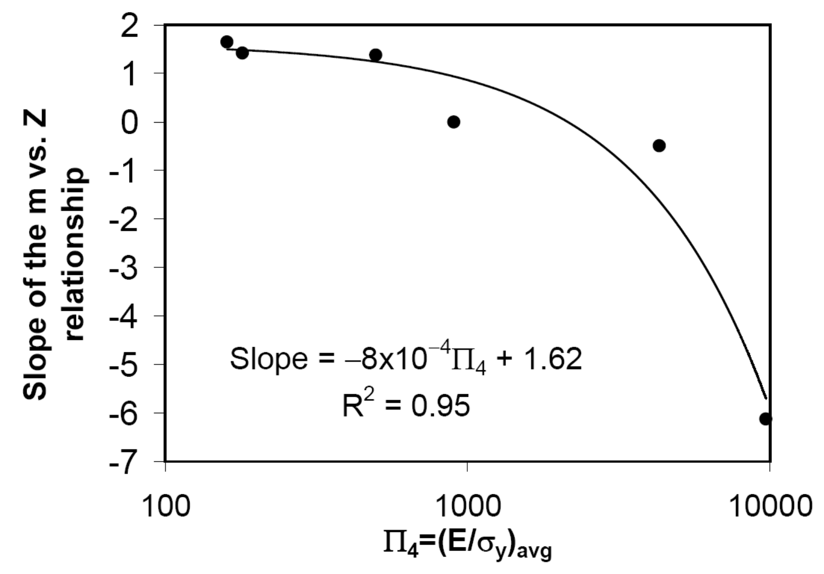
Z-dependence of *m*: ductile materials



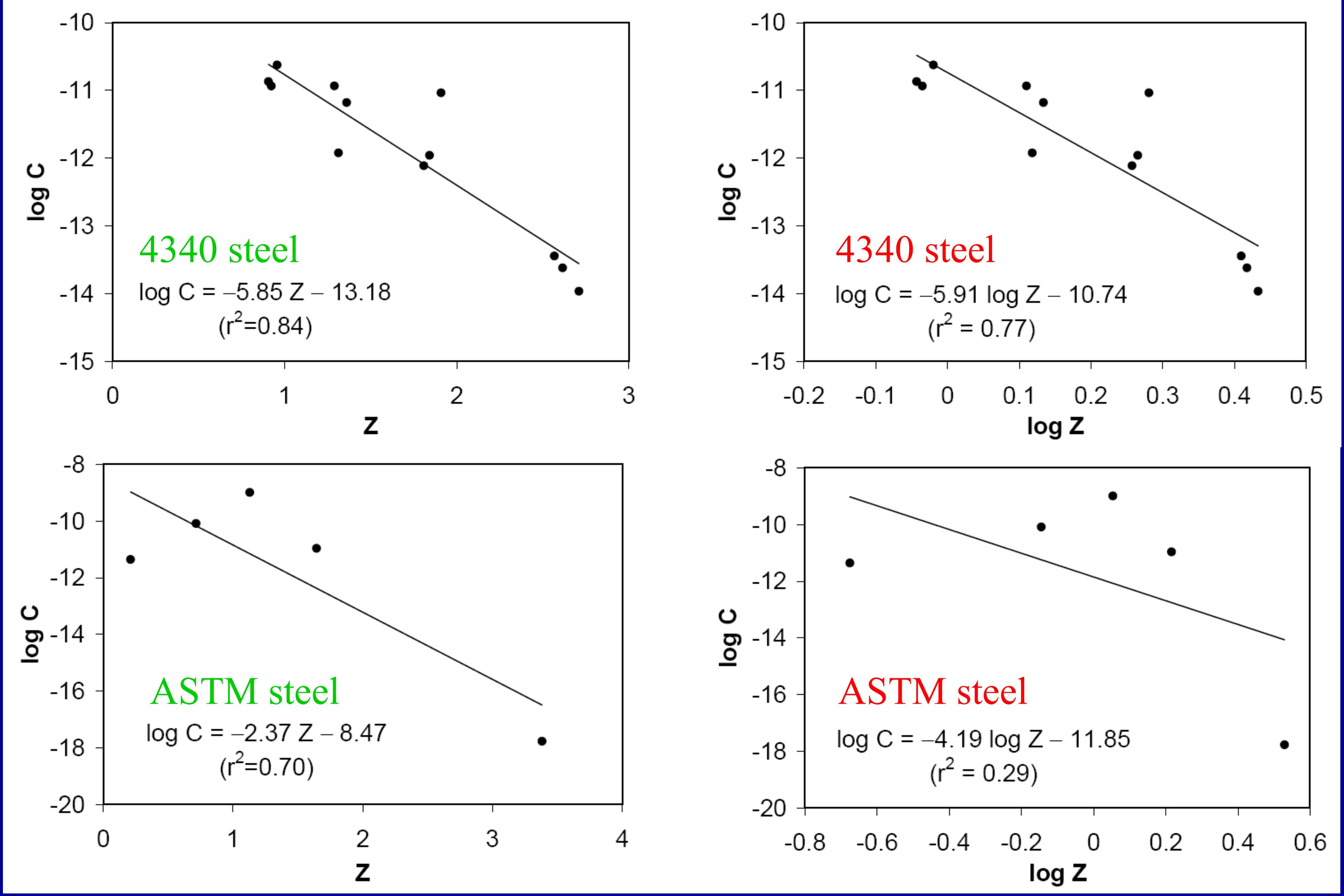
Z-dependence of *m*: quasi-brittle materials







