Stress Analysis of Bullet-Holes on the Boeing 737 Fuselage

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Date: January 11th, 2008

ES 240 Project

Overview

- Introduction / Objectives
- Variable / Parameter Definitions
- Case 1: Single Bullet Hole
- Case 2: Two Bullet Holes
- Case 3: Single Bullet Hole Near Window
- Case 4: Two Bullet Holes Near Window
- Case 5: Two Holes Approaching Each Other
- Extra: Fracture due to Crack-Propagation

Introduction / Objectives

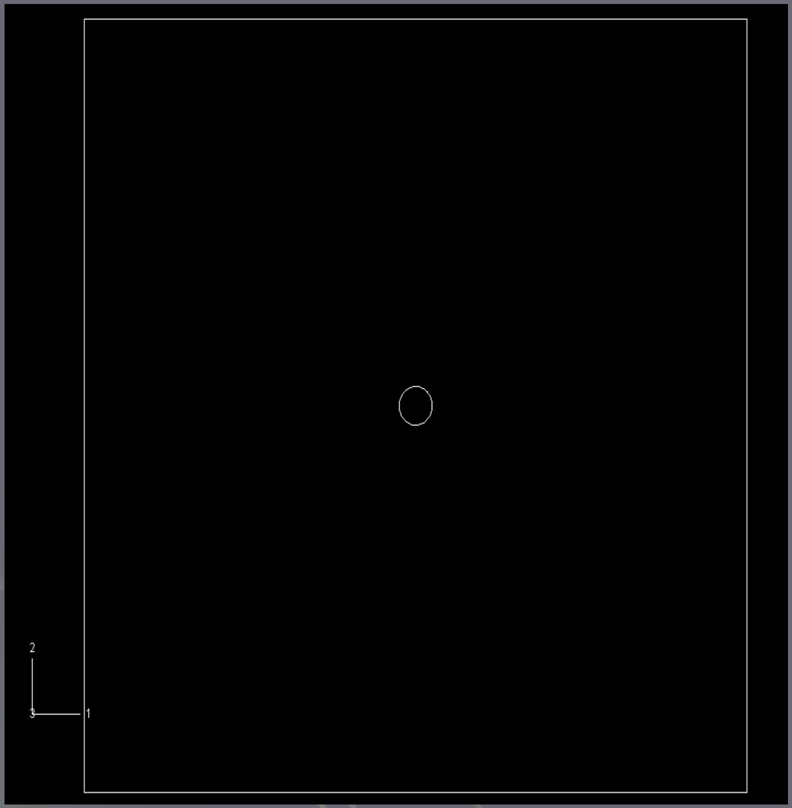
- Introduction:
  - How we came about doing this project
  - Why this analysis is important
  - How is this applicable to real-life engineering
- Objectives
  - Go through various case examples
  - Use ABAQUS CAE to model and view stress, analyze more complex scenarios
  - Compare results with theoretical calculations

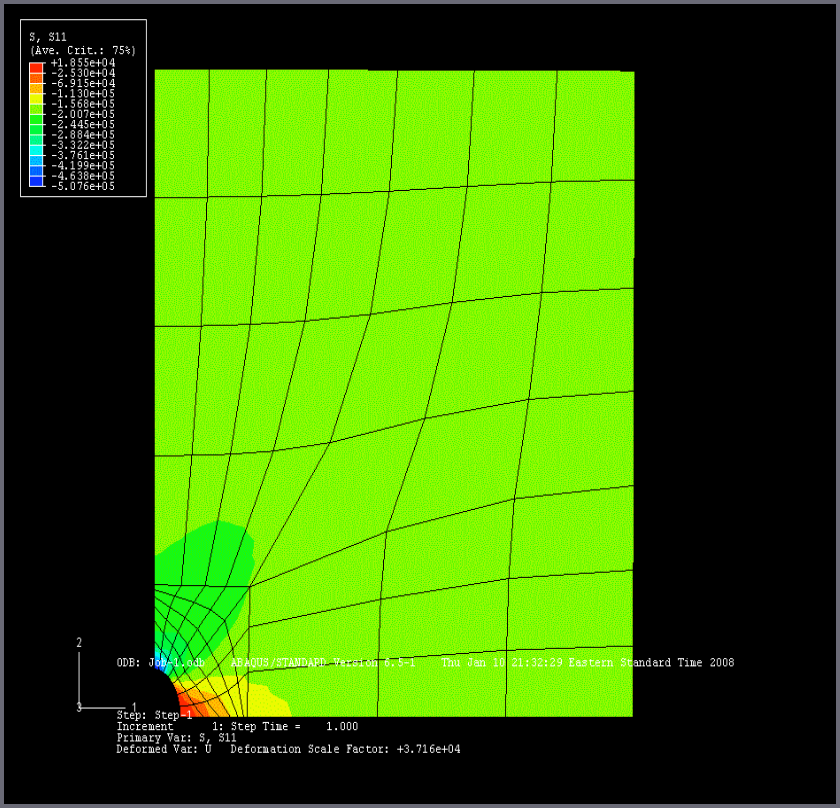
Variable / Parameter Definitions

- Material Constants:
  Aluminum 7079-T6
  - Young's Modulus:
    E = **71.7 GPa**
  - Poisson's Ratio:
    V = **0.33**
  - Density: **2740 kg/m^3**
- Physical Dimensions
  - Dimension of Fuselage:
    Radius = **5 m**
    Thickness = **0.2 m**
  - Dimension of Bullet Hole
    Diameter = **1**
  - Dimension of Window
    20 x 10

- General Assumptions and Simple Calculations:
  Uniaxial Load in Compression: **0.19 MPa**

Case 1: Single Bullet Hole





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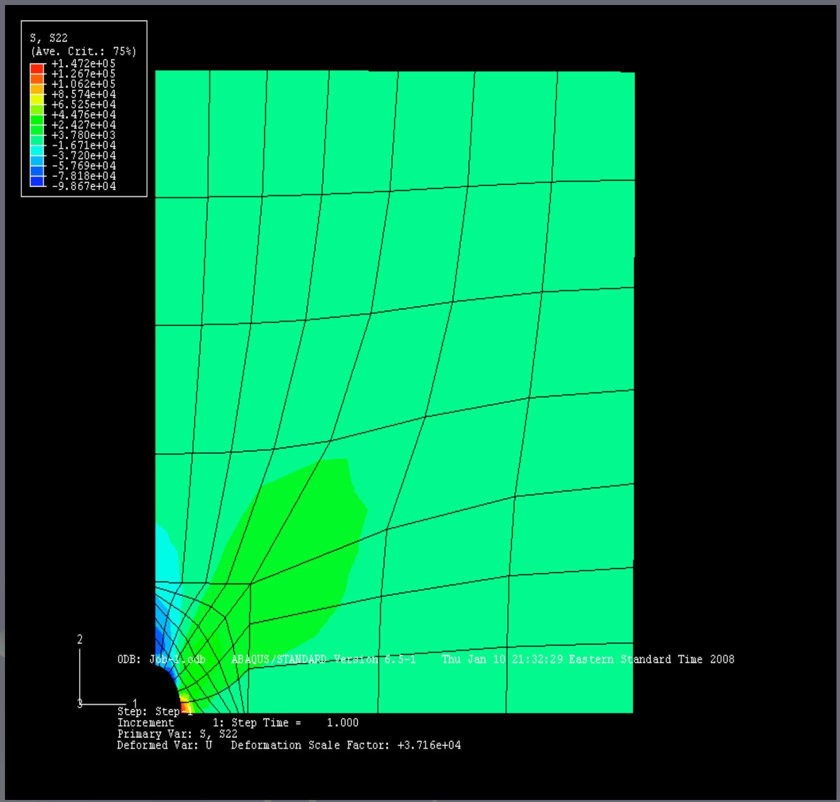
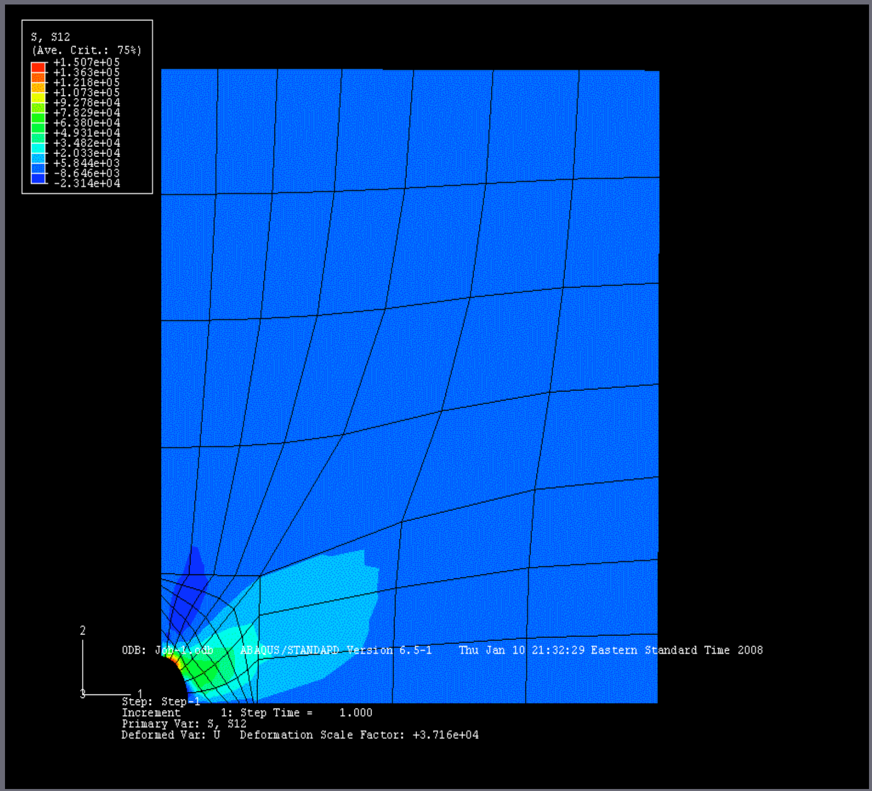
Case 1: ABAQUS CAE Analysis





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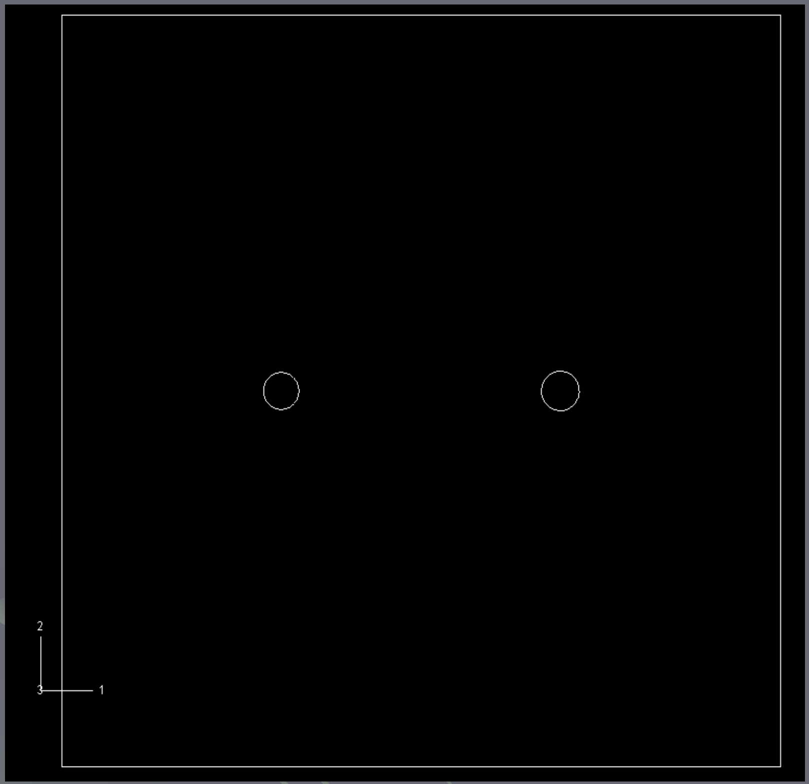
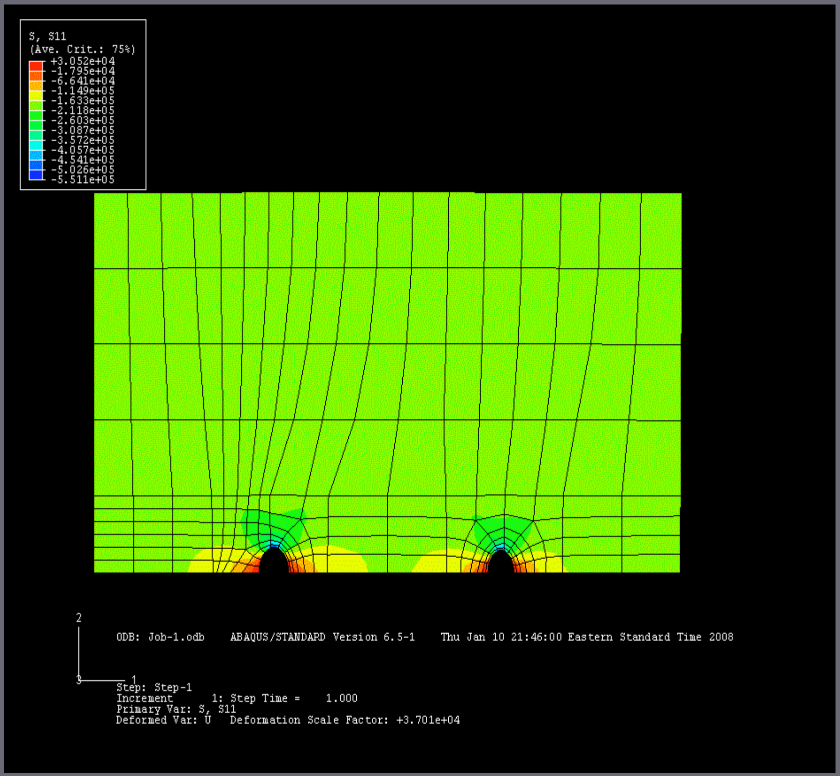
Case 2: Two Bullet Holes





