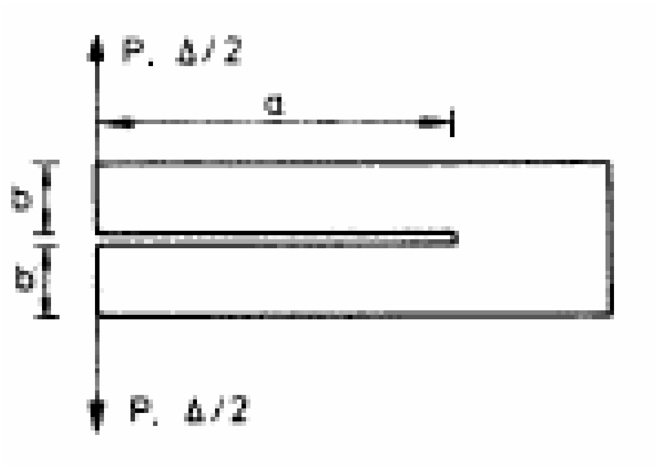
An illustration of the variation of stress intensity factor with crack length for prescribed load & prescribed displacement



Using results from slide on DCB specimen

$$K = 2\sqrt{3}\frac{Pa}{b^{3/2}}, \quad \Delta = \frac{4Pa^3}{\bar{E}b^3} \quad (P \text{ is the force/thickness})$$

$$K = \frac{\sqrt{3}\bar{E}b^{3/2}}{2a^2}\Delta$$

For fixed P, K increases as a increases.

For fixed Δ, K decreases as a increases.

With initial crack length, a_0, load up to P_0

with $\Delta_0 = 4P_0 a_0^3 / (\bar{E}b^3)$ and $K_0 = \frac{2\sqrt{3}P_0 a_0}{b^{3/2}}$.

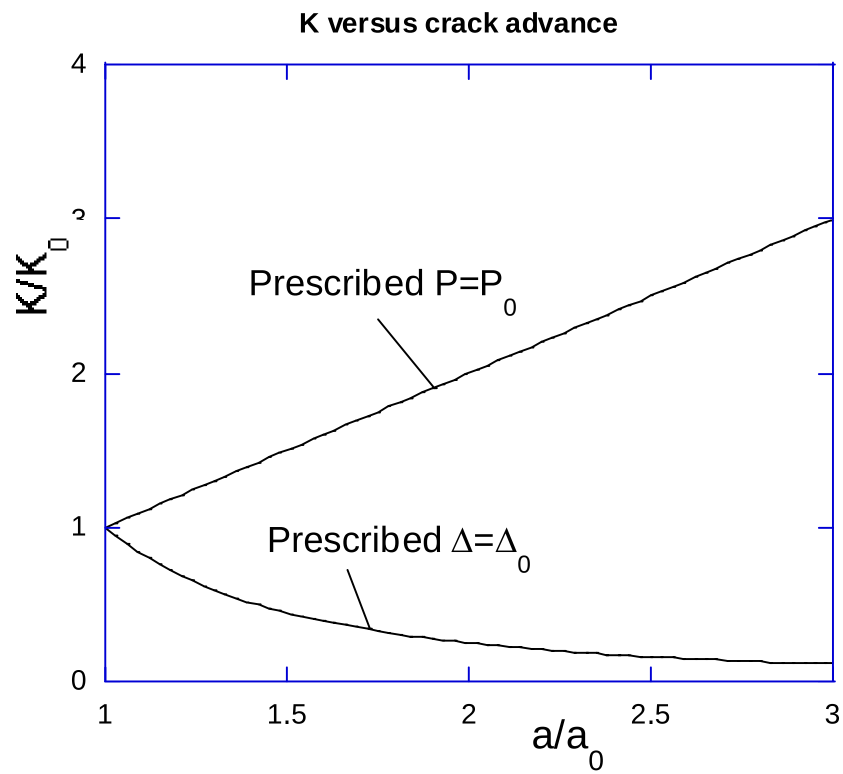
If we fix $P = P_0$ and increase the crack length,

