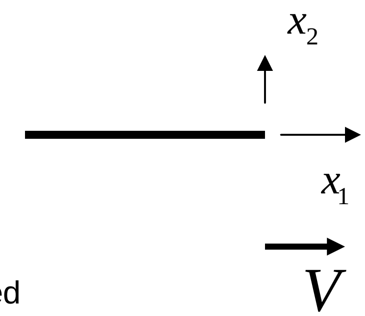
INTRODUCTION TO DYNAMIC FRACTURE

Basic reference: L.B. Freund (1990) *Dynamic fracture mechanics,* Camb. Univ. Press



Crack tip fields for isotropic elastic solid

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}}\Sigma_{ij}(\theta, m, \nu); \quad m = \frac{V}{c_r} \text{ (plane strain)}, \quad m = \frac{V}{c_s} \text{ (mode III)}$$

c_r: Raleigh wave speed; c_s: Shear wave speed

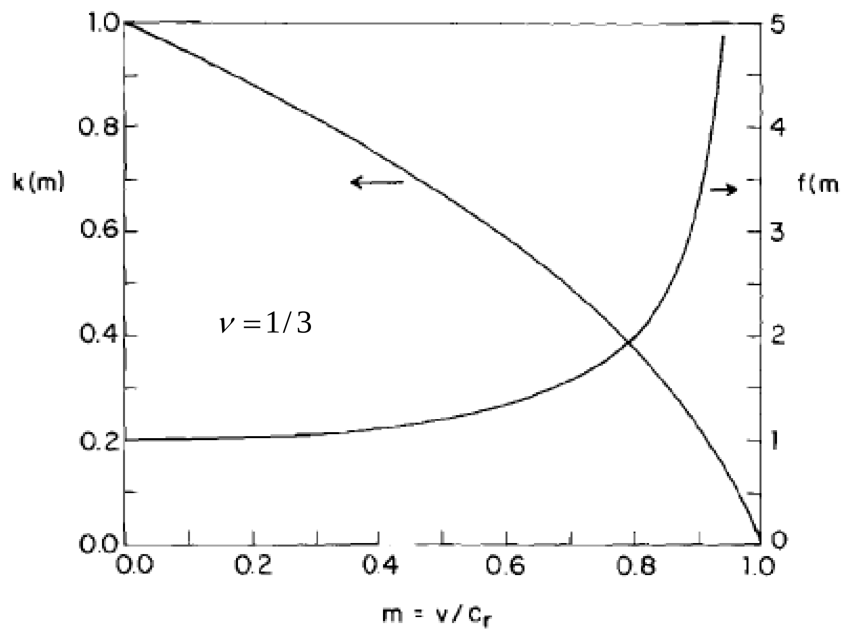
$$\sigma_{22} = \frac{K}{\sqrt{2\pi r}}, \; \theta = 0 \quad \text{(plane strain)}; \qquad \sigma_{23} = \frac{K}{\sqrt{2\pi r}}, \; \theta = 0 \quad \text{(mode III)}$$

$$\Sigma_{23} + i(1-m^2)^{1/2}\Sigma_{13} = \left(\cos\theta - i(1-m^2)^{1/2}\sin\theta\right)^{-1/2} \quad \text{for mode III}$$

Energy release rate:

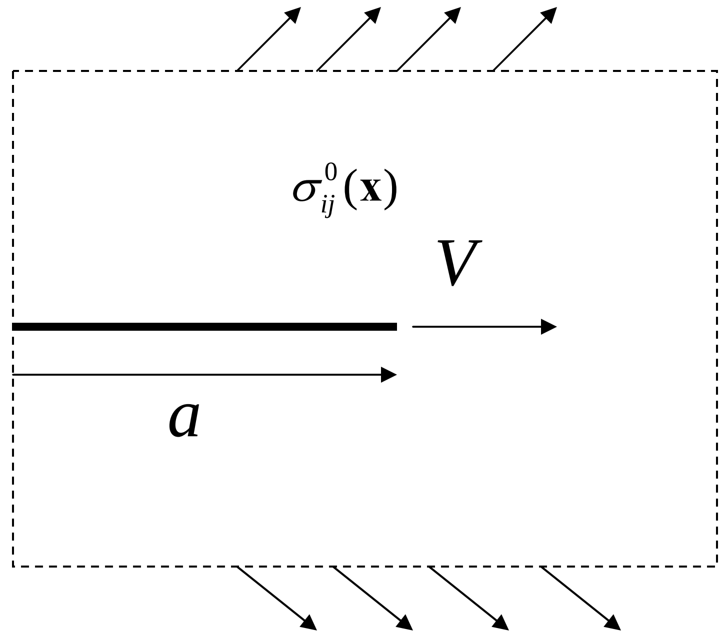
$$G = f(m)\frac{(1-\nu^2)K^2}{E} \quad \text{(plane strain, mode I)}$$