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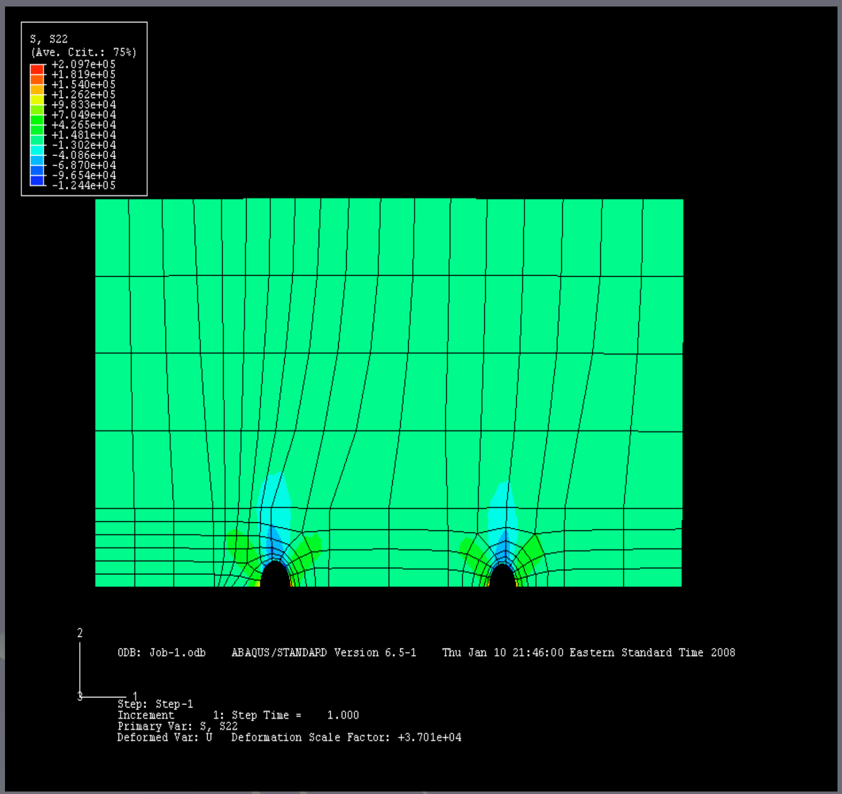

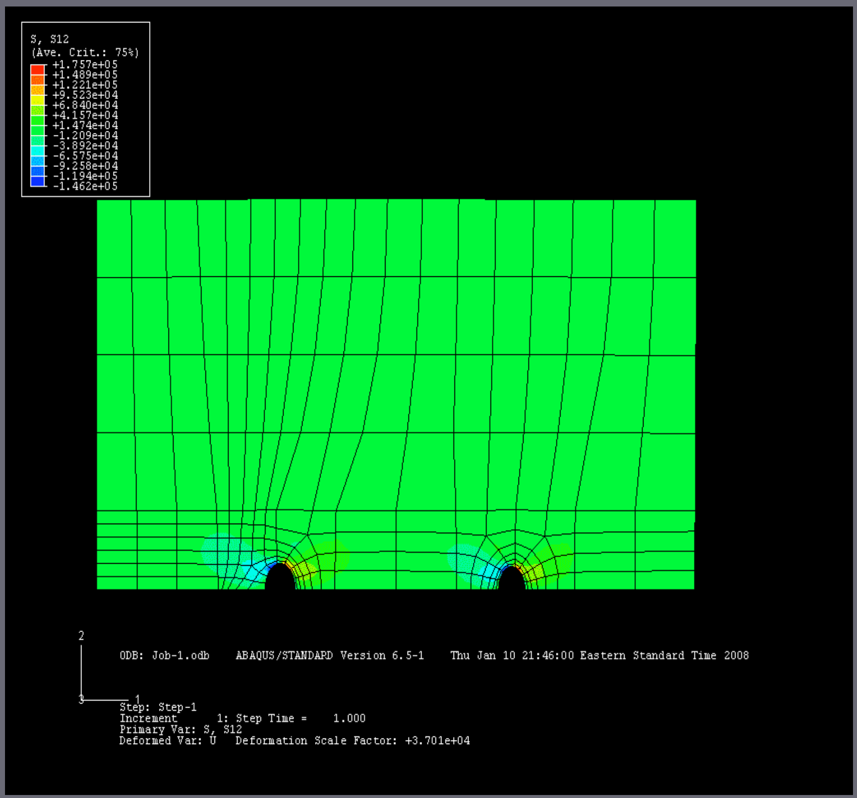

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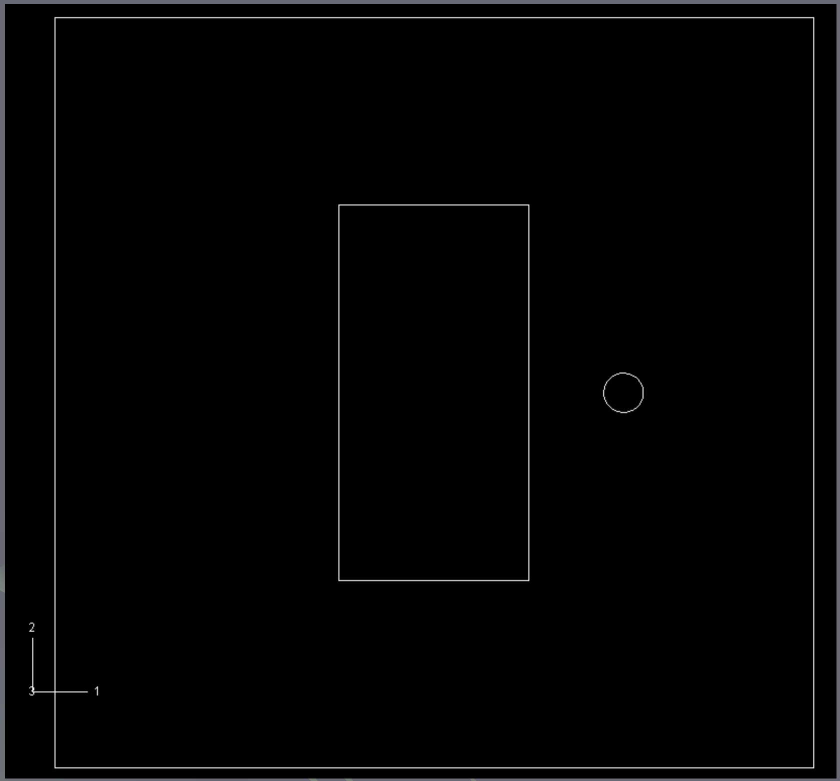
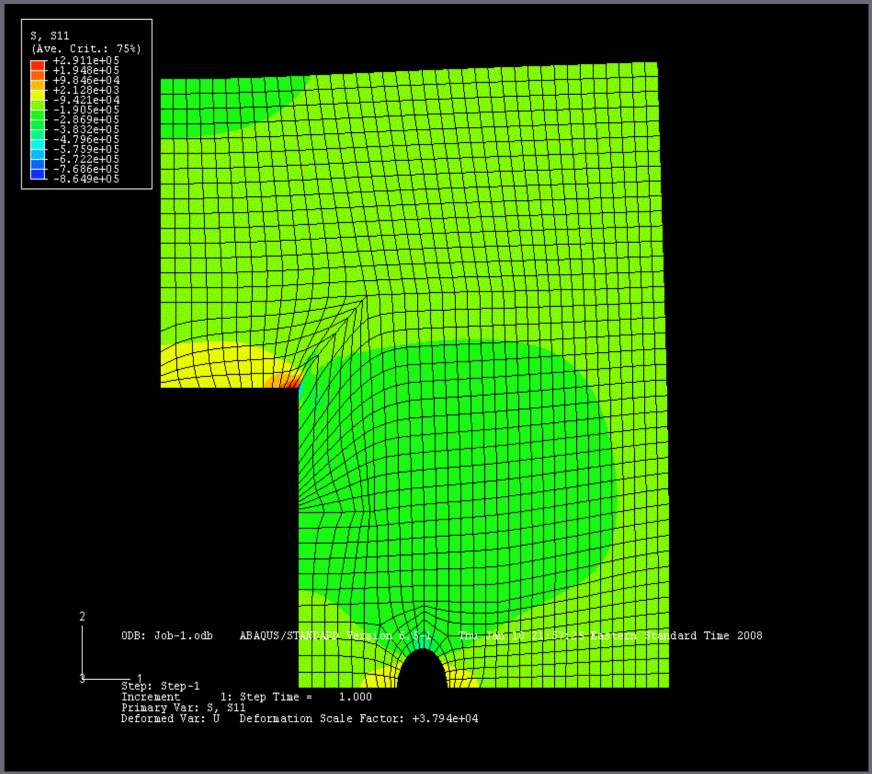
Case 3: Single Bullet Hole Near Window





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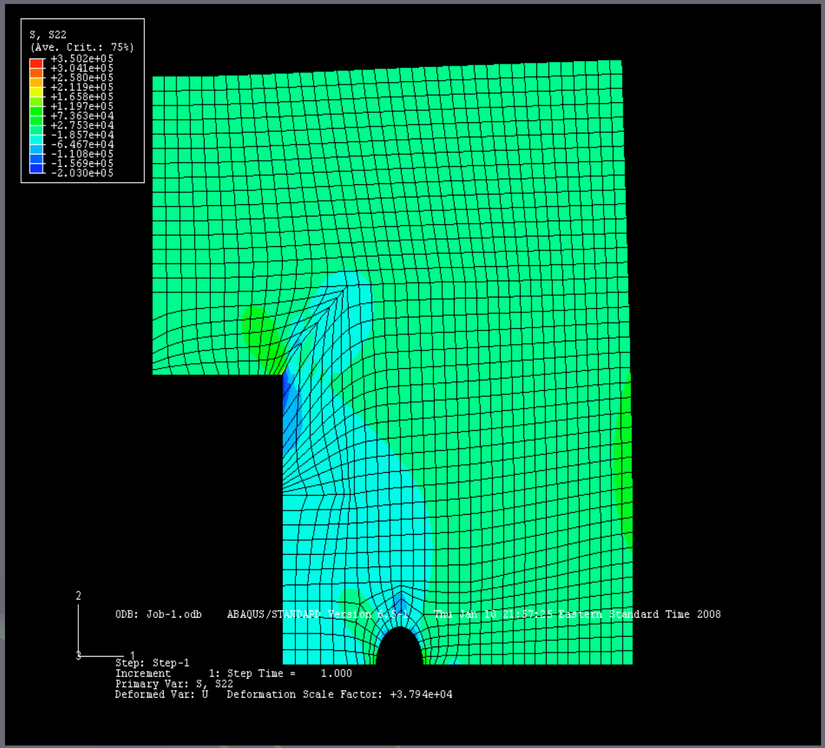

