"Pseudo"- Definitionen

$$\overset{\langle 4\rangle}{\mathbb{A}} \longmapsto a_{ijkl} \qquad \overset{\langle 4\rangle}{\mathbb{B}} \longmapsto b_{ijkl} \qquad \Longleftrightarrow \qquad \overset{\langle 4\rangle}{\mathbb{A}} \cdot\cdot \overset{\langle 4\rangle}{\mathbb{B}} \longmapsto a_{ijpq} b_{qpkl} \quad \text{SUMMIEREN nach p,q}$$

$$\overset{\langle 4\rangle}{\mathbb{A}} \longmapsto a_{ijkl} \qquad \mathbb{B} \longmapsto b_{ij} \qquad \Longleftrightarrow \qquad \overset{\langle 4\rangle}{\mathbb{A}} \cdot\cdot \mathbb{B} \longmapsto a_{ijpq} b_{qp} \quad \text{SUMMIEREN nach p,q}$$

$$\mathbb{A} \longmapsto a_{ij} \qquad \mathbb{B} \longmapsto b_{ij} \qquad \Longleftrightarrow \qquad \mathbb{A} \cdot\cdot \mathbb{B} \longmapsto a_{pq} b_{qp} \quad \text{SUMMIEREN nach p,q}$$

$$\overset{\langle 4\rangle}{\mathbb{E}}_s \longmapsto \frac{1}{2}(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl})$$

Symmetrierer Einheit für Doppelskalarmultiplikation vollständig symmetrischer Tensoren 4. Stufe, d.h. $a_{ijkl} = a_{jikl} = a_{ijlk} = a_{klij}$

Der Deviatorerzeuger

$$\mathbb{1}' = \overset{\langle 4\rangle}{\mathbb{E}}_s - \frac{1}{3}\mathbb{E} \circ \mathbb{E} \qquad \Rightarrow \qquad 1'_{ijkl} = \frac{1}{2}(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}) - \frac{1}{3}\delta_{ij}\delta_{kl}$$

Der Kugelenteil-Erzeuger

$$\mathbb{1}'' = \frac{1}{3}\mathbb{E} \circ \mathbb{E} \qquad \Rightarrow \qquad 1''_{ijkl} = \frac{1}{3}\delta_{ij}\delta_{kl}$$

$\overset{\langle 4\rangle}{\mathbb{E}}_s = \mathbb{1}' + \mathbb{1}'' \longleftrightarrow \frac{1}{2}(\delta_{il}\delta_{jk} + \delta_{ik}\delta$

Wichtige Eigenschaften

$$\mathbb{1}' \cdot\cdot \mathbb{1}' = \mathbb{1}' \qquad \mathbb{1}'' \cdot\cdot \mathbb{1}'' = \mathbb{1}'' \qquad \mathbb{1}' \cdot\cdot \mathbb{1}'' = \overset{\langle 4\rangle}{\mathbb{0}} \qquad \mathbb{1}' + \mathbb{1}'' = \overset{\langle 4\rangle}{\mathbb{E}}_s$$

Deviator eines Tensors 2. Stufe

$$\mathbb{A} \longmapsto a_{ij}, \qquad \mathbb{A}' = \mathbb{1}' \cdot\cdot \mathbb{A} \longmapsto a'_{ij} = a_{ij} - \frac{a_{pp}}{3}\delta_{ij}$$

Kugelanteil eines Tensors 2. Stufe

$$\mathbb{A} \longmapsto a_{ij}, \qquad \mathbb{A}'' = \mathbb{1}'' \cdot\cdot \mathbb{A} \longmapsto a''_{ij} = \frac{a_{pp}}{3}\delta_{ij}$$

vgl.: R. TROSTEL Me V, Me VII, 1

Maxwell-Körper

$$\dot{\mathbb{D}}_M = \overset{\langle 4\rangle}{\mathbb{S}}_{Me} \cdot\cdot\, \dot{\mathbb{S}}_M + \overset{\langle 4\rangle}{\mathbb{S}}_{Mv} \cdot\cdot\, \mathbb{S}_M \qquad \text{mit} \quad \overset{\langle 4\rangle}{\mathbb{S}}_{Me} = \frac{1}{2G_{Me}}\,\mathbb{1}' + \frac{1}{3K_{Me}}\,\mathbb{1}'' ;\; K_{Me} = \frac{2(1+\nu_{Me})G_{Me}}{3(1-2\nu_{Me})}$$

