Homogenization of both linear and nonlinear highly heterogeneous plate

Proposed by : Erasmo Carrera,

Politecnico di Torino
Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

erasmo.carrera@polito.it

Erick Pruchnicki

University and Unity of mechanic (EA 7512) of Lille .
Avenue paul Langevin
F 59655 Villeneuve d'ascq (France).

erick.pruchnicki@polytech-lille.fr

Composite plates are widely used in aeronautics applications because they offer excellent ratio between stiffness or strength performance and weight. The size of fine scale details in these heterogeneous plates is typically much smaller compared to the dimensions of the structure, thus making direct numerical analyses is prohibitively expensive. To avoid these large-scale computations, it is preferable to model these plates at the macroscale as a homogeneous continuum with effective properties obtained through a homogenization procedure. Based on asymptotic homogenization concepts (Caillerie, 1984 ; Kohn and Vogelius, 1984) discussed the homogenization of heterogeneous periodic linear elastic plates. These models are mathematically elegant and rigorous but only related to a simple engineering model (the Kirchhoff plate model). The Kirchhoff–Love plate model is the simplest and the most widely-used theory. Nevertheless this model neglects the contribution of out-of-plane stress components to the stress energy. However, when the plate slenderness ratio L/h (h is the plate thickness and L is the characteristic dimension of its mid-plane) decreases, out-of-plane stresses have an increasing influence on the plate deflection. Exactly as (Cecchi and Sab, 2007) did for Reissner-Mindlin homogenization of periodic plates, Lebée and Sab, 2012 propose a homogenization theory for their bending gradient theory (Lebée and Sab, 2011). **This method of homogenization concerns only the linear case and may be numerically implemented in order to compare this new and recent developments to our approach which is also valid in linear case (see below).** This approach correct the homogenization theory of Lewinski, 1991; Caillerie, 1984 ; Kohn and Vogelius, 1984 in order to take into account of out-of-plane stress components (transverse shearing and transverse normal stress). So they use implicitly the superposition principle and then theses theories are limited to linear elasticity.

The study of Petracca *et al*., 2017, focused on periodic brick-masonry walls, the macro-scale behavior obeys a Reissner-Mindlin and the local heterogeneous structures is assumed to be transverse isotropic. For macroscopic Reissner-Mindlin plate model, Terada *et al*., 2017 propose a new numerical plate testing (NPT) by adding a specific microscopic displacement terms such that the out-of-plane microscopic shear strain components, contain the macroscopic curvature associated with torsional deformation.

In this phd thesis, for the first part we propose to implement numerically the method recently proposed by Pruchnicki 2020 (and forthcomoing new theoretical development,

Pruchnicki (2021)) for which the macroscopic model is Nonlinear Cosserat type model. For numerical implementation of macroscopic Reissner-Mindlin (which is included model in the class of Cosserat type model) without locking we can for example refers to Krishna, 2019. So we avoid the Saint Venant-Kirchoff assumption considered in Coenen *et al.,* 2010; Cong *et al.,* 2015. The mechanical behaviour of the constituents of the plate is of non linear hyperelastic type (obviously we will begin with their elastic linear counterpart) . Then it is necessary to define the relation between the definitions of the macroscopic generalized strains and stresses for a plate continuum in terms of the microscopic ones. This macro-to-micro scale transition is performed by imposing the macroscopic generalized deformation gradient on the RVE (representative volume element) through the essential boundary conditions that may be periodic conditions. Upon solution of the microstructural boundary value problem, the macroscopic generalized stress resultants are expressed by averaging the computed RVE stress field. In our work, the through thickness dimension is directly combined with the in-plane homogenization.

For the second part, we propose to work on the US School homogenization idea (Steigmann, D.J. and Pipkin, A.C) which in particular can be used to model woven fabric (two orthogonal sets of yarns interlaced with one another (Parsons *et al.* [30])). The fibers of the yarn may have elliptical, rectangular or circular cross sections. In order to model the mechanical behaviour of woven fabric, Shirani *et al.* [31] assumed that the nonlinear spatial rods are of Kirchhoff type. Nevertheless a more accurate higher order model should be considered. In the spirit of the modelling on woven fabrics (Steigmann and Pipkin [32], Steigmann and Dell'Isola [33], Shirani *et al.* [31], Steigmann [34]) and following the idea of Pipkin, we can propose new models for woven fabrics (or also classical composite plate) by using new unidimensional modelling for curved homogeneous beams (Pruchnicki *et al.* (2021).