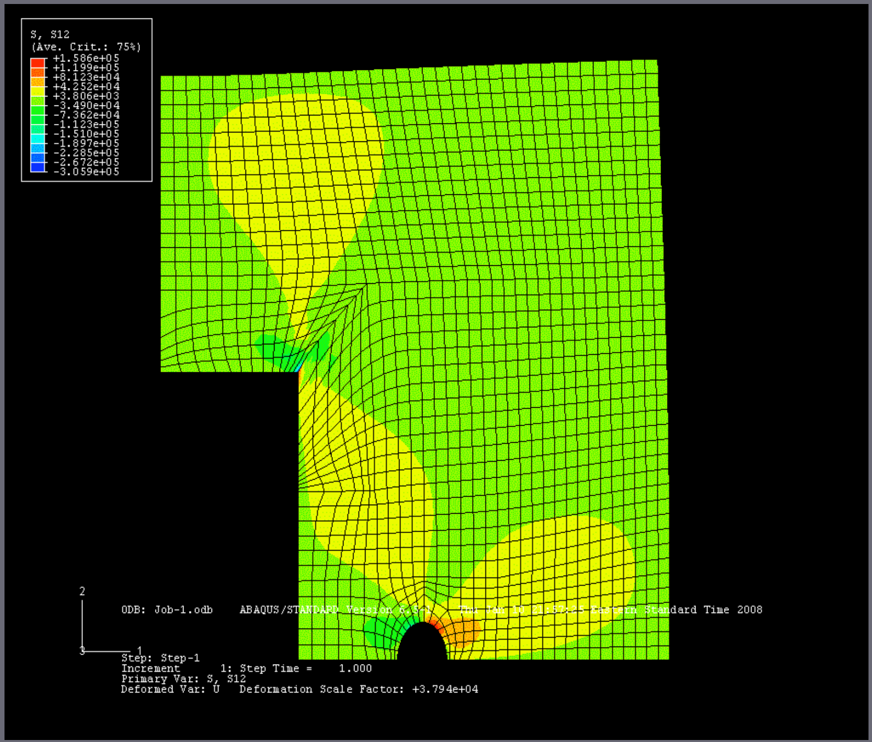

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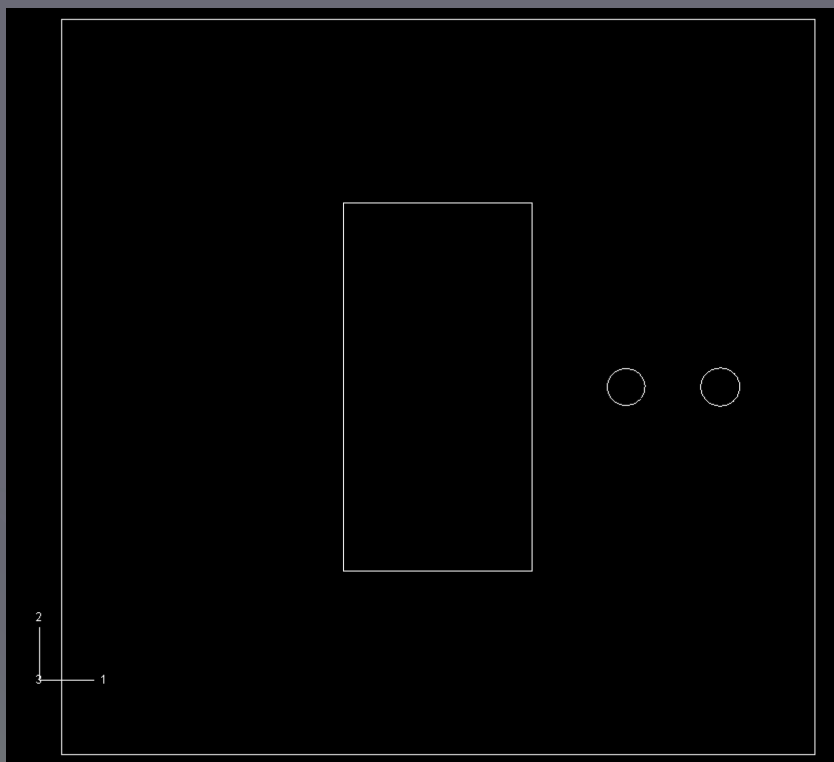
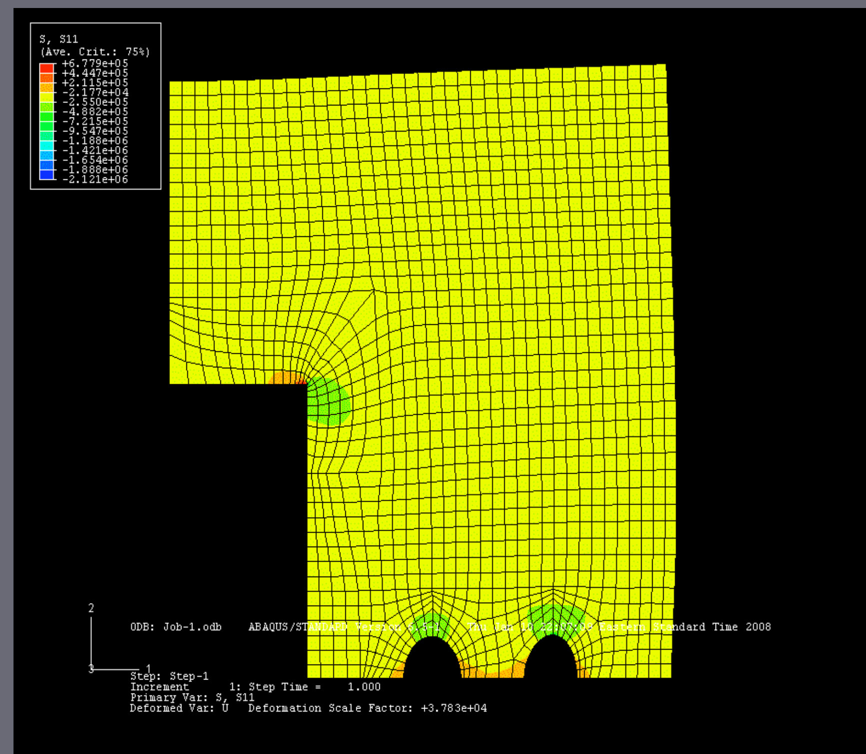
Case 4: Two Bullet Holes Near Window





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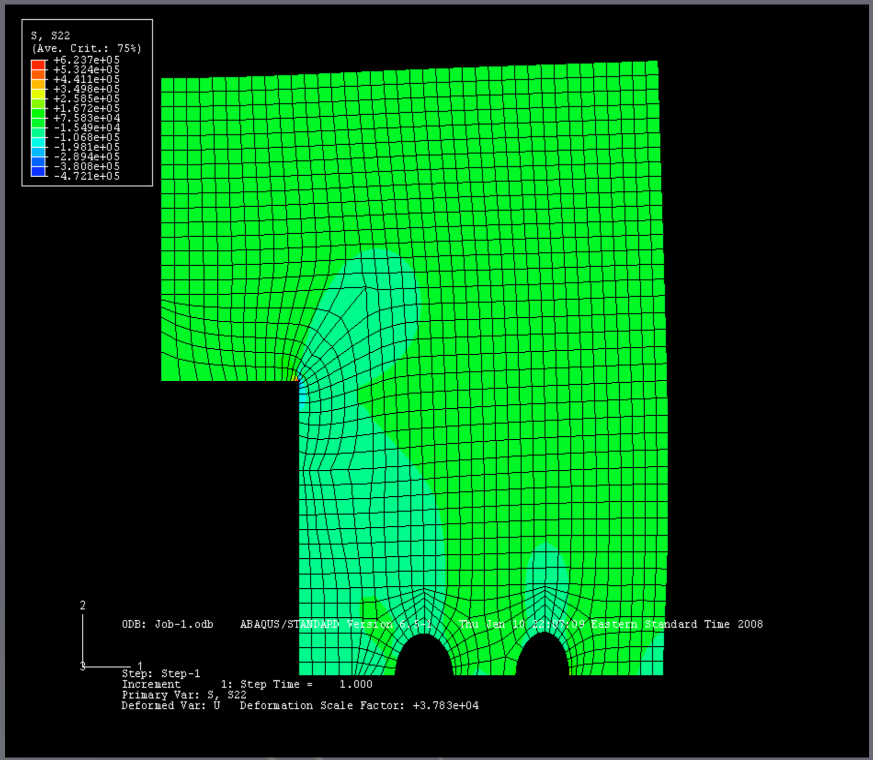
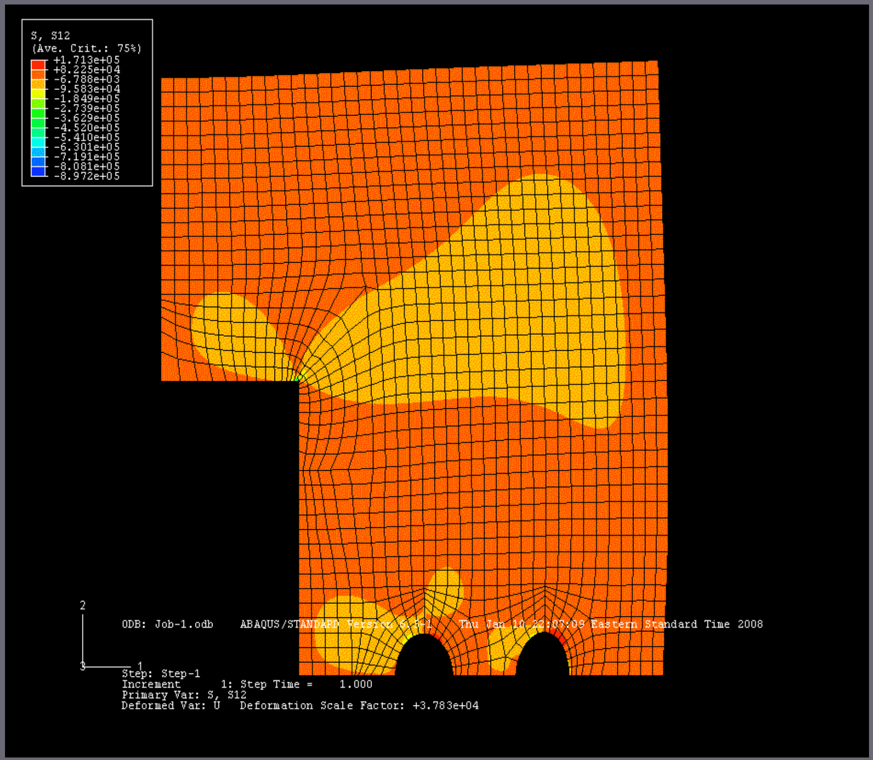
Case 4: ABAQUS CAE Analysis





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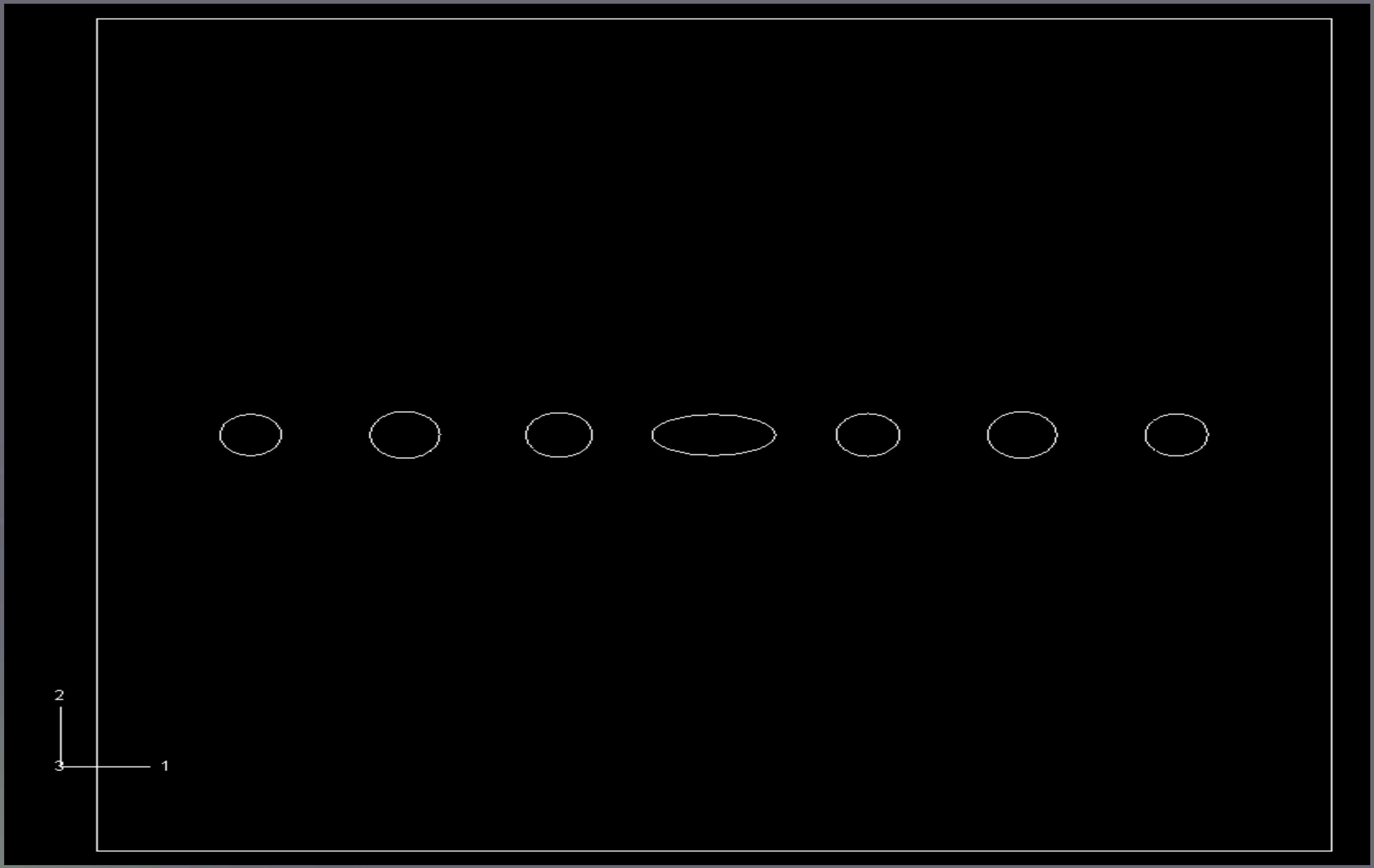
Case 5: Two Holes Approaching Each Other