$$G = (1-m^2)^{-1/2}\frac{K^2}{2\mu} \quad \text{(mode III, } \mu\text{=shear modulus)}$$



Details are in Freund and Hutch, 1985, JMPS 33, 169-191.

Elastio-dynamic crack solutions



Analytic dynamic crack solutions are very few--see Freund's book. Any solution provides K and G as a function of applied loads and crack tip velocity history.

The following fairly general result (Freund) is useful for understanding dynamic fracture. Consider an infinite block of material as depicted on the right which at rest is subject to a symmetric stress distribution $\sigma_{ij}^0(\mathbf{x})$ due to remote applied loads. Let K_S be the static stress intensity factor for a crack instantaneously at crack length a, and let $G_S = (1-\nu^2)K_S{}^2/E$.

The dynamic stress intensity factor and energy release rate for a dynamically running crack is given by the following result as long as no reflected stress waves reach the crack tip:

$$K = k(m)K_S, \qquad G = f(m)k(m)^2 G_S \cong (1-m)G_S$$

The reduction of the dynamic energy release rate with increasing crack tip velocity relative to the corresponding static rate reflects "inertial shielding" of the tip.

Generalization of the J-integral for **steady - state,** dynamic crack growth. With

$$W = \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij} \quad \& \quad T = \frac{1}{2}\rho \dot{u}_i \dot{u}_i = \frac{1}{2}\rho V^2 u_{i,1}{}^2$$

$$J = \int_\Gamma \left[\left(W + U\right) n_1 - \sigma_{ij} n_j u_{i,1} \right] ds$$

For elastic problems, $J = G$

J is path independent for steady-state problems but not for general crack motion. Nevertheless, by using contours closely surrounding the crack tip, one can use the line intregal to evaluate G.

Dynamic toughness

Crack growth criterion in terms of the dynamic stress intensity factor:

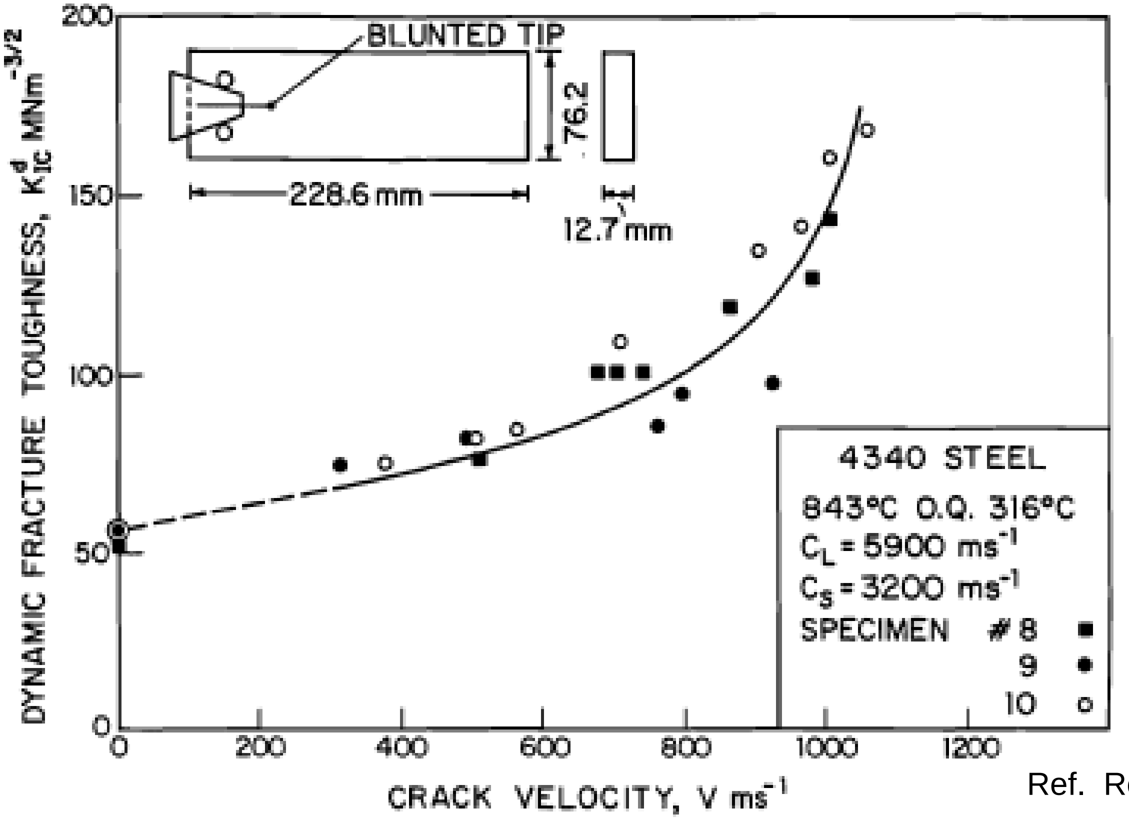
$$K(V) = K_{IC}(V)$$

where $K_{IC}(V)$ is called the **dynamic fracture toughness.**

Given a specimen that has been analyzed, with dynamic stress intensity as a function of crack tip velocity known, perform experiment to measure velocity as a function of dynamic stress intensity (very difficult!!). The result provides $K_{IC}(V)$.

The data shown is for a high strength steel (4340) in the range $0 < m < 1/3$.

Other steels also show a dramatic increase in toughness for $m \approx 1/3$.



A initially blunted crack tip is used to allow loads much in excess of the static fracture load. Thus, when the crack does advance the static stress intensity factor is much larger than the static KIC. The crack accelerates to a high velocity.

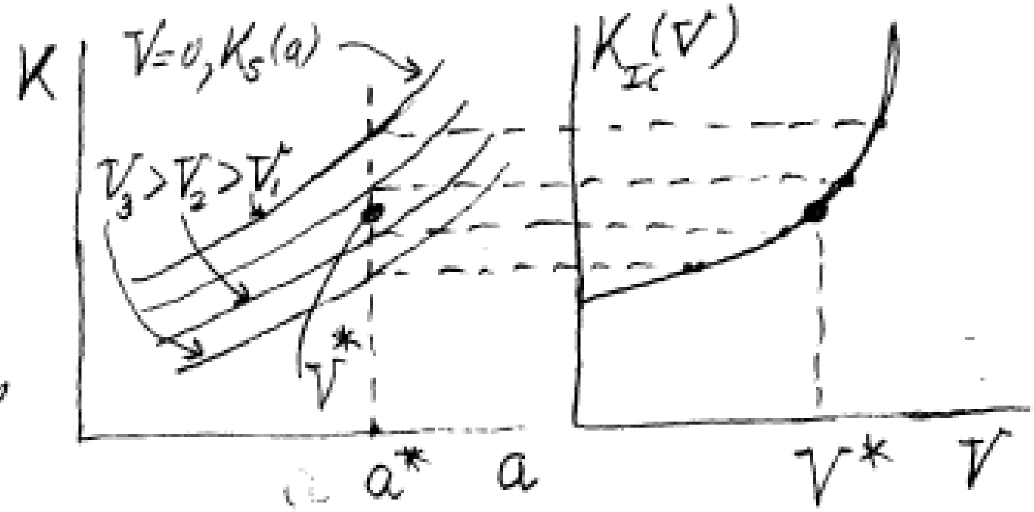
Ref. Rosakis, Duffy & Freund (1984) JMPS 32, 443.