$$\frac{K}{K_0} = \frac{a}{a_0}$$

However, if we fix $\Delta = \Delta_0$

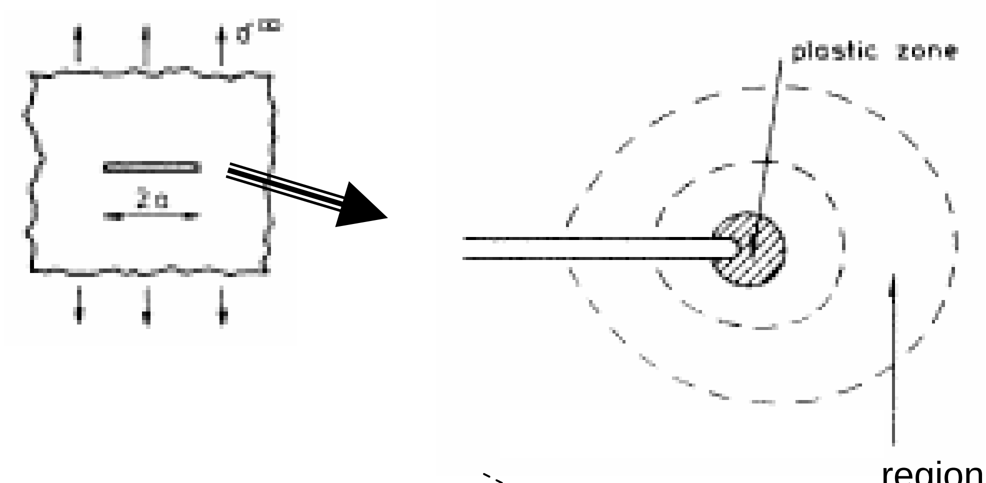
and increase the crack length,

$$\frac{K}{K_0} = \left(\frac{a_0}{a}\right)^2$$



See plot for very different trends

The concept of small scale yielding for cracks in metallic materials



$\sigma_Y \sim$ tensile yield stress

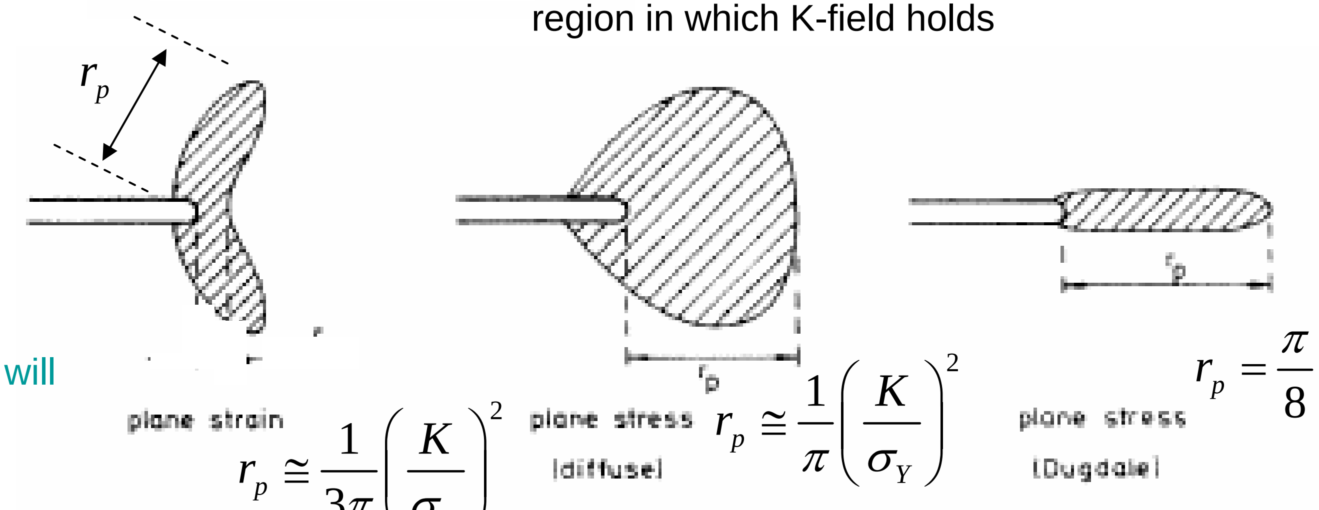
$L \sim$ crack length (a)