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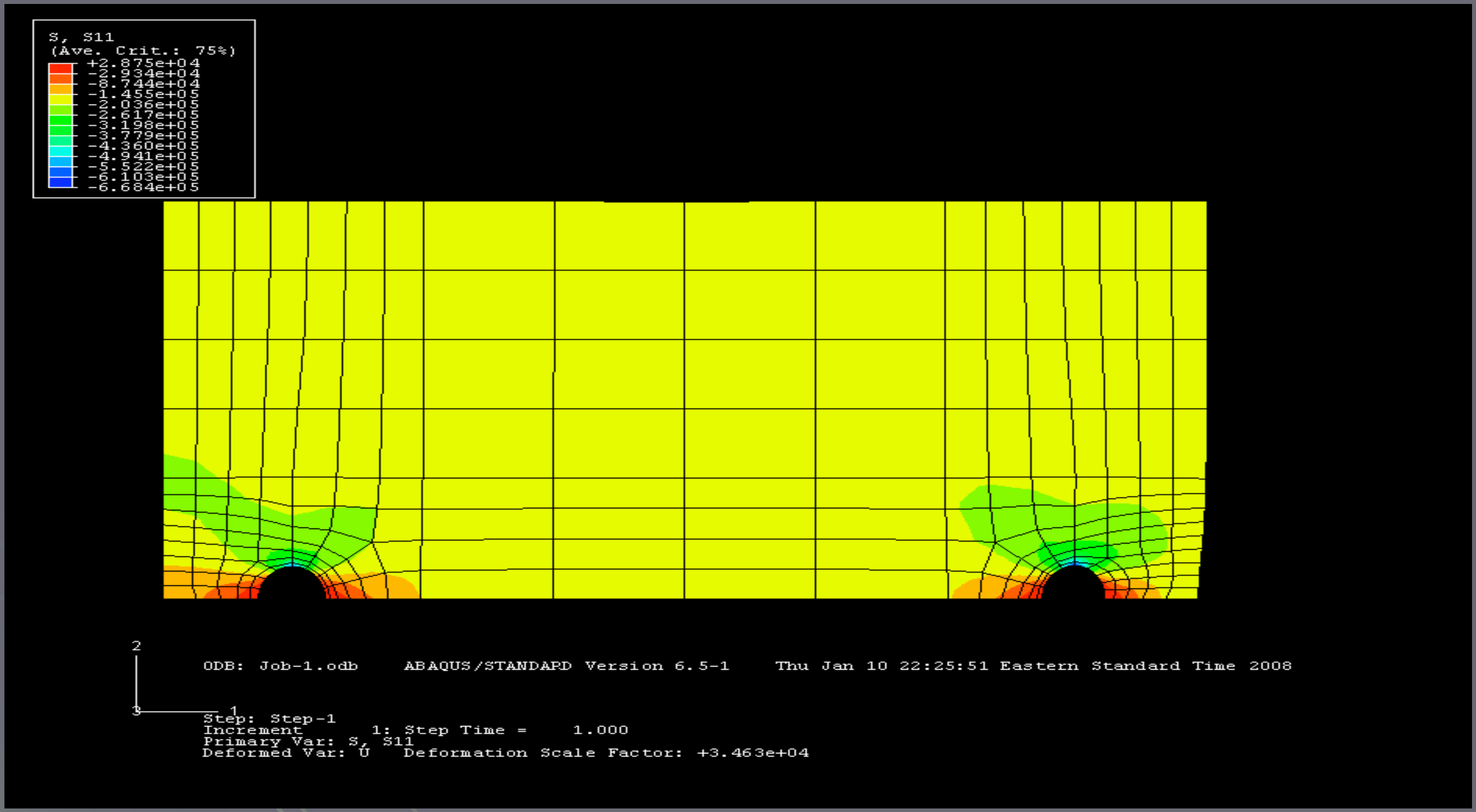

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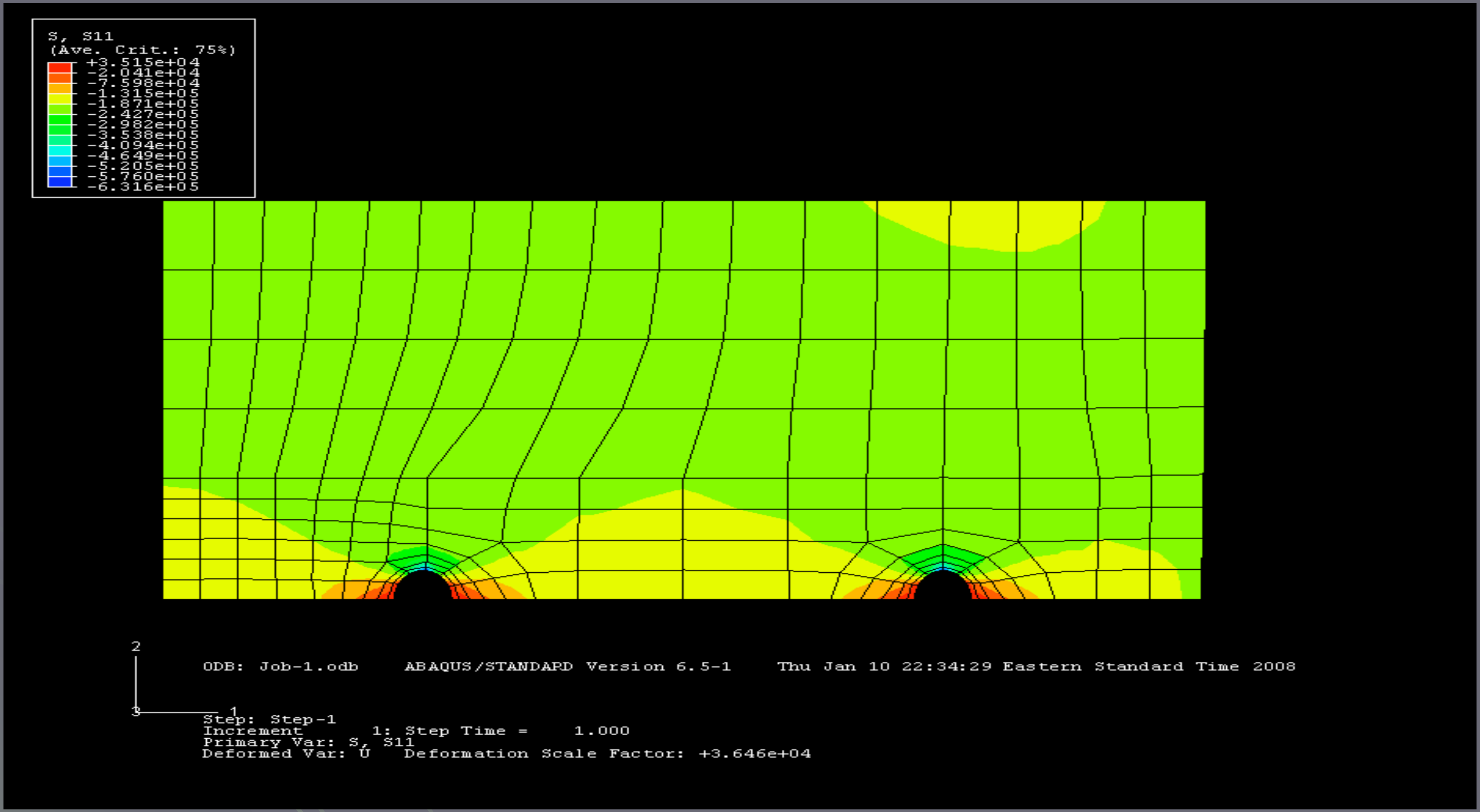

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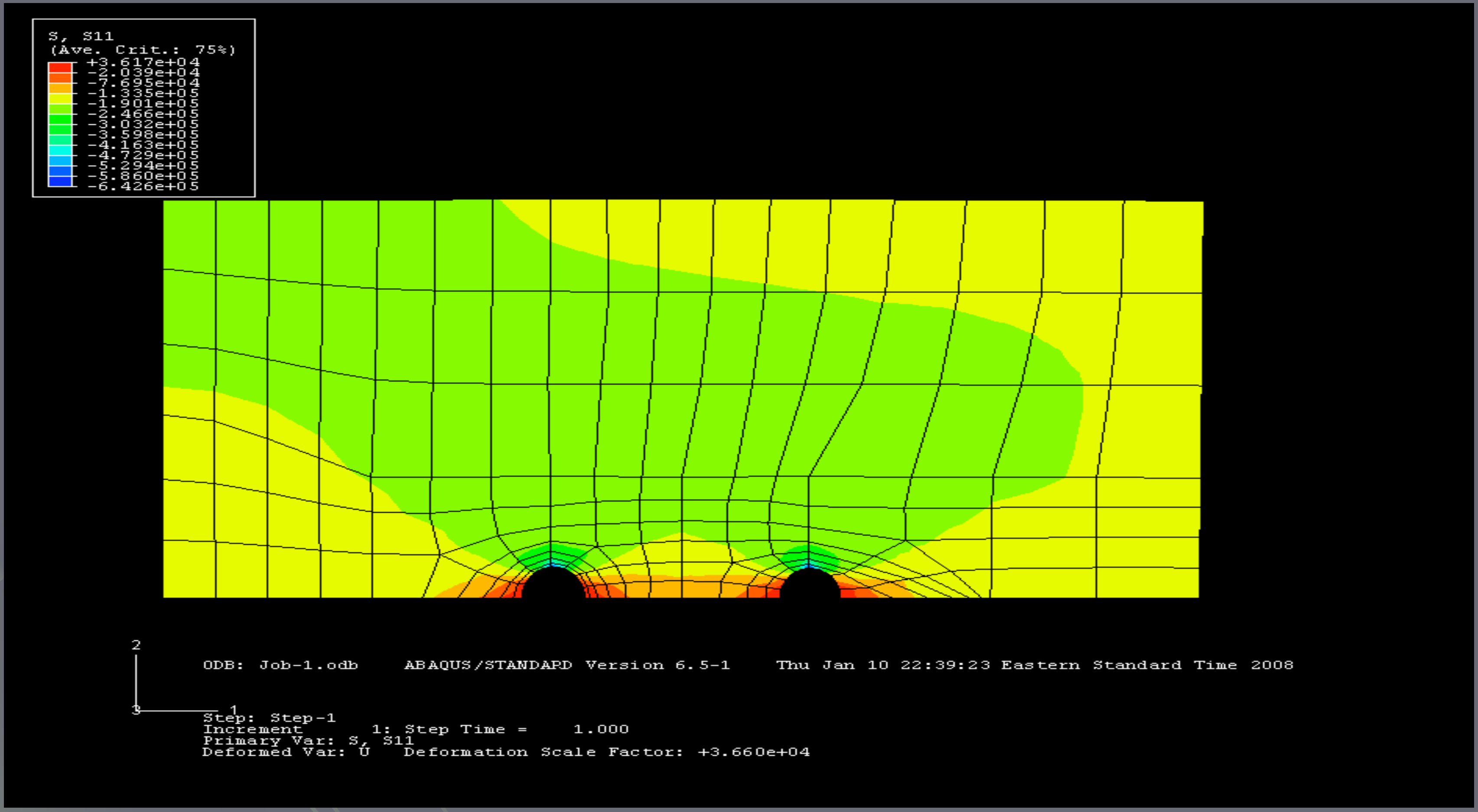