$$\text{und} \quad \overset{\langle 4\rangle}{\mathbb{S}}_{Mv} = \frac{1}{2\mu_{Mv}}\,\mathbb{1}' + \frac{1}{3K_{Mv}}\,\mathbb{1}'' ;\; K_{Mv} = \frac{2(1+\nu_{Mv})\mu_{Mv}}{3(1-2\nu_{Mv})}$$

im isotropen Fall

Elastischer Körper

$$\mathbb{D}_E = \overset{\langle 4\rangle}{\mathbb{S}}_E \cdot\cdot\, \mathbb{S}_E \qquad \text{mit} \quad \overset{\langle 4\rangle}{\mathbb{S}}_E = \frac{1}{2G_E}\,\mathbb{1}' + \frac{1}{3K_E}\,\mathbb{1}'' ;\; K_E = \frac{2(1+\nu_E)G_E}{3(1-2\nu_E)}$$

oder

$$\mathbb{S}_E = \overset{\langle 4\rangle}{\mathbb{C}}_E \cdot\cdot\, \mathbb{D}_E \qquad \text{mit} \quad \overset{\langle 4\rangle}{\mathbb{C}}_E = 2G_E\,\mathbb{1}' + 3K_E\,\mathbb{1}'' = \overset{\langle 4\rangle}{\mathbb{S}}_E{}^{-1}$$

im isotropen Fall

PARALLEL-Schaltung : [illegible]

$$\Downarrow$$

$$\mathbb{D}_E = \mathbb{D}_M = \mathbb{D} \qquad \text{und } \mathbb{S} = \mathbb{S}_M + \mathbb{S}_E \quad \Rightarrow \quad \mathbb{S}_M = \mathbb{S} - \mathbb{S}_E$$

$$\Downarrow$$

$$\dot{\mathbb{D}} = \overset{\langle 4\rangle}{\mathbb{S}}_{Me} \cdot\cdot\, (\mathbb{S} - \mathbb{S}_E)^{\cdot} + \overset{\langle 4\rangle}{\mathbb{S}}_{Mv} \cdot\cdot\, (\mathbb{S} - \mathbb{S}_E) =$$

$$= \overset{\langle 4\rangle}{\mathbb{S}}_{Me} \cdot\cdot \left[\mathbb{S} - \overset{\langle 4\rangle}{\mathbb{C}}_E \cdot\cdot\, \mathbb{D} \right]^{\cdot} + \overset{\langle 4\rangle}{\mathbb{S}}_{Mv} \cdot\cdot \left[\mathbb{S} - \overset{\langle 4\rangle}{\mathbb{C}}_E \cdot\cdot\, \mathbb{D} \right]$$

$$\Downarrow$$

Anisotropes Materialgesetz (symbolische Schreibweise)

$$\left[\overset{\langle 4\rangle}{\mathbb{E}}_s + \overset{\langle 4\rangle}{\mathbb{S}}_{Me} \cdot\cdot\, \overset{\langle 4\rangle}{\mathbb{C}}_E \right] \cdot\cdot\, \dot{\mathbb{D}} + \overset{\langle 4\rangle}{\mathbb{S}}_{Mv} \cdot\cdot\, \overset{\langle 4\rangle}{\mathbb{C}}_E \cdot\cdot\, \mathbb{D} = \overset{\langle 4\rangle}{\mathbb{S}}_{Me} \cdot\cdot\, \dot{\mathbb{S}} + \overset{\langle 4\rangle}{\mathbb{S}}_{Mv} \cdot\cdot\, \mathbb{S}$$

