5. Fracture Toughness

References:

J. W. Hutchinson, *Notes on Nonlinear Fracture Mechanics* (http://imechanica.org/node/755);
Alan Zehnder, *Lecture Notes on Fracture Mechanics* (http://hdl.handle.net/1813/3075).

Griffith's energy criterion for fracture. For a "perfectly brittle" material with a surface energy density γ, a necessary condition for crack initiation and growth is: $G \geq 2\gamma$. Griffith proposed that fracture would occur when the energy release rate G attains the critical value, $G_c = 2\gamma$. Thus the criterion for fracture initiation in a perfectly brittle material can be written as: $G = G_c$, where the left hand side is regarded as being applied and therefore a function of geometry and load, while the right hand side G_c is a material parameter. The same condition applies for quasi-static crack growth in a perfectly brittle material, which means that the energy release rate is a constant during the quasi-static growth.

Using the relationship between energy release rate and stress intensity factor (mode I only), the condition for fracture initiation and quasi-static propagation can be rewritten as: $K = K_c$, where $K_c = \sqrt{E' G_c}$.

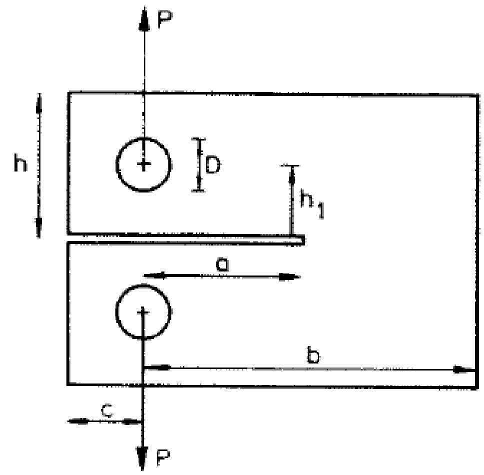
Both G_c and K_c are called *fracture toughness*, which may be used interchangeably but differentiated from each other by their units. G_c is also called fracture energy sometimes.

Role of plasticity under small-scale yielding conditions. Orowan and Irwin (1950s) argued that the same fracture criterion ($G = G_c$ or $K = K_c$) would still apply in the presence of plastic yielding of the material at the crack tip under small-scale yielding conditions if G_c is modified to include the plastic energy dissipation per unit area of crack growth. For most metals, G_c interpreted this way is several orders of magnitude larger than 2γ, except under extremely brittle conditions such as at very low temperatures. Realizing that it is virtually impossible to calculate G_c as a basic material property, Orowan and Irwin proposed that G_c be an experimentally determined quantity using the fracture condition in conjunction with measured load at which a crack starts to grow in a specimen.

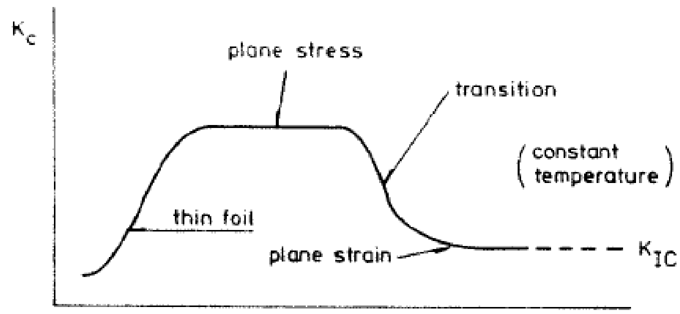
A partition of labor. The fracture criterion effectively illustrates two aspects in fracture mechanics. On one side, we ought to be able to determine the energy release rate or stress intensity factors as the driving force for fracture. Under the small scale yielding condition, this can be done by solving a boundary value elasticity problem (analytically or numerically). On the other side, we shall measure the fracture toughness (G_c or K_c) as a material property.

Compact tension specimen. For the standard ASTM compact tension specimen (h = 0.6b, h_1 = 0.275b, D = 0.25b, c = 0.25b, thickness w), the stress intensity factor at the crack tip is given by