Exponential vs. power-law *Z*-dependence of *C*



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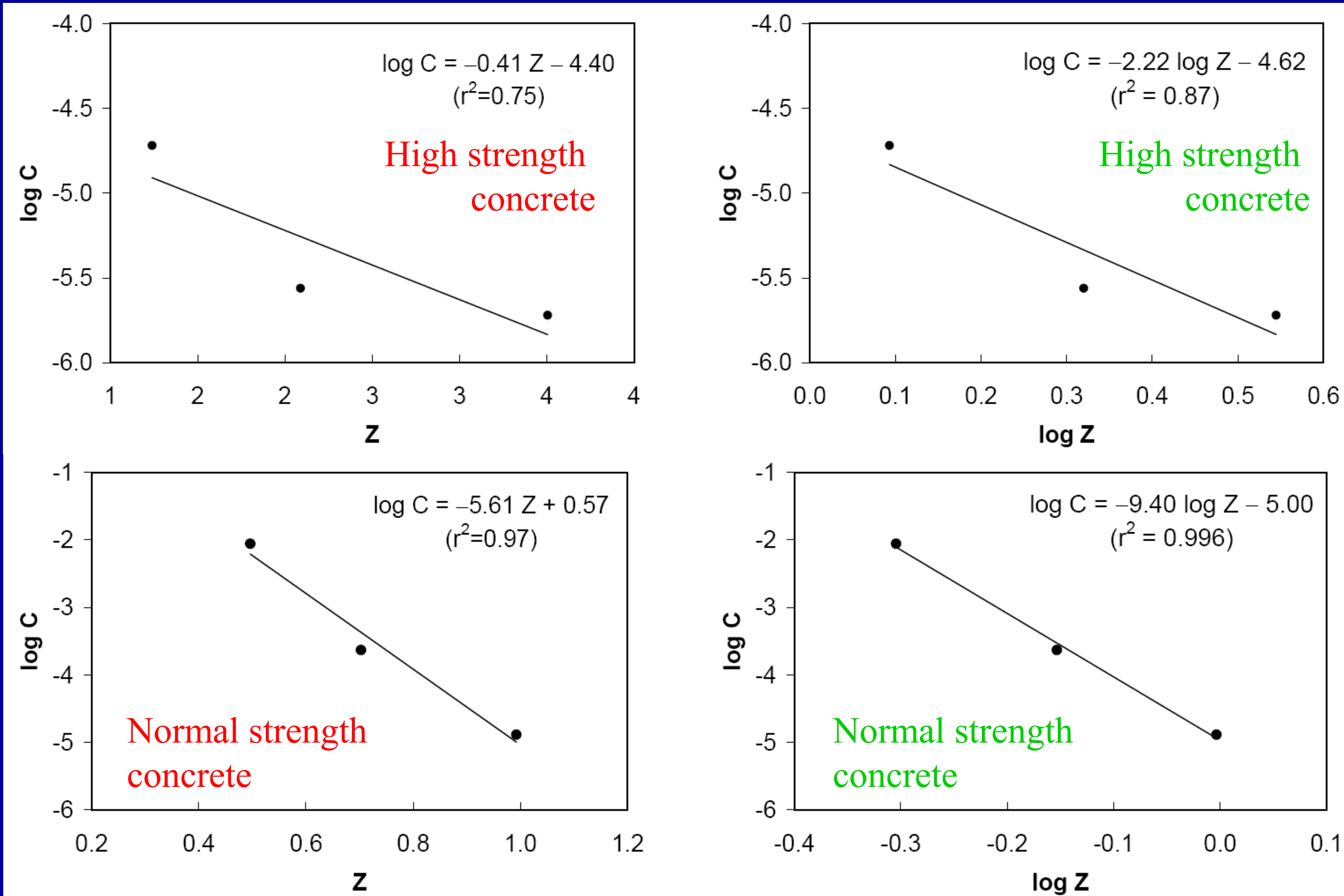
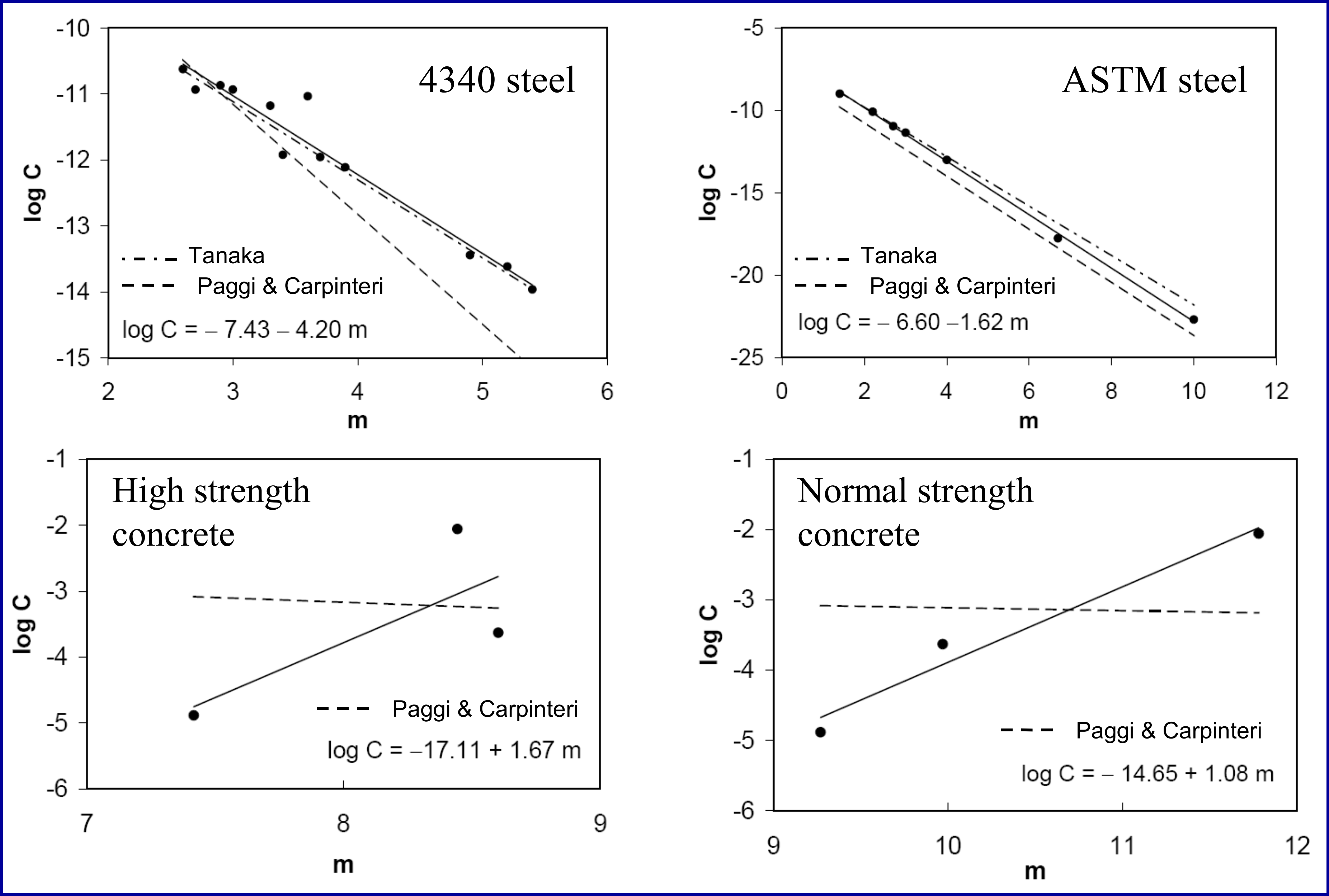