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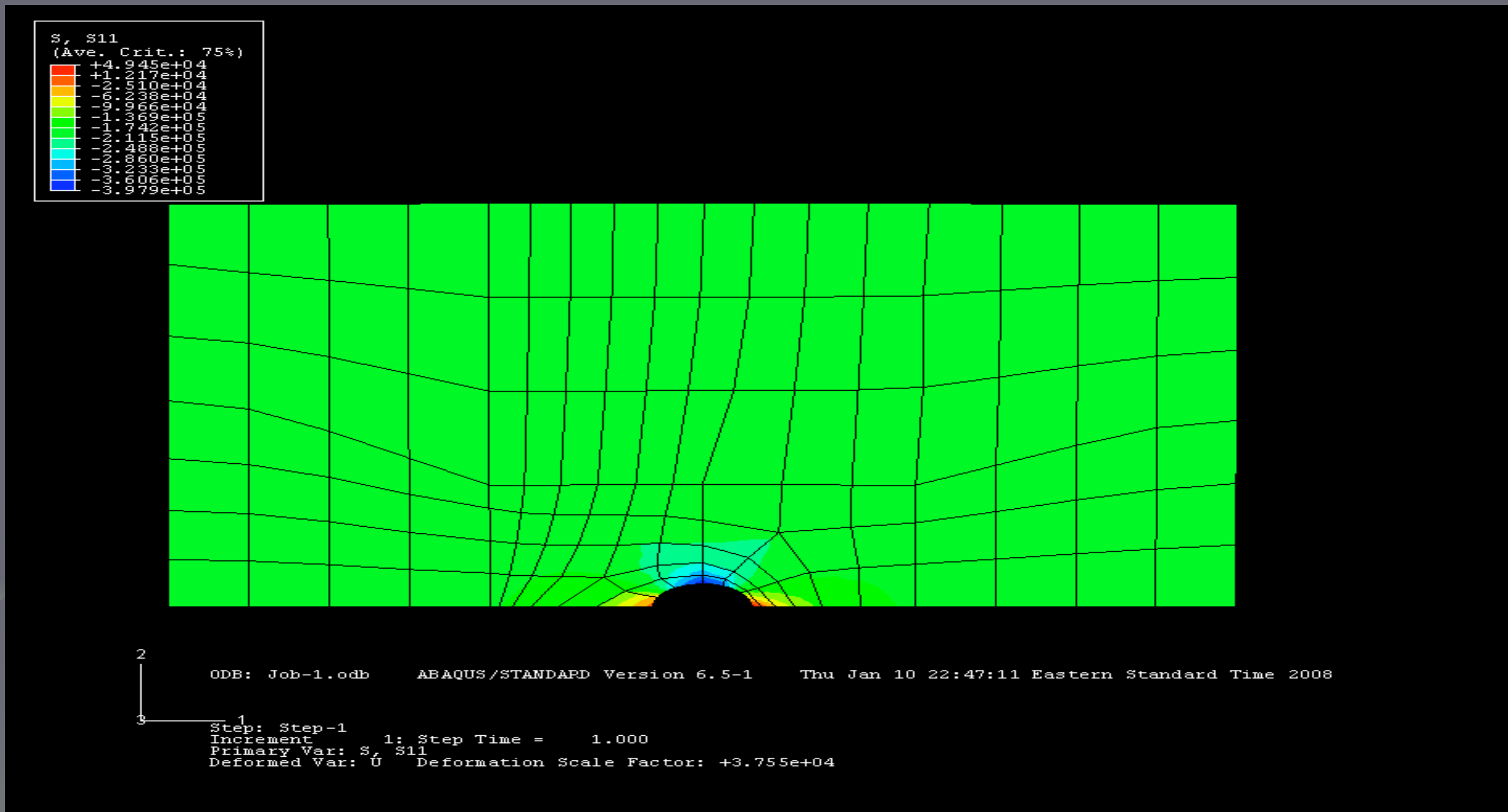

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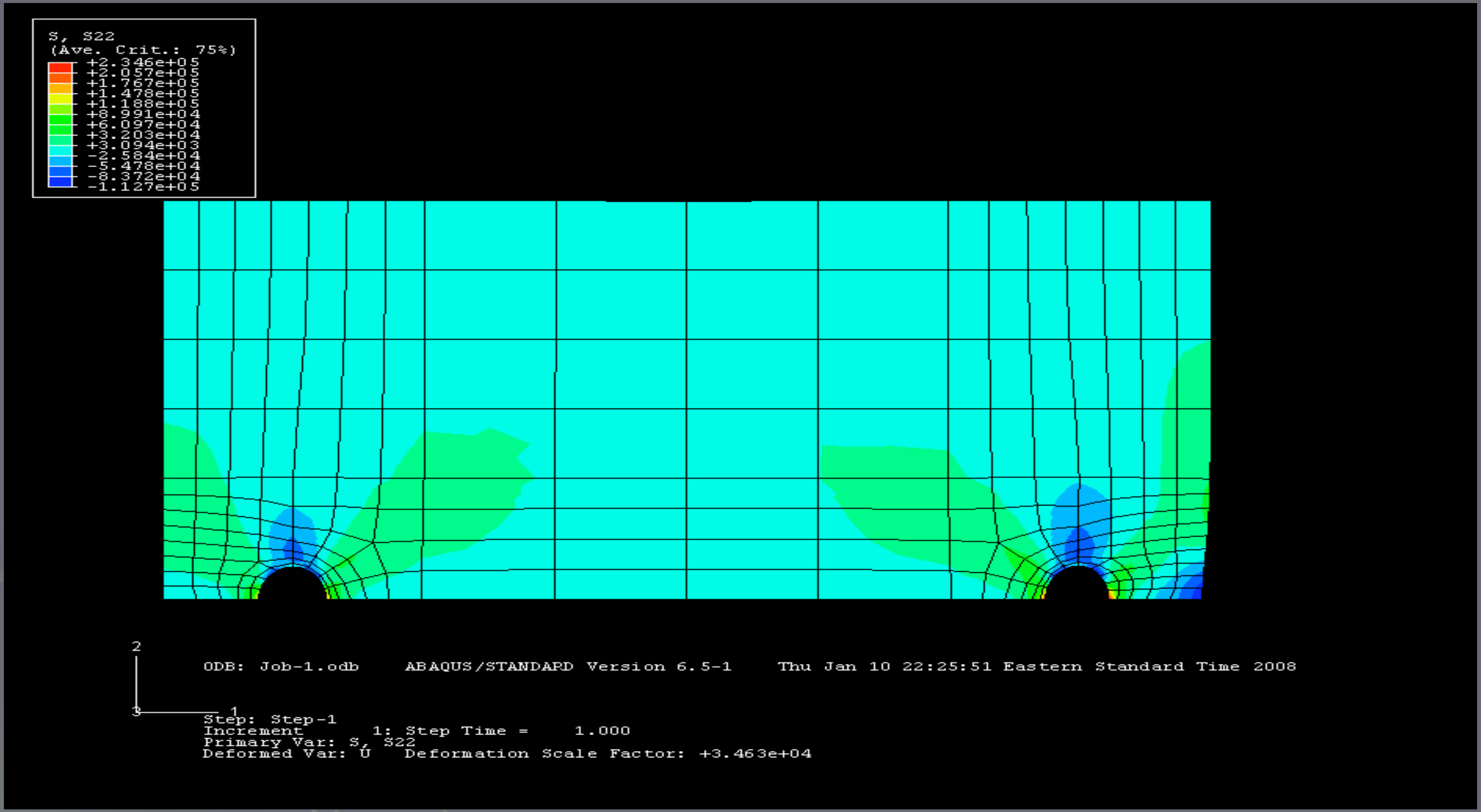

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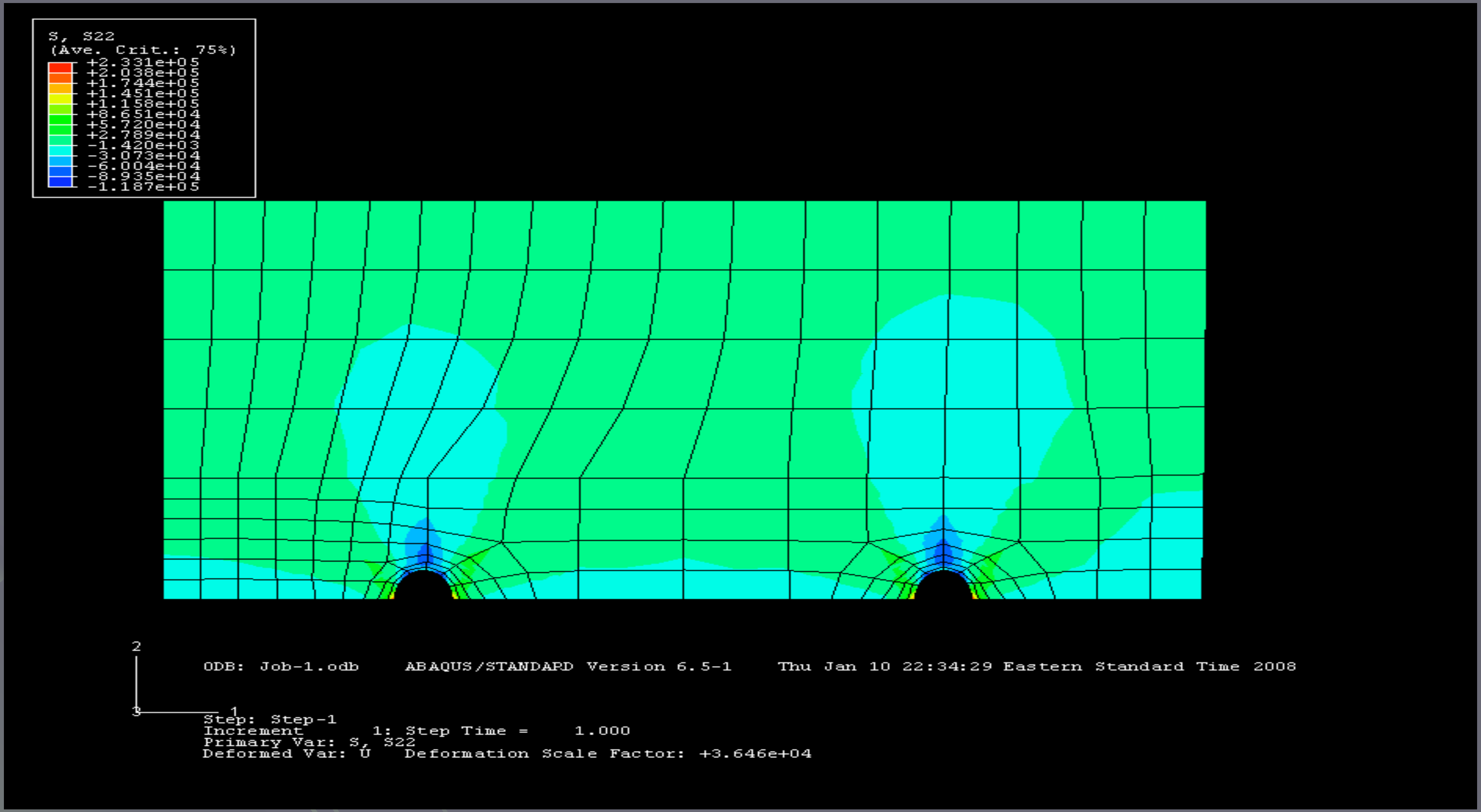