Isotropes Materialgesetz

Berechnung der auftretenden Koefizienten

$$\overset{\langle 4\rangle}{\mathbb{E}}_s + \overset{\langle 4\rangle}{\mathbb{S}}_{Me} \cdot\cdot \overset{\langle 4\rangle}{\mathbb{C}}_E = \mathbb{1}' + \mathbb{1}'' + \left[\frac{1}{2G_{Me}} \mathbb{1}' + \frac{1}{3K_{Me}} \mathbb{1}'' \right] \cdot\cdot \left[2G_E \mathbb{1}' + 3K_E \mathbb{1}'' \right] =$$

$$\left[1 + \frac{G_E}{G_{Me}} \right] \mathbb{1}' + \left[1 + \frac{K_E}{K_{Me}} \right] \mathbb{1}''$$

$$\overset{\langle 4\rangle}{\mathbb{S}}_{Mv} \cdot\cdot \overset{\langle 4\rangle}{\mathbb{C}}_E = \left[\frac{1}{2\mu_{Mv}} \mathbb{1}' + \frac{1}{3K_{Mv}} \mathbb{1}'' \right] \cdot\cdot \left[2G_E \mathbb{1}' + 3K_E \mathbb{1}'' \right] = \frac{G_E}{\mu_{Mv}} \mathbb{1}' + \frac{K_E}{K_{Mv}} \mathbb{1}''$$

Isotropes Materialgesetz (symbolische Schreibweise)

$$\left[\left[1 + \frac{G_E}{G_{Me}} \right] \mathbb{1}' + \left[1 + \frac{K_E}{K_{Me}} \right] \mathbb{1}'' \right] \cdot\cdot \dot{\mathbb{D}} + \left[\frac{G_E}{\mu_{Mv}} \mathbb{1}' + \frac{K_E}{K_{Mv}} \mathbb{1}'' \right] \cdot\cdot \mathbb{D} =$$

$$\left[\frac{1}{2G_{Me}} \mathbb{1}' + \frac{1}{3K_{Me}} \mathbb{1}'' \right] \cdot\cdot \dot{\mathbb{S}} + \left[\frac{1}{2\mu_{Mv}} \mathbb{1}' + \frac{1}{3K_{Mv}} \mathbb{1}'' \right] \cdot\cdot \mathbb{S}$$

Isotropes Materialgesetz (indizistische Schreibweise)

$$\left[1 + \frac{G_E}{G_{Me}} \right] \left[\dot{\epsilon}_{ij} - \frac{\dot{\epsilon}_{pp}}{3} \delta_{ij} \right] + \left[1 + \frac{K_E}{K_{Me}} \right] \frac{\dot{\epsilon}_{pp}}{3} \delta_{ij} + \frac{G_E}{\mu_{Mv}} \left[\epsilon_{ij} - \frac{\epsilon_{pp}}{3} \delta_{ij} \right] + \frac{K_E}{K_{Mv}} \frac{\epsilon_{pp}}{3} \delta_{ij} =$$

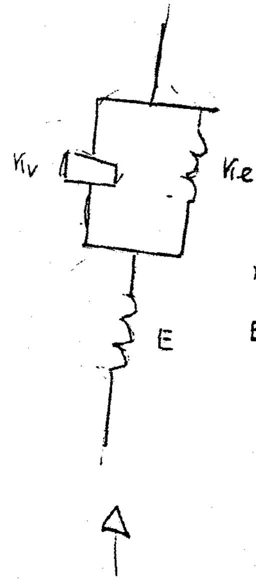
$$= \frac{1}{2G_{Me}} \left[\dot{\sigma}_{ij} - \frac{\dot{\sigma}_{pp}}{3} \delta_{ij} \right] + \frac{1}{3K_{Me}} \frac{\dot{\sigma}_{pp}}{3} \delta_{ij} + \frac{1}{2\mu_{Mv}} \left[\sigma_{ij} - \frac{\sigma_{pp}}{3} \delta_{ij} \right] + \frac{1}{3K_{Mv}} \frac{\sigma_{pp}}{3} \delta_{ij}$$

oder

$$\left[1 + \frac{G_E}{G_{Me}} \right] \dot{\epsilon}_{ij} + \left[\frac{K_E}{K_{Me}} - \frac{G_E}{G_{Me}} \right] \frac{\dot{\epsilon}_{pp}}{3} \delta_{ij} + \frac{G_E}{\mu_{Mv}} \epsilon_{ij} + \left[\frac{K_E}{K_{Mv}} - \frac{G_E}{\mu_{Mv}} \right] \frac{\epsilon_{pp}}{3} \delta_{ij} =$$