or other relevant length

region in which K-field holds

Mode I small scale yielding plastic zones

Details of these shapes will be discussed later



$$r_p \cong \frac{1}{3\pi}\left(\frac{K}{\sigma_Y}\right)^2$$

$$r_p \cong \frac{1}{\pi}\left(\frac{K}{\sigma_Y}\right)^2$$

$$r_p = \frac{\pi}{8}\left(\frac{K}{\sigma_Y}\right)^2$$

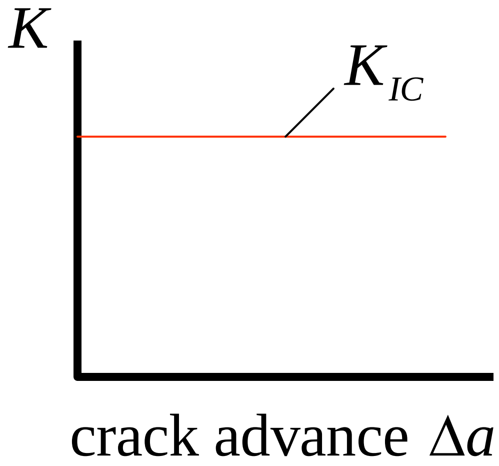
Small scale yielding requires that r_p be sufficiently small compared to L such that there exists a region surrounding the plastic zone in which the K-field is accurate.

For the crack of length $2a$ in an infinite sheetor slab, this condition requires

$$r_p << a/4.$$

Griffith-Irwin Criterion for Mode I Crack Advance in an Ideally Brittle Material

(pg. 14 of notes)



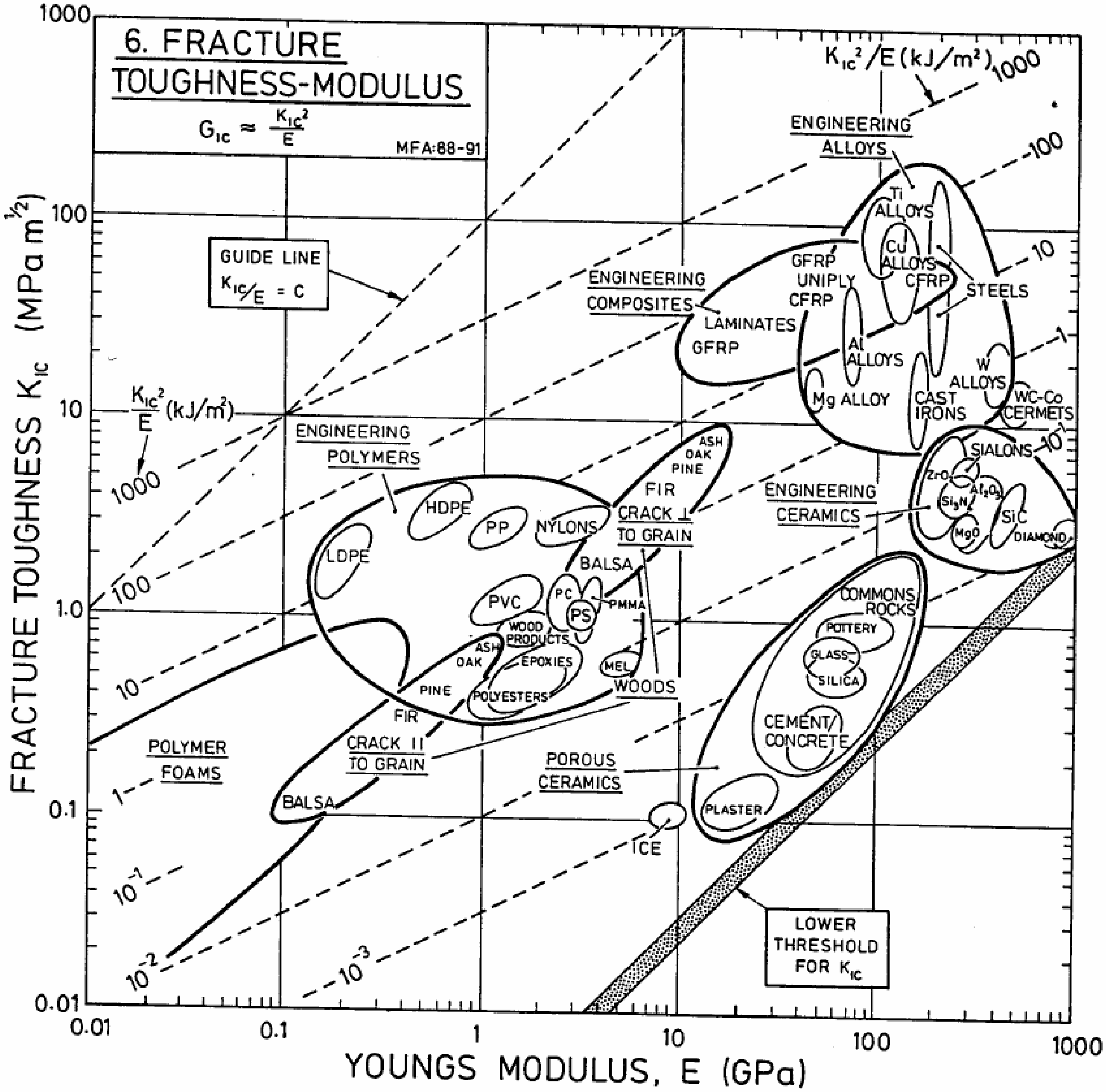
Quasi-static advance occurs when $K = K_{IC}$