GRAPHICAL "SOLUTION" FOR MONOTONIC $K_{Ic}(V)$

For $K_S(a)$ <u>increasing</u> with a

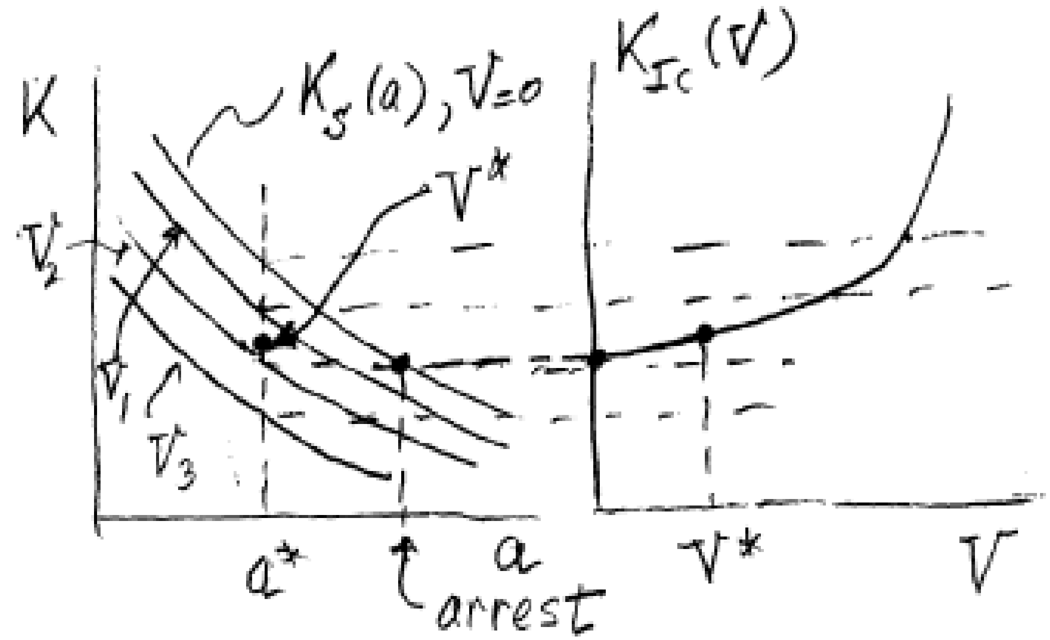
Recall, $K \approx (1-m)K_S$

In this case, V increases as a increases.