Fehler

$$= \frac{1}{2G_{Me}} \dot{\sigma}_{ij} + \left[\frac{1}{3K_{Me}} - \frac{1}{2G_{Me}} \right] \frac{\dot{\sigma}_{pp}}{3} \delta_{ij} + \frac{1}{2\mu_{Mv}} \sigma_{ij} + \left[\frac{1}{3K_{Mv}} - \frac{1}{2\mu_{Mv}} \right] \frac{\sigma_{pp}}{3} \delta_{ij}$$



Indizes
E, Ke, Kv

Kelvin Körper

$$\mathbb{S}_K = \overset{\langle 4\rangle}{\mathbb{C}}_{Ke} \cdot\cdot\, \mathbb{D}_K + \overset{\langle 4\rangle}{\mathbb{C}}_{Kv} \cdot\cdot\, \dot{\mathbb{D}}_K \qquad \text{mit}$$

$$\overset{\langle 4\rangle}{\mathbb{C}}_{Ke} = 2G_{Ke}\mathbb{1}' + 3K_{Ke}\mathbb{1}'' \qquad \overset{\langle 4\rangle}{\mathbb{C}}_{Kv} = 2\mu_{Kv}\mathbb{1}' + 3K_{Kv}\mathbb{1}''$$

im isotropen Falle

Hintereinanderschaltung mit einer elastischen Komponente. = Standard-Körper

$$\mathbb{S}_K = \mathbb{S}_E = \mathbb{S} \quad \mathbb{D} = \mathbb{D}_E + \mathbb{D}_K \quad \text{für } \mathbb{D}_E \text{ s.o.}$$

$$\mathbb{D}_K = \mathbb{D} - \mathbb{D}_E$$

Einsetzen in die Materialgleichung d. Kelvin Körpers.

Materialgleichung für den anisotropern Fall

$$\mathbb{S} = \overset{\langle 4\rangle}{\mathbb{C}}_{Ke} \cdot\cdot\, (\mathbb{D} - \mathbb{D}_E) + \overset{\langle 4\rangle}{\mathbb{C}}_{Kv} \cdot\cdot\, (\mathbb{D} - \mathbb{D}_E)^{\cdot} =$$
$$= \overset{\langle 4\rangle}{\mathbb{C}}_{Ke} \cdot\cdot \left[\mathbb{D} - \overset{\langle 4\rangle}{\mathbb{S}}_E \cdot\cdot\, \mathbb{S} \right] + \overset{\langle 4\rangle}{\mathbb{C}}_{Kv} \cdot\cdot \left[\mathbb{D} - \overset{\langle 4\rangle}{\mathbb{S}}_E \cdot\cdot\, \mathbb{S} \right]^{\cdot}$$

$$\left[\overset{\langle 4\rangle}{\mathbb{E}}_s + \overset{\langle 4\rangle}{\mathbb{C}}_{Ke} \cdot\cdot\, \overset{\langle 4\rangle}{\mathbb{S}}_E \right] \cdot\cdot\, \mathbb{S} + \overset{\langle 4\rangle}{\mathbb{C}}_{Kv} \cdot\cdot\, \overset{\langle 4\rangle}{\mathbb{S}}_E \cdot\cdot\, \dot{\mathbb{S}} = \overset{\langle 4\rangle}{\mathbb{C}}_{Ke} \cdot\cdot\, \mathbb{D} + \overset{\langle 4\rangle}{\mathbb{C}}_{Kv} \cdot\cdot\, \dot{\mathbb{D}}$$