K_{IC} is called the **fracture toughness** (units $Pa\sqrt{m}$)

$$G_{IC} = \frac{(1-\nu^2)K_{IC}^{\ 2}}{E}$$ is also called the fracture toughness (units J/m^2)

We will discuss experimental methods to measure fracture toughness later. To use LEFM, the test must satisfy small scale yielding conditions in plane strain. In the table Below we list representative values of yield strength, fracture toughness and plastic zone size for four metals ranging from relatively low toughness to high toughness.

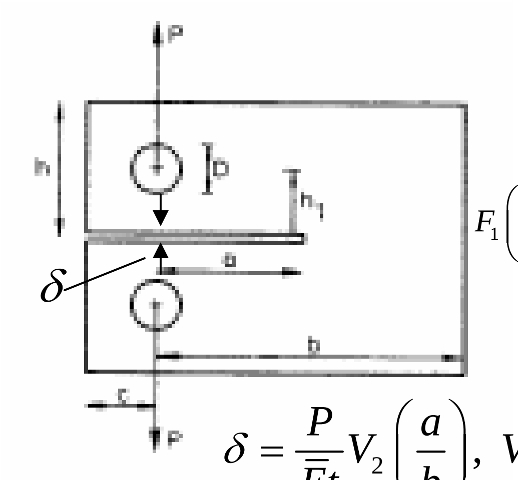
material	T (^{o}C)	σ_Y (MN/m^2)	K_{IC} $(MPa\sqrt{m})$	G_{IC} (J/m^2)	r_p (mm)
$6061-T651(Al)$	20	269	33	14,000	5
$7075-T651(Al)$	20	620	36	16,600	0.35
AISI 4340 (Steel)	0	1500	33	4900	0.05
A533B (Steel)	93	620	200	180,000	260
Typical ceramic	20	??	0.3 – 0.6	5 – 20	??



From *Material Selection in Mechanical Design,* M.F. Ashby, Pergamon Press

Crack Advance in Ideally in a Brittle 4340 Steel Compact Tension Specimen

ASTM compact tensile specimen



$$K_I = \frac{P}{bt}\sqrt{a}\;F_1\left(\frac{a}{b}\right)$$

$$F_1\left(\frac{a}{b}\right) \cong 11.7\left(\frac{0.6 - a/b}{0.2}\right) + 17.6\left(\frac{a/b - 0.4}{0.2}\right)$$