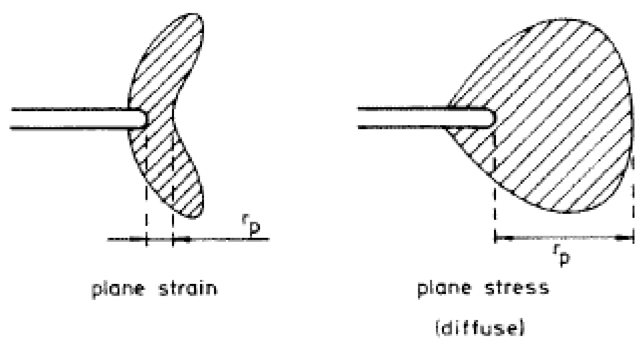
$$K_I = \frac{P\sqrt{a}}{wb} F\left(\frac{a}{b}\right)$$



where $F(x) = 29.6 - 185.5x + 655.7x^2 - 1017x^3 + 63.9x^4$ for $0.4 < x < 0.6$. The initial crack is created by fatigue crack growth from the notch under a cyclic load of low magnitude. During the test, the applied load is recorded along with the displacement at the loading point. The critical load for the onset of crack growth (P_c) is determined from the peak load at the load-displacement curve. The fracture surface is examined after the test, and the initial crack length a is measured with the crack tip located by observing the fatigue beachmarks. The fracture toughness K_c is then calculated by using the measured critical load and crack length in the above equation.



Using a set of specimens with different thickness w, a plot of K_c as a function of w is depicted in the figure, with all tests conducted at the same temperature. With possible exception for some very thin foils, the lowest value of K_c is found for thick specimens, for which plane strain conditions prevail along most of the crack front in the interior of the specimen. For sufficiently large thickness, K_c is found to approach a constant that is denoted by K_{Ic} and commonly referred to as the *fracture toughness* of the material under mode I, plane strain conditions. More details of the standard K_{Ic} test can be found in ASTM publications (e.g., Fracture toughness testing methods, ASTM STP 381, 1965).



The size of the plastic zone under plane strain conditions is given by

$$r_p = \frac{1}{3\pi}\left(\frac{K}{\sigma_y}\right)^2$$

with σ_y being the tensile yield stress. The plastic zone size increases as the load (K) increases, until it reaches the critical condition ($K = K_{Ic}$) for the initiation of crack growth. The plastic zone size remains a constant during quasi-static crack growth (steady state). Thus, the maximum plastic zone size under the quasi-staic, plane strain conditions is

$$r_{pc} = \frac{1}{3\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2.$$

Since both σ_y and K_{Ic} are considered material properties, r_{pc} represents a characteristic length of the material. Size matters in fracture mechanics!

Small scale yielding conditions require that the crack length $a >> r_{pc}$. For the compact tension specimen,

$$a \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

is usually sufficient. Furthermore, plane strain conditions require that the specimen thickness $w >> r_{pc}$. The ASTM standard for the K_{Ic} tests requires

$$w \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2.$$

Extensive data for K_{Ic} is now available. Representative data for several metals is listed in the table below, along with epoxy and glass for comparison. The combination of high toughness and low yield stress in some metals leads to relatively large plastic zone sizes at fracture and also values of K_c much higher than K_{Ic} under plane stress conditions. It should also be noted that the fracture toughness of many metals is a strong function of temperature, characterized by a ductile-to-brittle transition as the temperature decreases (low fracture toughness at low temperature).

Material	T (°C)	σ_y (MPa)	K_{Ic} (MPa-m$^{1/2}$)	r_{pc} (mm)
6061-T651 (Al)	20	269	33	5
7075-T651 (Al)	20	620	36	0.35
AISI 4340	0	1500	33	0.05
A 533-B	93	620	200	11
Epoxy	20	~50	~0.5	0.01
Silica glass	20	7200	~1.0	2×10^{-6}

Example: design against yield or fracture. Consider a cantilever beam to hold a dead load W at the free end. The design parameters include the cross section (assumed to be rectangle of width B and height H) and length L as well as relevant material properties. By the elementary theory for linear elastic beams, the maximum tensile stress is

$$\sigma_{\max} = \frac{6WL}{BH^2}$$

which occurs at the fixed end of the cantilever. If a material of low yield strength, high fracture toughness is to be used, a safe design requires no yielding, i.e., $\sigma_{\max} < \sigma_y$. On the other hand, if a material of high yield strength, low fracture toughness is considered, fracture condition must be evaluated for a safe design. Assume a small edge crack near the fixed end of the cantilever (worst scenario), the stress intensity factor is approximately

$$K = 1.12\sigma_{\max}\sqrt{\pi c}$$

where the crack size c is assumed to be small compared to the other dimensions of the beam (B, H, L). Thus, a requirement for the beam design against fracture is: $K < K_{Ic}$, or $\sigma_{\max} < \frac{K_{Ic}}{1.12\sqrt{\pi c}}$. For any arbitrary material, the yield condition prevails if $\sigma_y < \frac{K_{Ic}}{1.12\sqrt{\pi c}}$, and the fracture condition prevails if $\sigma_y > \frac{K_{Ic}}{1.12\sqrt{\pi c}}$. The defect size c depends on the material processing and may be estimated.