Isotropes Materialgesetz; man setze isotrope Materialtensoren ein

Berechnung der Koefizienten

$$\overset{\langle 4\rangle}{\mathbb{E}}_s + \overset{\langle 4\rangle}{\mathbb{C}}_{Ke} \cdot\cdot\, \overset{\langle 4\rangle}{\mathbb{S}}_E = \mathbb{1}' + \mathbb{1}'' + \left[2G_{Ke}\mathbb{1}' + 3K_{Ke}\mathbb{1}'' \right] \cdot\cdot \left[\frac{1}{2G_E}\mathbb{1}' + \frac{1}{3K_E}\mathbb{1}'' \right] =$$
$$= \left[1 + \frac{G_{Ke}}{G_E} \right] \mathbb{1}' + \left[1 + \frac{K_{Ke}}{K_E} \right] \mathbb{1}''$$

$$\overset{\langle 4\rangle}{\mathbb{C}}_{Kv} \cdot\cdot\, \overset{\langle 4\rangle}{\mathbb{S}}_E = \left[2\mu_{Kv}\mathbb{1}' + 3K_{Kv}\mathbb{1}'' \right] \cdot\cdot \left[\frac{1}{2G_E}\mathbb{1}' + \frac{1}{3K_E}\mathbb{1}'' \right] =$$
$$= \frac{\mu_{Kv}}{G_E}\mathbb{1}' + \frac{K_{Kv}}{K_E}\mathbb{1}''$$

$$\left[\left[1 + \frac{G_{Ke}}{G_E} \right] \mathbb{1}' + \left[1 + \frac{K_{Ke}}{K_E} \right] \mathbb{1}'' \right] \cdot\cdot\, \mathbb{S} + \left[\frac{\mu_{Kv}}{G_E} \mathbb{1}' + \frac{K_{Kv}}{K_E} \mathbb{1}'' \right] \cdot\cdot\, \dot{\mathbb{S}} =$$

$$= \left[2G_{Ke}\mathbb{1}' + 3K_{Ke}\mathbb{1}'' \right] \cdot\cdot\, \mathbb{D} + \left[2\mu_{Kv}\mathbb{1}' + 3K_{Kv}\mathbb{1}'' \right] \cdot\cdot\, \dot{\mathbb{D}}$$

Isotropes Materialgesetz (indizistischer Schreibweise)

(man setze die Deviatoren und Kugel-Anteile ein)

$$\left[1 + \frac{G_{Ke}}{G_E} \right] \left[\sigma_{ij} - \frac{\sigma_{pp}}{3} \delta_{ij} \right] + \left[1 + \frac{K_{Ke}}{K_E} \right] \frac{\sigma_{pp}}{3} \delta_{ij} + \frac{\mu_{Kv}}{G_E} \left[\dot{\sigma}_{ij} - \frac{\dot{\sigma}_{pp}}{3} \delta_{ij} \right] +$$

$$+ \frac{K_{Kv}}{K_E} \frac{\dot{\sigma}_{pp}}{3} \delta_{ij} =$$

$$= 2G_{Ke} \left[\epsilon_{ij} - \frac{\epsilon_{pp}}{3} \delta_{ij} \right] + 3K_{Ke} \frac{\epsilon_{pp}}{3} \delta_{ij} + 2\mu_{Kv} \left[\dot{\epsilon}_{ij} - \frac{\dot{\epsilon}_{pp}}{3} \delta_{ij} \right] + 3K_{Kv} \frac{\dot{\epsilon}_{pp}}{3} \delta_{ij}$$

oder

$$\left[1 + \frac{G_{Ke}}{G_E} \right] \sigma_{ij} + \left[\frac{K_{Ke}}{K_E} - \frac{G_{Ke}}{G_E} \right] \frac{\sigma_{pp}}{3} \delta_{ij} + \frac{\mu_{Kv}}{G_E} \dot{\sigma}_{ij} + \left[\frac{K_{Kv}}{K_E} - \frac{\mu_{Kv}}{G_E} \right] \frac{\dot{\sigma}_{pp}}{3} \delta_{ij} =$$

$$= 2G_{Ke}\epsilon_{ij} + (3K_{Ke} - 2G_{Ke}) \frac{\epsilon_{pp}}{3} \delta_{ij} + 2\mu_{Kv} \dot{\epsilon}_{ij} + (3K_{Kv} - 2\mu_{Kv}) \frac{\dot{\epsilon}_{pp}}{3} \delta_{ij}$$

Standard-Körper