(see earlier slide)
And from Tada, pg. 62

choose : $b = 2in = 0.0508m$

$h = 0.6b$

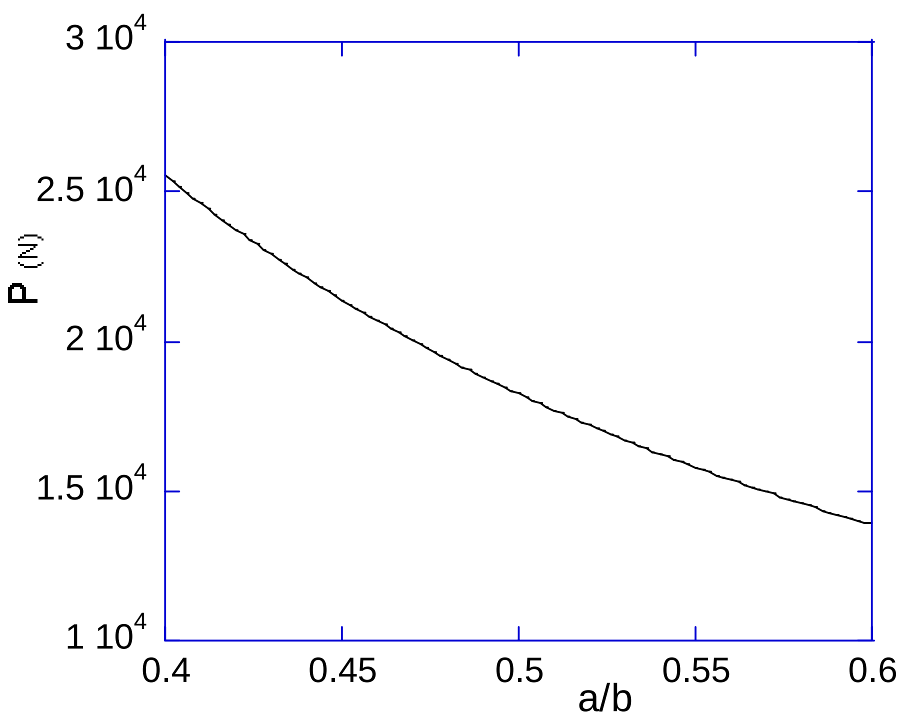
$h_1 = 0.275b$

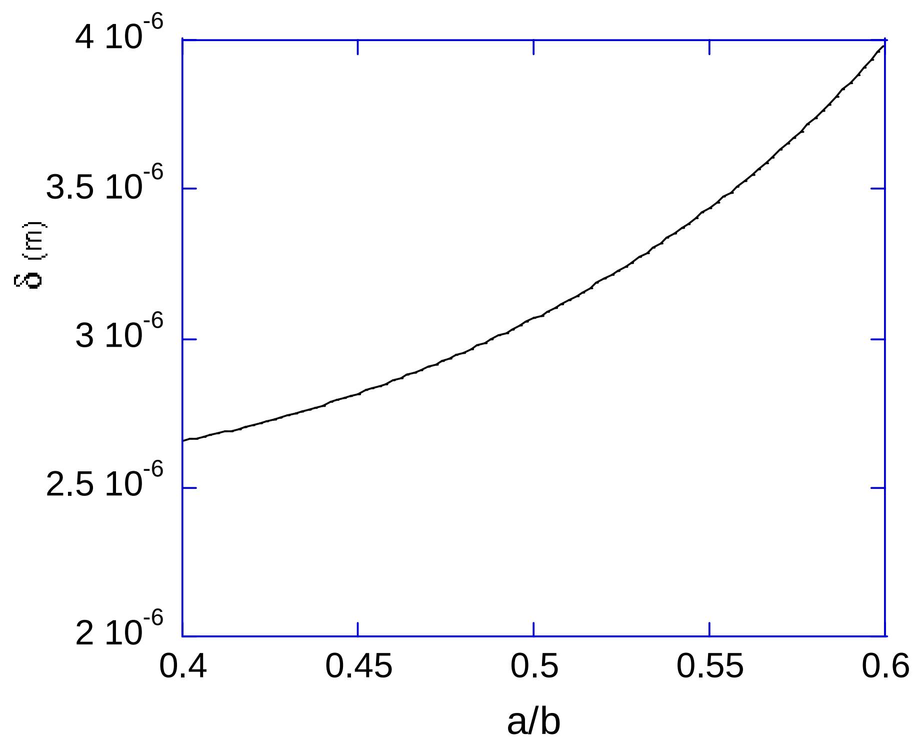
$c = D = 0.25b$

$thickness \equiv t = b/2$

$K_{IC} = 33MPam^{-1/2},\; E = 200GPa,\; \nu = 0.29$

$$\delta = \frac{P}{\bar{E}t}V_2\left(\frac{a}{b}\right),\; V_2(x) = \left(\frac{1+x}{1-x}\right)^2\left(2.163 + 12.219x - 20.065x^2 - 0.9925x^3 + 20.609x^4 - 9.9314x^5\right)$$