Correlation between *C* and *m*



So what we predict with state-of-the-art numerical methods?

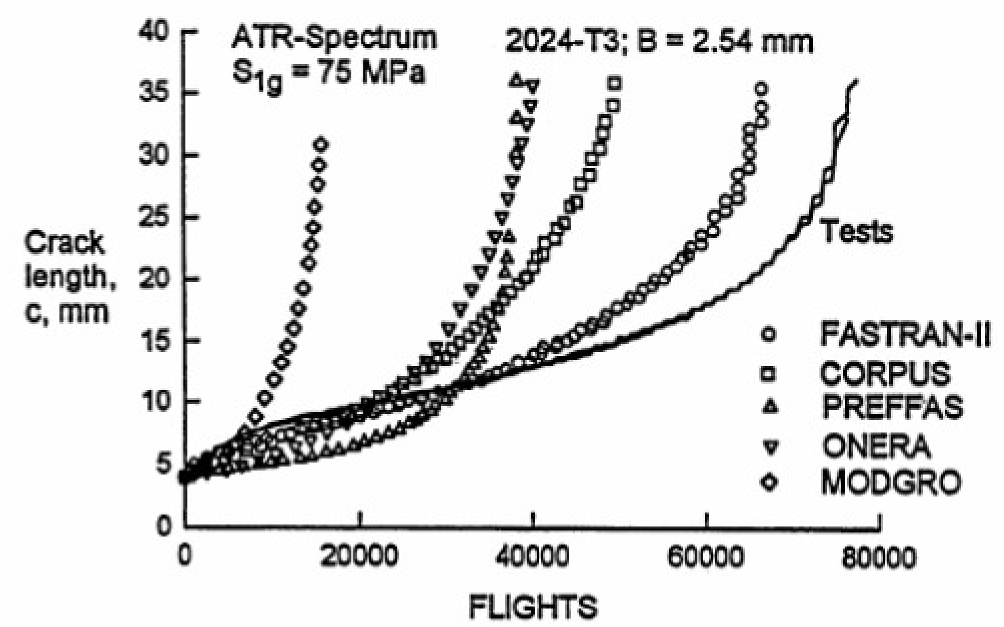


Figure 10. Comparison of fatigue life predictions of various empirical models by Lazzeri, Pieracci and Salvetti (1995); Lazzeri and Salvetti (1996).

Are these "predictions"?

Some free speculations

So what is the way forward?