References

BERDICHEVSKY V., 1979, Variational asymptotic method of constructing a theory of shells, *Journal of Applied Mathematics and Mechanics*, **43,** 4, 664–687.

CAILLERIE D., 1984, Thin elastic and periodic plate, *Mathematical Models and Methods in Applied Sciences,* **6**, 1, 159–191

CECCHI A., SAB K., 2007, A homogenized Reissner–Mindlin model for orthotropic periodic plates: application to brickwork panels, *International Journal of Solids and Structures*, **44**, 18–19, 6055–6079.

Cherdantsev, M. and Cherednichenko, K.D., 2015, Bending of thin periodic plates. Calculus of Variations. 54 :4079-4117.

COENEN E.W.C., KOUZNETSOVA V.G., Geers M.G.D., 2010, Computational homogenization for heterogeneous thin sheets. *International Journal for Numerical Methods in Engineering*, **83**, 8-9,1180–1205.

CONG Y., NEZAMBADI S., ZARHOUNI H., YVONNET J., 2015, Multiscale computational homogenization of heterogeneous shells at small strains with extensions to finite displacements and buckling. *International Journal for Numerical Methods in Engineering,* **104**, 4, 235–259.

KOHN R.V., VOGELIUS M., 1984, A new model for thin plates with rapidly varying thickness, *International Journal of Solids and Structures*, **20**, 4, 333–350.

Krishna, H. 2019. Development of a new methodogy for finding deformation of Reissner-Mindlin plate using isogeometric analysis. Project report Master of Technology, Department of Mechanical Engineering. Engineering College, Barton Hill, Thiruvananthapuram.

LEBEE A., SAB K., 2011, A bending-gradient model for thick plates. Part I: Theory. *International Journal of Solids and Structures*, **48**, 20, 2878–2888.

LEBEE A., SAB K., 2012, Homogenization of thick periodic plates: Application of the Bending-Gradient plate theory to a folded core sandwich panel, *International Journal of Solids and Structures*, **49**, 19-20, 2778–2792.

LEWINSKI T., 1991, Effective models of composite periodic plates .1: asymptotic solution. *International Journal of Solids and Structures,* **27**, 9, 1155–1172.

PETRACCA M., PELA L., ROSSI R., OLLER S., CAMATA G., SPACONE E., 2017, Multiscale computational first order homogenization of thick shells for the analysis of out-of-plane loaded masonry walls, *Computer Methods in Applied Mechanics and Engineering,* 315, 273–301.

Pruchnicki, E., 2020. Non linear homogenization of heterogeneous periodic plates of Reissner-Mindlin type. Journal of Theoretical and Applied Mechanics, 58(2):317–323.

Pruchnicki, 2021. Linear and nonlinear homogenization for Cosserat plate model.

which will be submitted soon.

TERADA K., HIRAYAMA N., YAMAMOTO K., MURAMATSU M., MATSUBARA S., NISH, S-N., 2016, Numerical plate testing for linear two-scale analyses of composite plates with in-plane periodicity. *International Journal of Numerical Methods in Enging* 2016; **105**, 2, 111–137.

Parsons, E.M, Weerasooriya, T, Sarva, S and Socrate, S. Impact of woven fabric: Experiments and mesostructure-based continuum-level simulations. *Journal of the Mechanics and Physics of Solids* 2010; 58 : 1995–2021.

Shirani, M, Luo, C and Steigmann, D.J. Cosserat elasticity of lattice shells with kinematically independent flexure and twist. *Continuum Mechanics and Thermodynamics* 2018; 1-11, doi.org/10.1007/s00161-018-0679-x.

Steigmann, D.J. and Pipkin, A.C. Equilibrium of elastic nets. *Phil. Trans. R. Soc. Lond* 1991; A335 : 419-454.

Steigmann, D.J. and Dell'Isola, F. Mechanical response of fabric sheets to three-dimensional bending, twisting and stretching. *Acta Mechanica Sinica* 2015; 31 : 373-382.

Steigmann, D.J. Continuum theory for elastic sheets formed by inextensible crossed elasticae. *International Journal of Non linear Mechanic* 2018; doi.org/10.1016/j.ijnonlinmec.2018.03.012.

Pruchnicki, E, Chen, X and Dai, H.H. New refined model for curved linear anisotropic rods with circular cross section. Applications in Engineering Science. 6, June 2021, 100046.