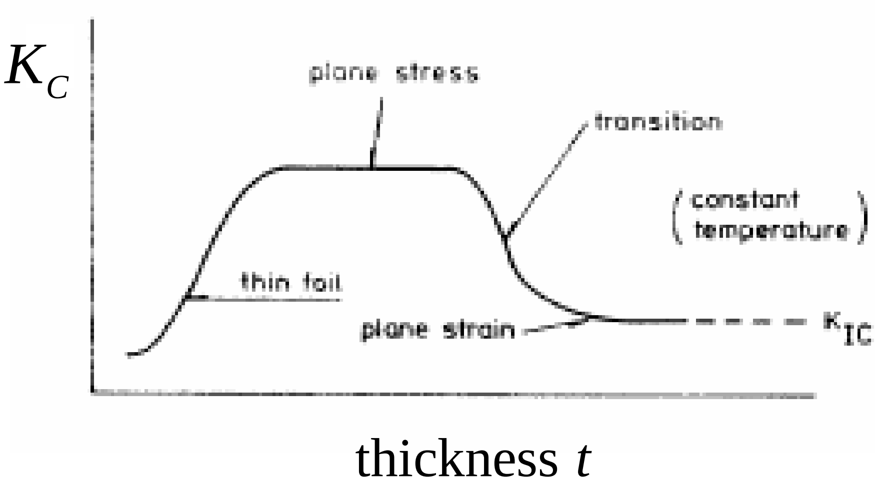




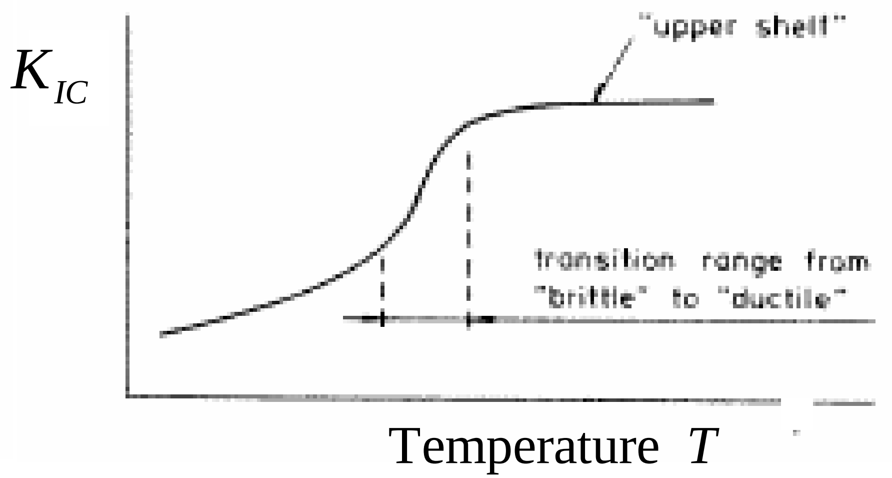
Thickness and Temperature Dependence of Fracture Toughness

(pg. 16 & 17 of notes)