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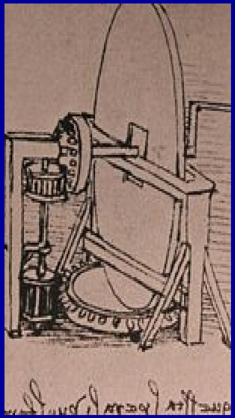
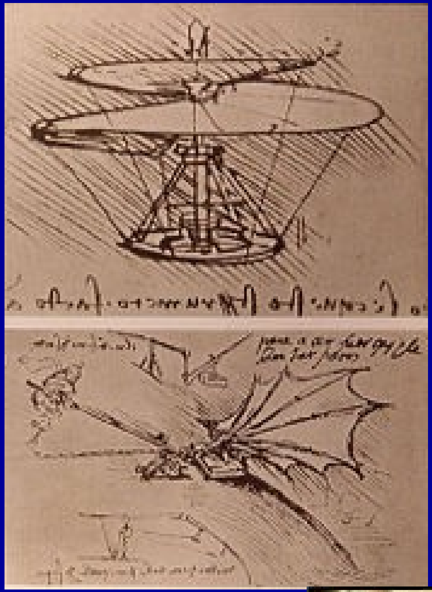
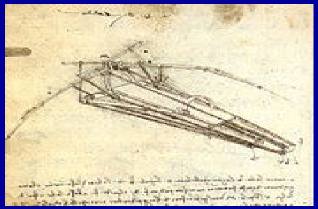
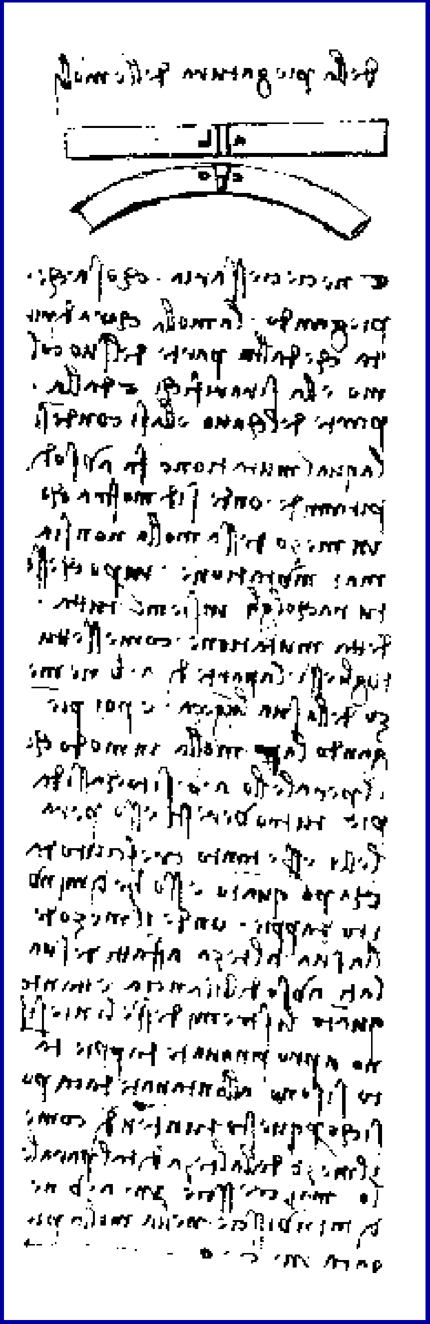
In engineering, how often a pure power law can be the solution?

Remember the lesson of the greatest Engineer of all times....









But LEONARDO da VINCI -- code Madrid Da Vinci's discussion of the deformation of a beam/spring with rectangular cross-section. (Image taken from the book "The Unknown Leonardo," McGraw Hill Co., New York, 1974.)

LEONARDO HAD FOUND BEAM THEORY MUCH BEFORE BERNOULLI, EULER, and THE WRONG THEORY OF GALILEO !!!

For the specific case he considered of a rectangular cross-section, da Vinci argues for equal tensile and compressive strains at the outer fibers, the existence of a neutral surface, and a linear strain distribution. Of course, da Vinci did not have available to him Hooke's law and the calculus. If he did, it is conceivable that he would have derived the formulas listed above, so that beam theory would be referred to as Da Vinci Beam Theory.

Mechanical engineering web 4/18/03 The Da Vinci-Euler-Bernoulli Beam Theory? by Roberto Ballarini
http://www.memagazine.org/contents/current/webonly/webex418.html

LEONARDO da VINCI ALWAYS USED ANALOGIES

"Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion". [Trans. Piomelli in Lumley, J.L., 1997. Some comments on turbulence. *Phys. Fluids A* **4**, 203-211.]

Code Madrid unknown since 1967, and still largely unknown....

Leonardo, Nonlinearity and Integrated Systems

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ABSTRACT

In one of his lesser-known studies of flow, Leonardo da Vinci in 1513 came upon yet another question he could not answer: When blood hits the wall of the heart, does the flow split in two? In 1977, this question was answered by Albert Libchaber in an experiment that became a cornerstone of chaos theory. Can Leonardo's question, Libchaber's solution and notions of integrated systems be drawn together to create a whole? While this trajectory has its limitations, the journey has some rewards, taking in Leonardo's cosmology, chaos theory, poststructuralist philosophy, the Polynesian worldview, the Internet and the weather.