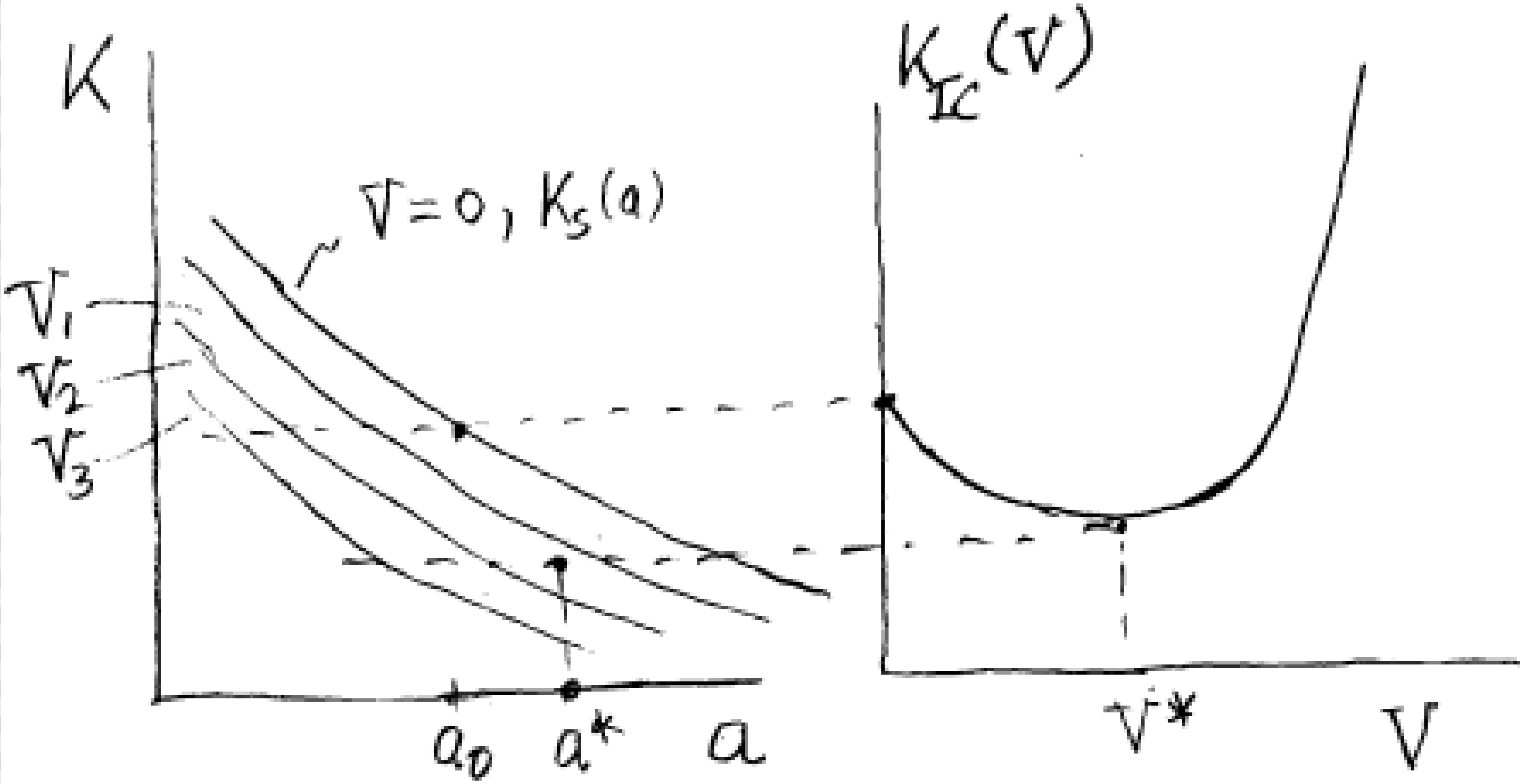


For $K_S(a)$ <u>decreasing</u> with a

Note that the crack arrests where $K_S(a) = K_{Ic}(0)$



GRAPHICAL "SOLUTION" FOR NON-MONOTONIC $K_{Ic}(V)$

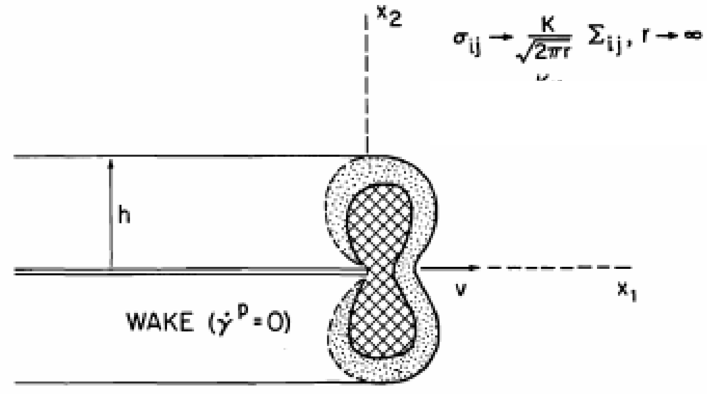


Crack arrests at $K_S(a) < K_{Ic}(0)$

Crack jumps ahead when KS=KIC(0) and arrests.

Basic plasticity aspects of dynamic toughness in small scale yielding Mode III- steady-state growth.

Ref. Freund and Douglas (1982) JMPS 30, 59.



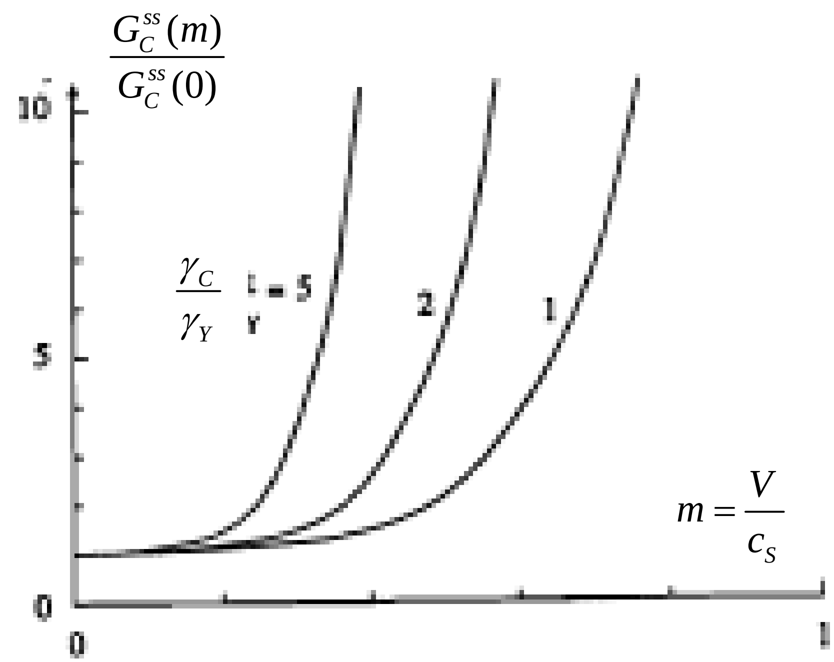
Elastic - perfectly plastic $(\tau_Y, \gamma_Y = \tau_Y / \mu)$.