Schematic of Experimental Thickness dependence



Schematic of Experimental Temperature dependence for a steel



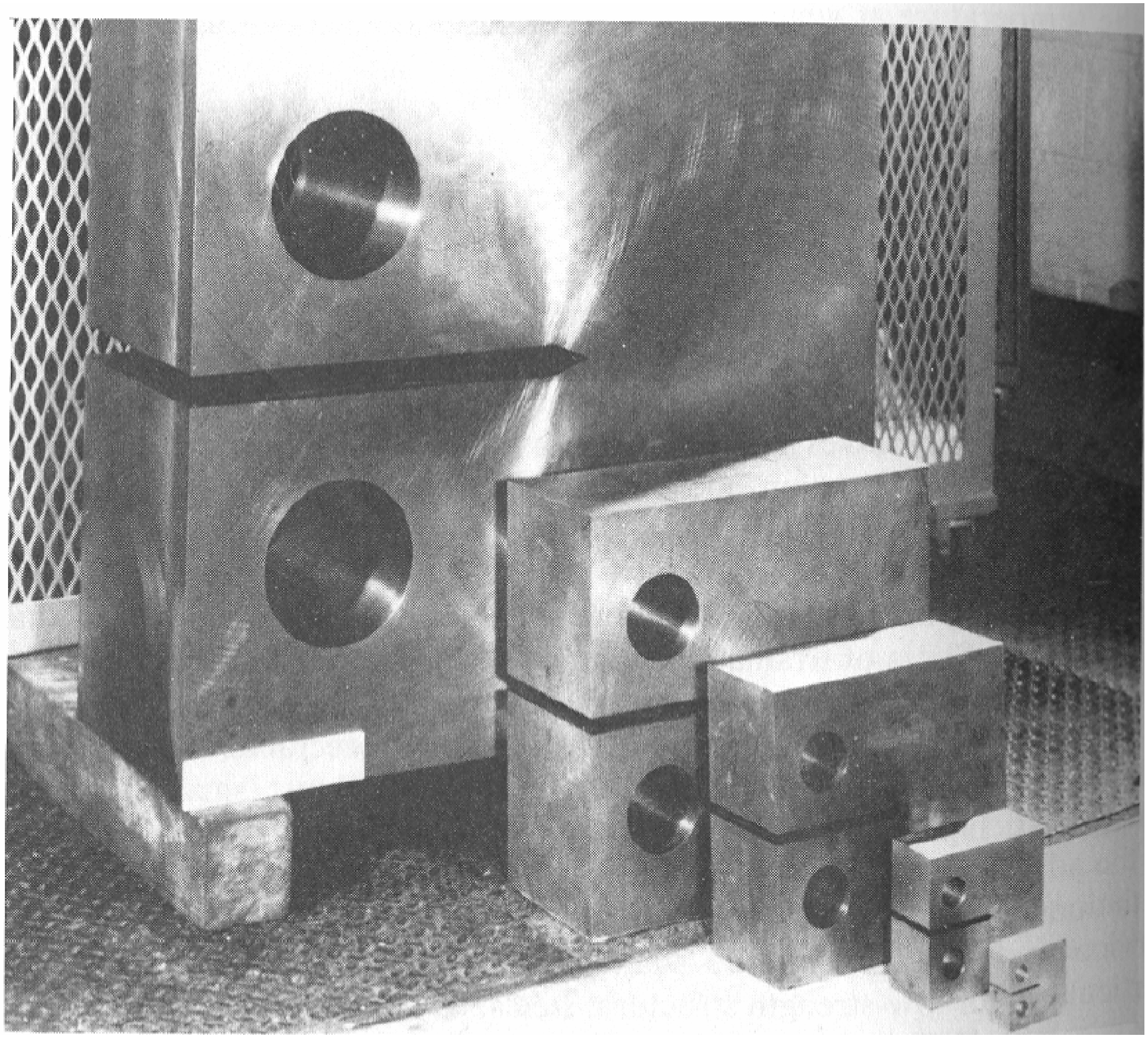
Lower temperature range governed by cleavage

"Upper shelf" range governed by ductile void nucleation, growth an coalescence mechanism

The mechanism transition is referred to as the brittle to ductile toughness transition.

ASTM requirement for valid K_{IC} test :

$$t > 2.5\left(\frac{K_{IC}}{\sigma_Y}\right)^2 \cong 25 r_p \quad \& \quad t > 2.5\left(\frac{K_{IC}}{\sigma_Y}\right)^2 \cong 25 r_p$$



Compact tension specimens for valid KIC testing. The smallest specimen is about 2 inches wide. The largest specimen was used to obtain the toughness of a very tough pressure vessel steel (A533B), as required by LEFM.