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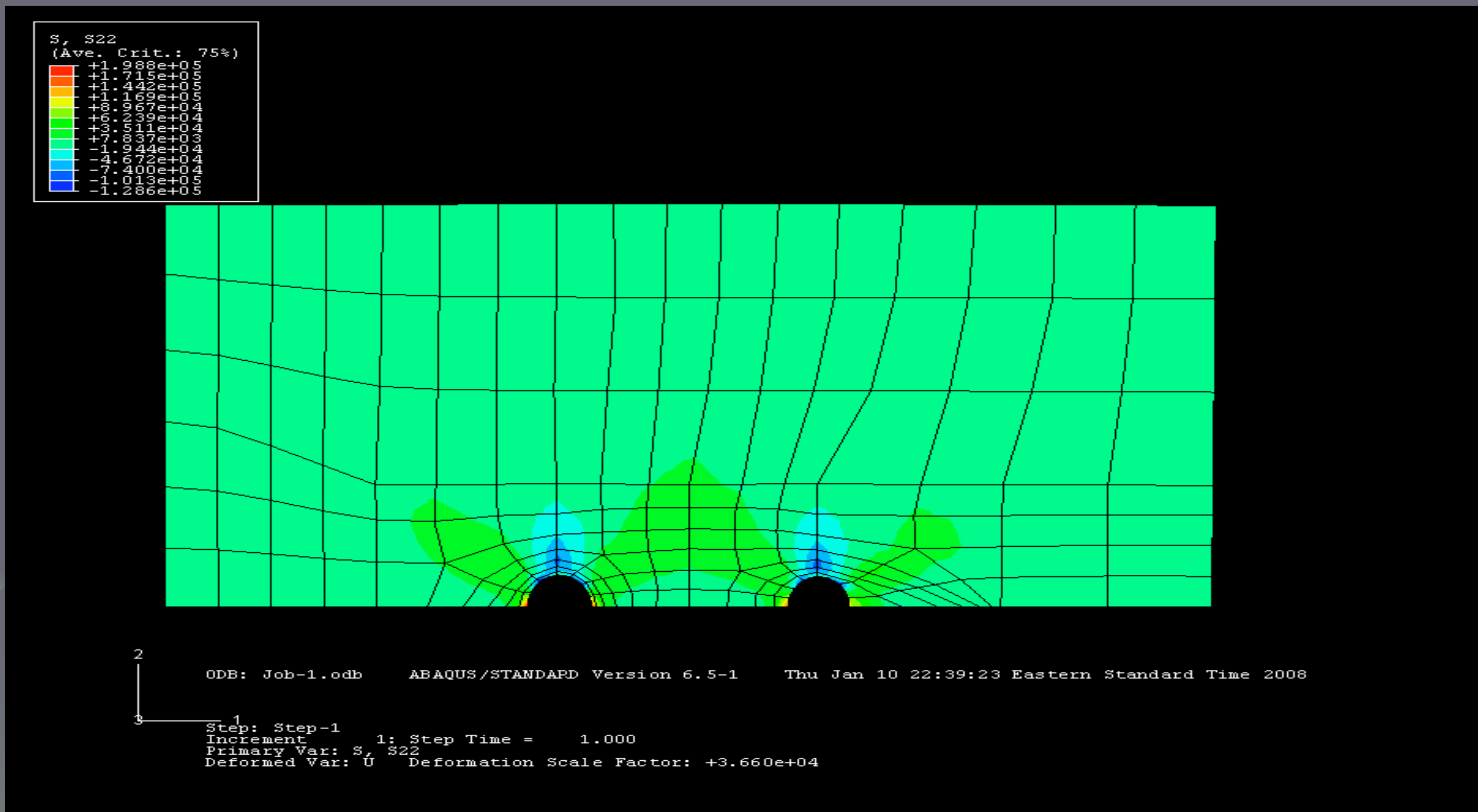

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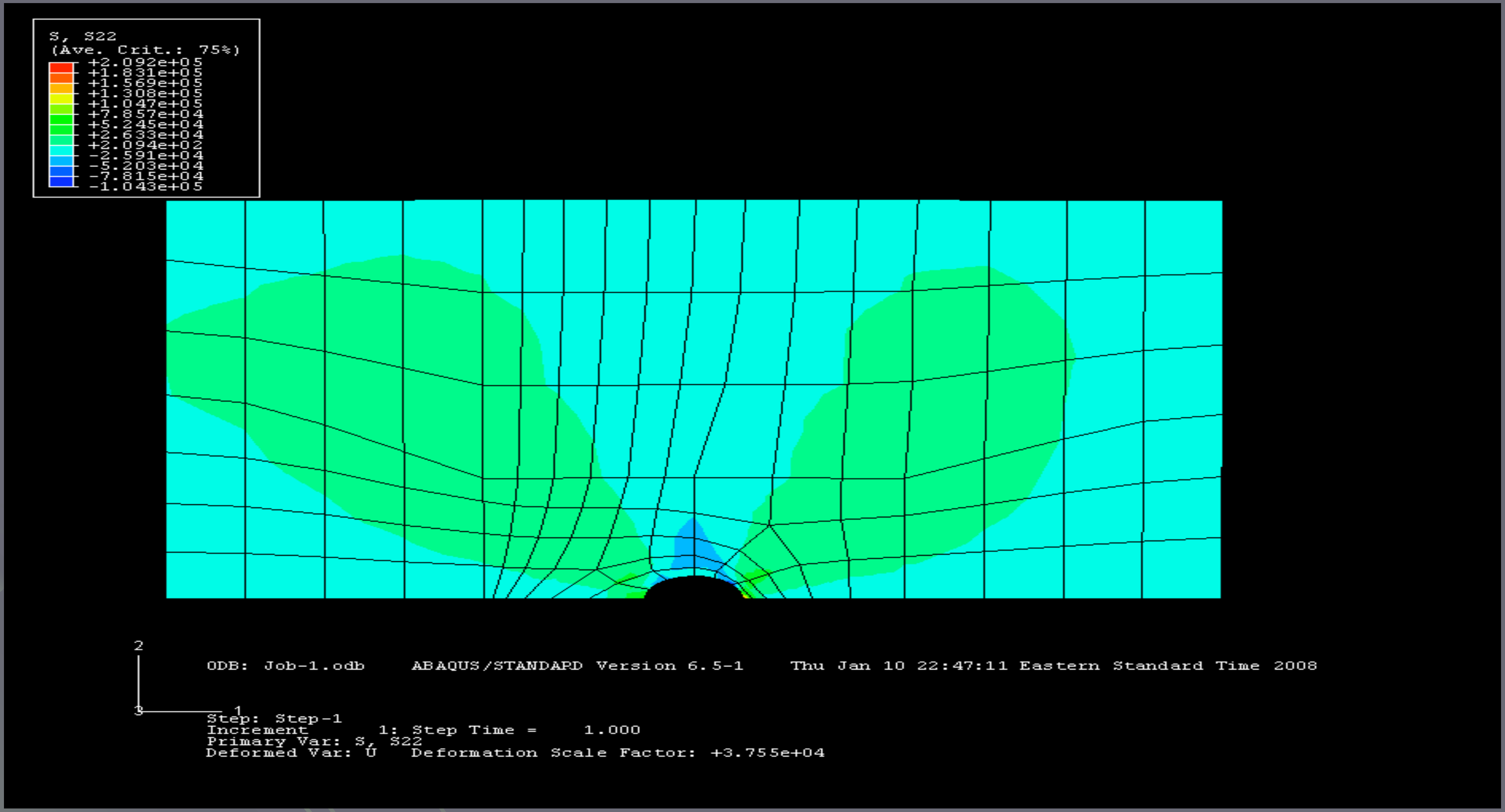

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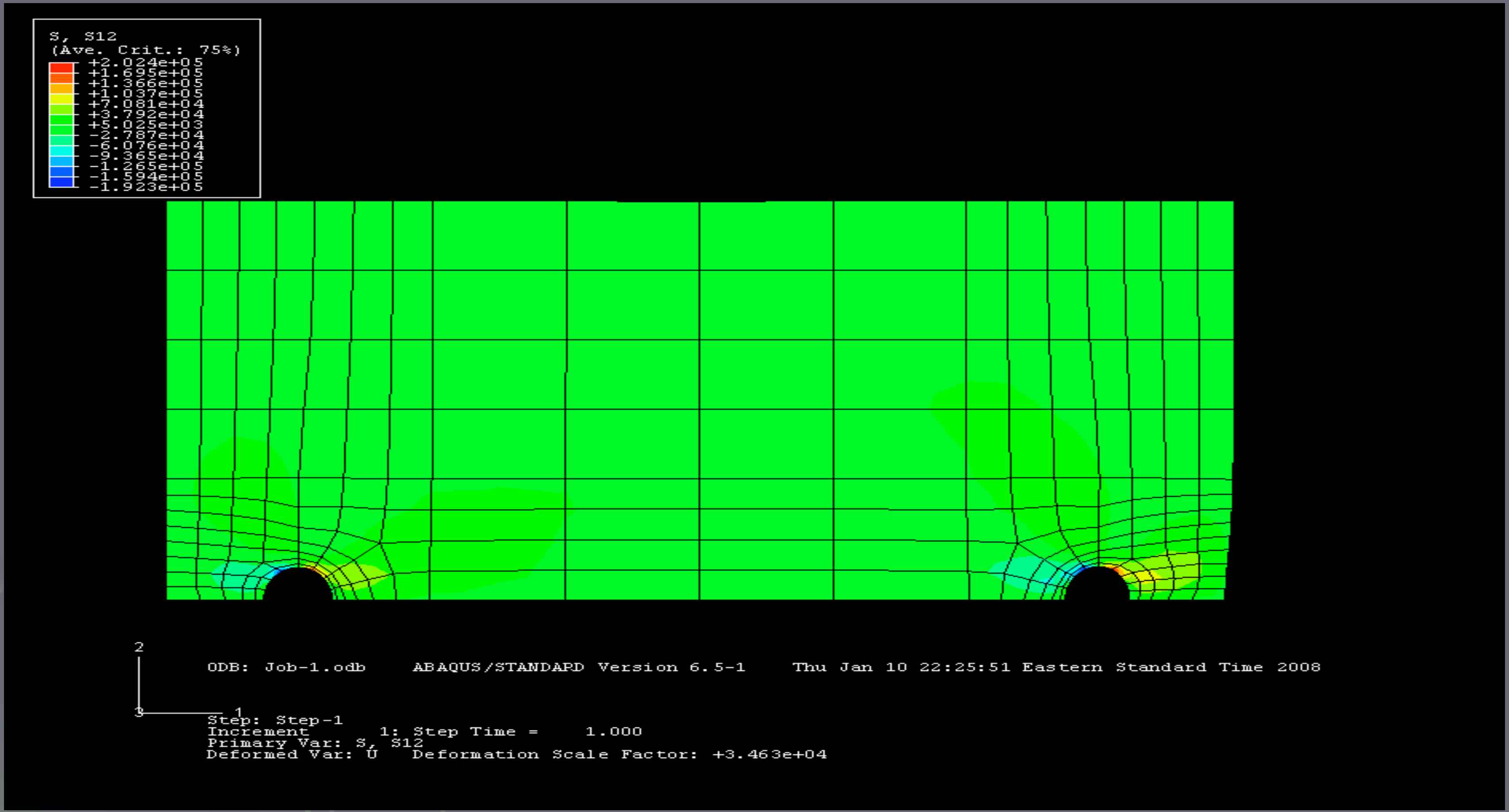

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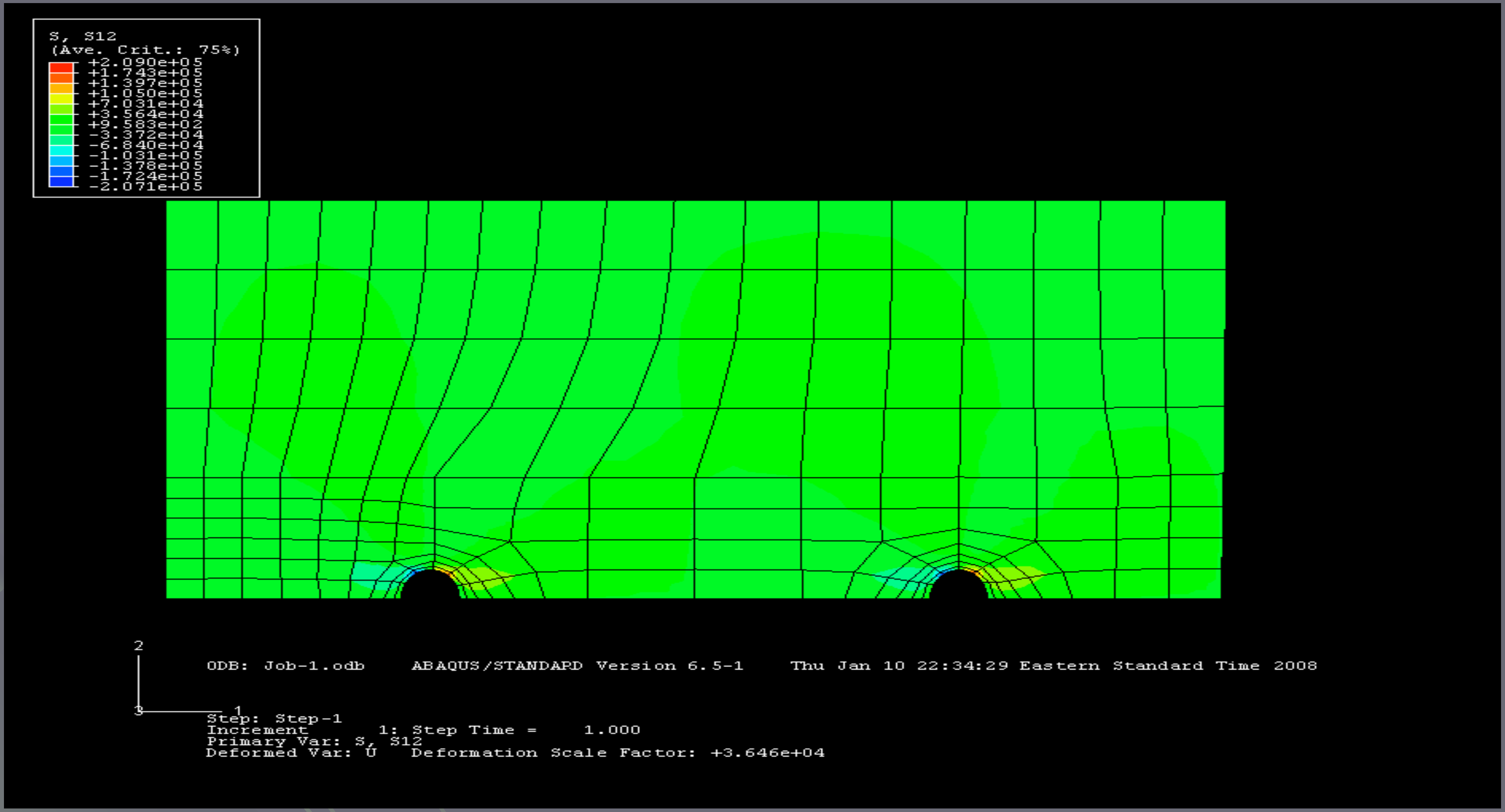

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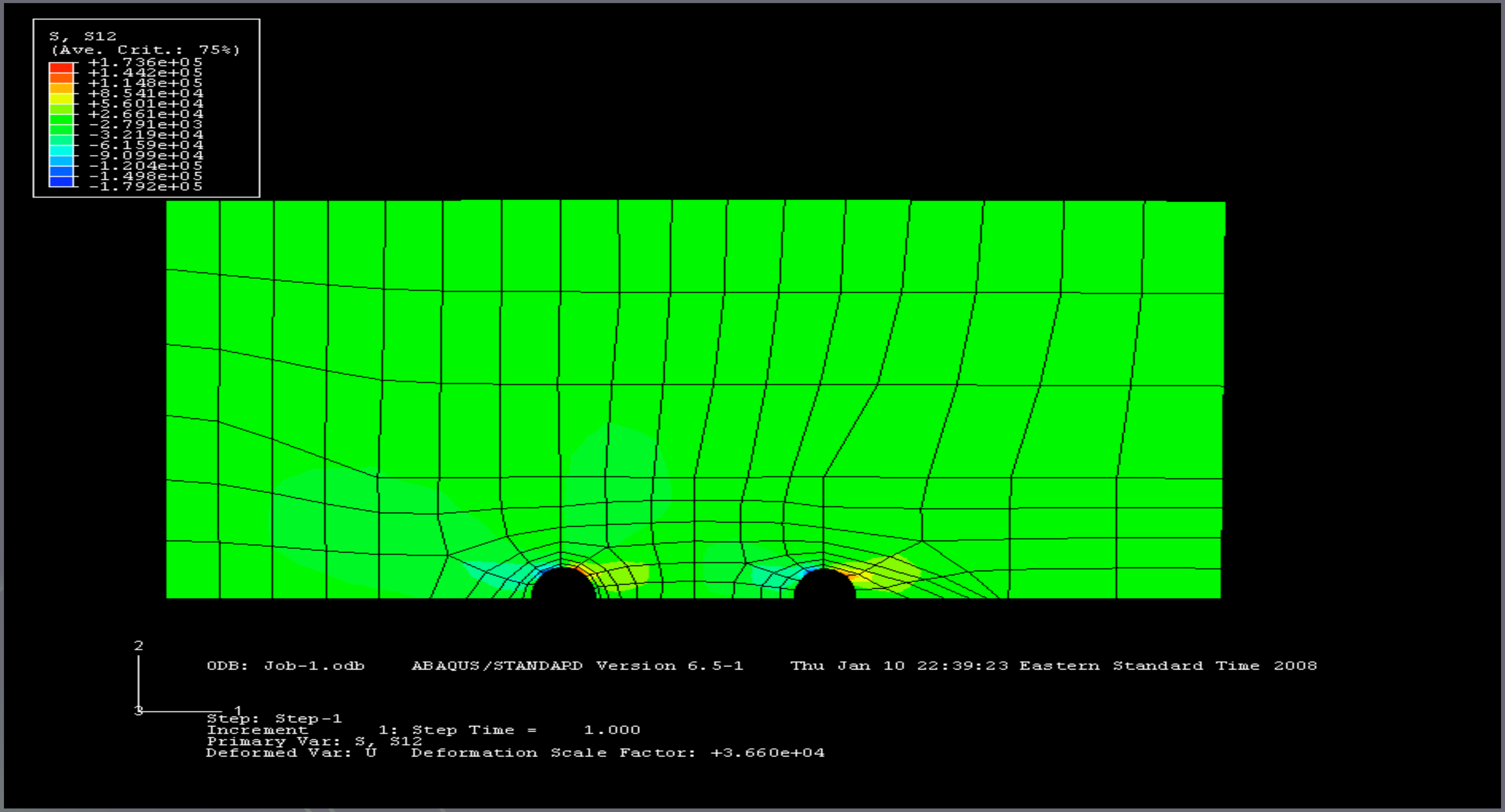

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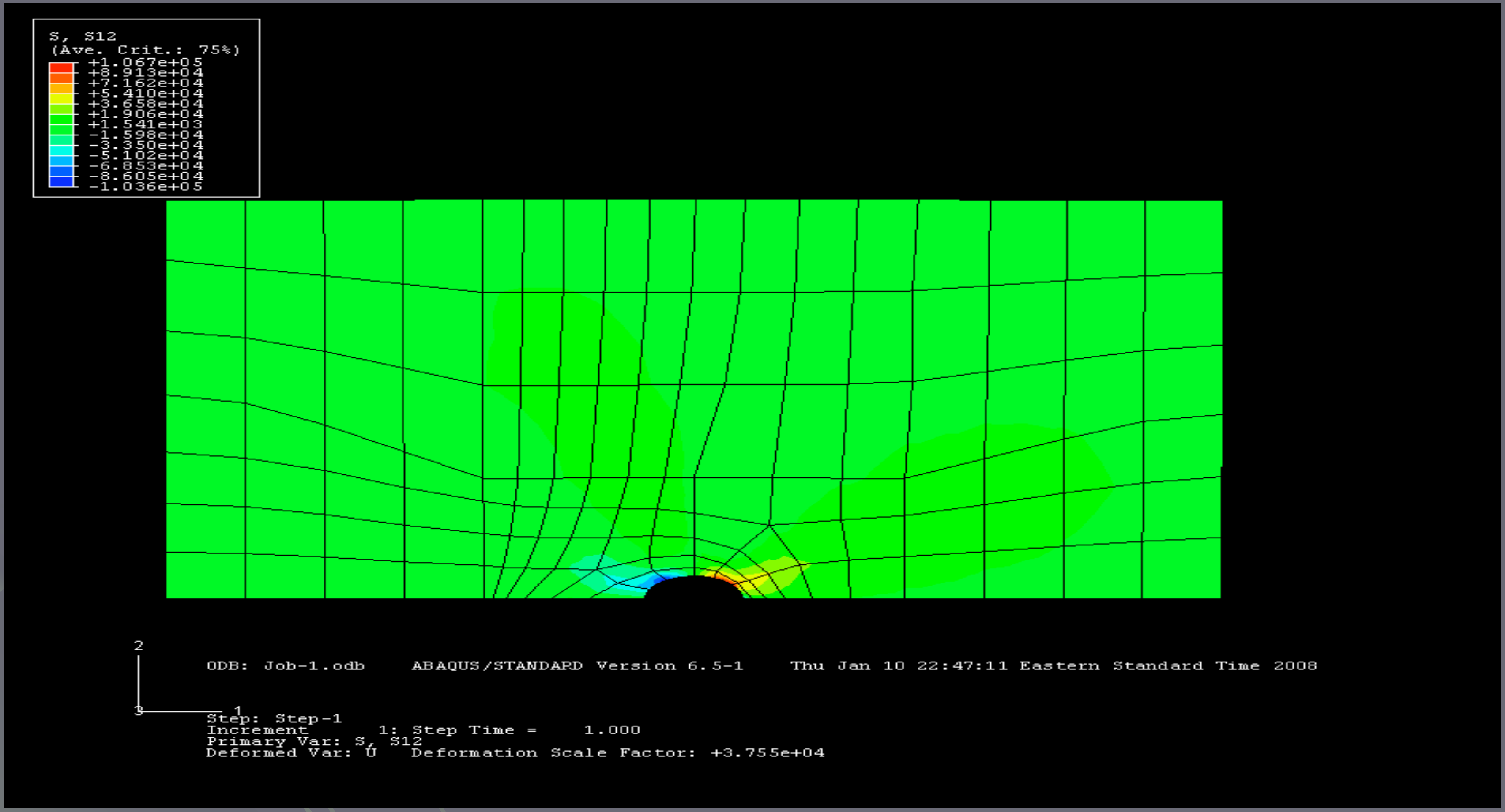

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Orthotropic Stress Equations (Ellipse)

Stress-Strain Relations:

$$\varepsilon_{XX} = \frac{1}{E_X}\sigma_{XX} - \frac{\upsilon_{YX}}{E_Y}\sigma_{YY}$$

$$\varepsilon_{YY} = \frac{1}{E_Y}\sigma_{YY} - \frac{\upsilon_{XY}}{E_X}\sigma_{XX}$$

$$\varepsilon_{XY} = \frac{1}{\mu_{XY}}\sigma_{XY}$$

Compatibility Equation:

$$\frac{\partial^2 \varepsilon_{XX}}{\partial y^2} + \frac{\partial^2 \varepsilon_{YY}}{\partial x^2} = \frac{\partial^2 \varepsilon_{XY}}{\partial x \partial y}$$

Airy Stress Function:

$$\sigma_{XX} = \frac{\partial^2 F}{\partial y^2} \qquad \sigma_{YY} = \frac{\partial^2 F}{\partial x^2} \qquad \sigma_{XY} = -\frac{\partial^2 F}{\partial x \partial y}$$

Differential Equation 1:

$$\frac{1}{E_Y}\frac{\partial^4 F}{\partial x^4} + \left(\frac{1}{\mu_{XY}} - 2\frac{\upsilon_{XY}}{E_X}\right)\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{1}{E_X}\frac{\partial^4 F}{\partial y^4} = 0$$

Differential Equation Simplified:

$$\frac{\partial^4 F}{\partial x^4} + 2\kappa \frac{\partial^4 F}{\partial x^2 \partial \eta^2} + \frac{\partial^4 F}{\partial \eta^4} = 0$$

where:

$$\kappa = \frac{\sqrt{E_X E_Y}}{2}\left(\frac{1}{\mu_{XY}} - \frac{2\upsilon_{XY}}{E_X}\right)$$

$$\eta = \xi y \quad \text{and} \quad \xi = \sqrt[4]{\frac{E_X}{E_Y}}$$

Orthotropic Stress Equations (Ellipse)

Solve for Roots of Equation:

$$r_1 = ie^{\frac{\phi}{2}} \qquad r_2 = ie^{\frac{-\phi}{2}}$$

$$r_3 = -ie^{\frac{\phi}{2}} \qquad r_4 = -ie^{\frac{-\phi}{2}}$$

Biharmonic Equation Form:

$$\left(\frac{\partial^2}{\partial x^2} + \alpha^2 \frac{\partial^2}{\partial \eta^2}\right)\left(\frac{\partial^2}{\partial x^2} + \beta^2 \frac{\partial^2}{\partial \eta^2}\right) F = 0$$

where: $\alpha^2 = e^{\phi}$

$\beta^2 = e^{-\phi}$

General Solution Form:

$$F = \mathrm{Re}\left[F_1\left(x + i\alpha\eta\right) + F_2\left(x + i\beta\eta\right)\right] + F_P\left(x,\eta\right)$$

Specific Solution Form:

$$F = \mathrm{Re}\left\{\begin{matrix} \frac{A}{2y_1^2}\left[\frac{1}{2}\left(z_1 - w_1\right)^2 + y_1^2 \log(z_1 + w_1)\right] + \\ \frac{B}{2y_2^2}\left[\frac{1}{2}\left(z_2 - w_2\right)^2 + y_2^2 \log(z_2 + w_2)\right] \end{matrix}\right\} + \frac{P\eta^2}{2\xi^2}$$

where:

$$z_1 = x + i\alpha\eta$$

$$z_2 = x + i\beta\eta$$

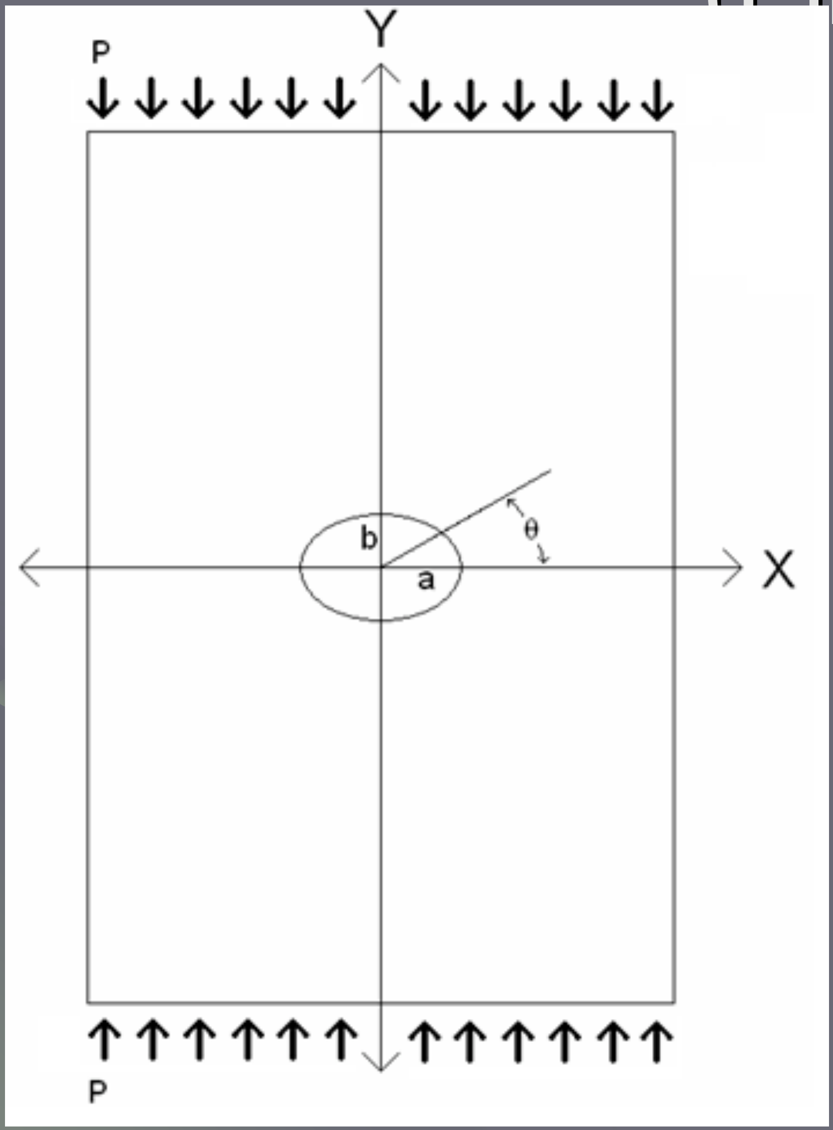
$$w_1 = \sqrt{z_1^2 - y_1^2}$$

$$w_2 = \sqrt{z_2^2 - y_2^2}$$

$$y_1^2 = a^2 - \alpha^2\xi^2 b^2$$

$$y_2^2 = a^2 - \beta^2\xi^2 b^2$$

Orthotropic Stress Equations (Ellipse)



$$x = a\cos\theta$$

$$\eta = \xi y = \xi b\sin\theta$$

$$P = pressure$$

Use Boundary Conditions to solve for constants of integration and unknown variables…

Orthotropic Stress Equations (Ellipse)

$$\frac{\partial^2 F}{\partial x^2} = \sigma_{YY} = \mathrm{Re}\left\{\frac{Pb}{\xi(\alpha-\beta)}\left[\frac{(a+\alpha\xi b)}{w_1(z_1+w_1)} - \frac{(a+\beta\xi b)}{w_2(z_2+w_2)}\right]\right\}$$

$$\frac{\partial^2 F}{\partial y^2} = \sigma_{XX} = \mathrm{Re}\left\{\frac{P\xi b}{(\alpha-\beta)}\left[\frac{-\alpha^2(a+\alpha\xi b)}{w_1(z_1+w_1)} + \frac{\beta^2(a+\beta\xi b)}{w_2(z_2+w_2)}\right]\right\} + P$$

$$\frac{\partial^2 F}{\partial x \partial \eta} = \sigma_{XY} = \mathrm{Re}\left\{\frac{Pb}{(\alpha-\beta)}\left[\frac{-\alpha i(a+\alpha\xi b)}{w_1(z_1+w_1)} + \frac{\beta i(a+\beta\xi b)}{w_2(z_2+w_2)}\right]\right\}$$

To obtain the stress equations for a circle,

set a=b in the stress equations for an ellipse.

Isotropic Stress Equations (Ellipse)

For an isotropic material:

$$\alpha = \beta = \xi = 1$$

Therefore, specific solution (from previous) becomes:

$$F = R\left\{ \frac{Pb}{2\xi\left(e^{\frac{\phi}{2}} - e^{\frac{-\phi}{2}}\right)} \left(\begin{aligned} & -\frac{1}{a - e^{\frac{\phi}{2}}\xi b}\left(\frac{1}{2}(z_1 - w_1)^2 + y_1^{\,2}\log(z_1 + w_1)\right) + \\ & \frac{1}{a - e^{-\frac{\phi}{2}}\xi b}\left(\frac{1}{2}(z_2 - w_2)^2 + y_2^{\,2}\log(z_2 + w_2)\right) \end{aligned} \right) \right\} + \frac{P\eta^2}{2\xi^2}$$

Isotropic Stress Equations (Ellipse)

$$F = \lim_{\phi \to 0} \left[R \left\{ \frac{Pb}{\xi\left(e^{\frac{\phi}{2}} + e^{\frac{-\phi}{2}}\right)} \left(\begin{aligned} & - \frac{1}{a - e^{\frac{\phi}{2}}\xi b} \left(\begin{aligned} & \frac{(z_1 - w_1)}{2}\left(ie^{\frac{\phi}{2}}\eta - \frac{z_1 ie^{\frac{\phi}{2}}\eta + e^{\phi}\xi^2 b^2}{w_1} \right) \\ & + \frac{{y_1}^2}{2} \frac{ie^{\frac{\phi}{2}}\eta + \dfrac{z_1 ie^{\frac{\phi}{2}}\eta + e^{\phi}\xi^2 b^2}{w_1}}{z_1 + w_1} - e^{\phi}\xi^2 b^2 \log(z_1 + w_1) \end{aligned} \right) \\ & - \frac{e^{\frac{\phi}{2}}\xi b}{2\left(a - e^{\frac{\phi}{2}}\xi b\right)^2} \left(\frac{(z_1 - w_1)^2}{2} + {y_1}^2 \log(z_1 + w_1) \right) \\ & + \frac{1}{a - e^{\frac{-\phi}{2}}\xi b} \left(\begin{aligned} & \frac{(z_2 - w_2)}{2}\left(-ie^{\frac{-\phi}{2}}\eta + \frac{z_2 ie^{\frac{-\phi}{2}}\eta + e^{-\phi}\xi^2 b^2}{w_2} \right) \\ & + \frac{{y_2}^2}{2} \frac{-ie^{\frac{-\phi}{2}}\eta - \dfrac{z_2 ie^{\frac{-\phi}{2}}\eta + e^{-\phi}\xi^2 b^2}{w_2}}{z_2 + w_2} + e^{-\phi}\xi^2 b^2 \log(z_2 + w_2) \end{aligned} \right) \\ & - \frac{e^{\frac{-\phi}{2}}\xi b}{2\left(a - e^{\frac{-\phi}{2}}\xi b\right)^2} \left(\frac{1}{2}(z_2 - w_2)^2 + {y_2}^2 \log(z_2 + w_2) \right) \end{aligned} \right) \right\} \right] + \frac{P\eta^2}{2\xi^2}$$

Isotropic Stress Equations (Ellipse)

$$F = R\left\{\frac{Pb}{2}\left(\begin{array}{l} -\dfrac{1}{a-b}\left((z-w)\dfrac{\left(iyw - iyz - b^2\right)}{w}\right) - \dfrac{b}{2(a-b)^2}(z-w)^2 \\ -\dfrac{a+b}{w}\left(iy + \dfrac{b^2}{z+w}\right) - b\log(z+w) + \dfrac{y^2}{b} \end{array}\right)\right\}$$

where:

$$w = \sqrt{z^2 - a^2 + b^2}$$

For simplification, set a=b to obtain stress equations for a circle:

$$F = R\left\{\frac{Pa}{2}\left(\frac{a^3}{z^2} - \frac{a^3}{2z^2} - \frac{2ay}{z} - \frac{a^3}{z^2} - a\log 2z + \frac{y^2}{a}\right)\right\}$$

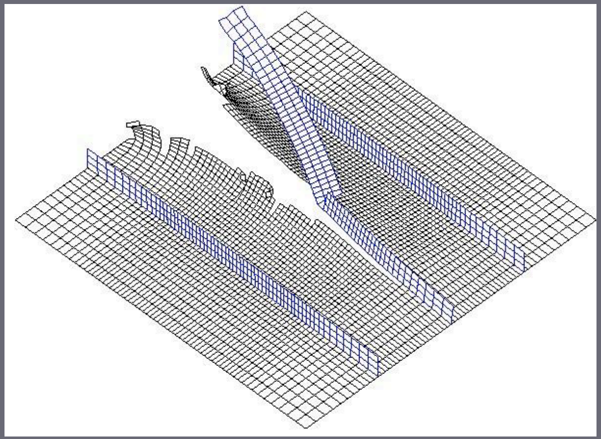
In Polar Coordinates:

$$F = \frac{P}{4}\left(-\frac{a^4}{r^2}\cos 2\theta - 4a^2\sin^2\theta + 2r^2\sin^2\theta - 2a^2\log r\right)$$

$$= \frac{P}{4}\left(r^2 - 2a^2\log r - \frac{\left(r^2 - a^2\right)^2}{r^2}\cos 2\theta\right)$$

Extra: Fracture due to Crack-Propagation

- Fast Fracture of aircraft fuselage due to Crack Propagation
- Linear-Elastic Fracture Mechanics (LEFM)



Condition for Fast Fracture

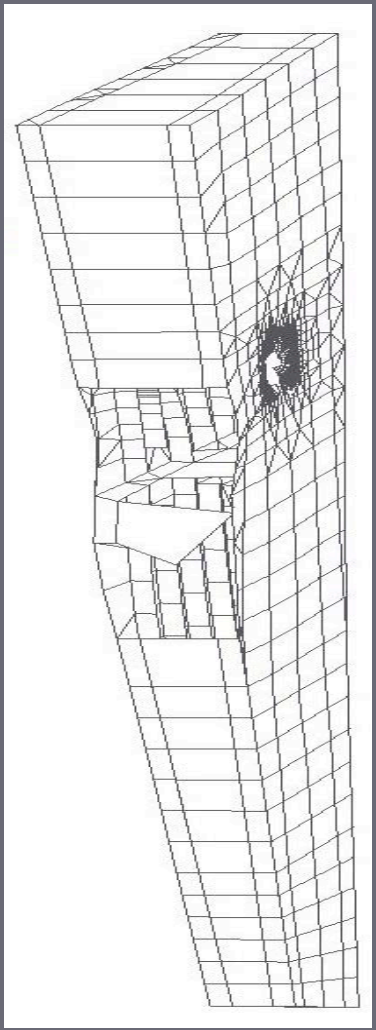
$$K_{1C} = Y\sigma\sqrt{\pi * a}$$

Plane Strain:

$$a_c = \frac{1}{\pi}\left(\frac{K_{1C}}{Y\sigma}\right)^2$$

Plane Stress:

$$r_Y = \frac{1}{2\pi}\left(\frac{K_c}{\sigma_Y}\right)$$



References

- ES 240 Lecture Notes provided by Professor Joost Vlassak
- “Theory of Plates and Shells” by Timoshenko and Woinowsky-Krieger
- “Elementary Engineering Fracture Mechanics” by David Broek
- “Engineering Materials 3: Materials Failure Analysis” by D.R.H. Jones