Propragation criterion: $\gamma^P = \gamma_C$ at $x_1 = x_C$ (ahead of tip)

Quasi-static (V=0): $G_C^{ss}(0) = 1.69\left(\frac{x_C \tau_Y^{\,2}}{\mu}\right)\exp\left(\left(1 + 2\frac{\gamma_C}{\gamma_Y}\right)^{1/2} - 1\right)$

Freund &Douglas derived the result for the dynamic energy release in steady-state, $G_C^{ss}(m)$.

The plot of $G_C^{ss}(m)/G_C^{ss}(0)$ for various values of γ_C/γ_Y is shown to the right.

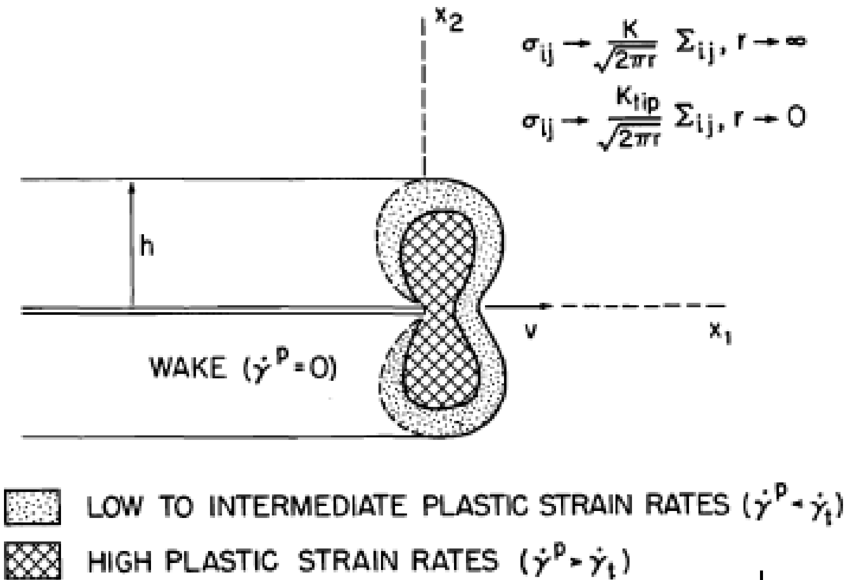


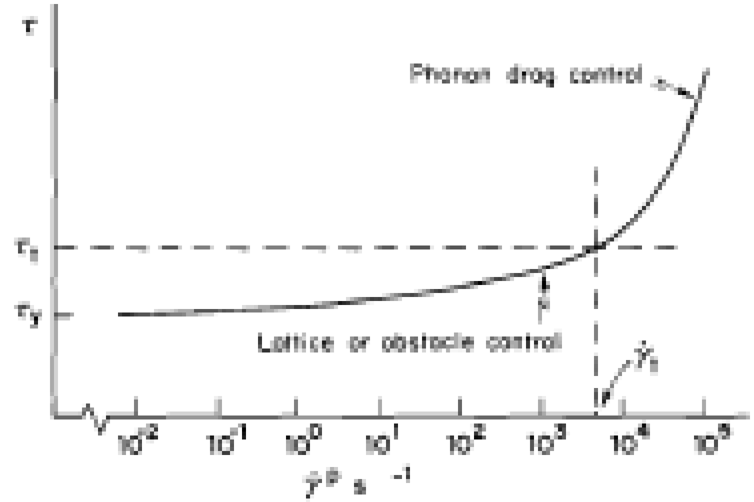
No material rate-dependence in this model. The dependence on crack velocity is due entirely to inertia.

For the same material model, similar trends have been obtained for plane strain, mode I.

Basic plasticity aspects of dynamic toughness in small scale yielding
Mode I- steady-state growth with material rate-dependence

- Ref. Freund and hutch. (1985) JMPS 33, 169-191. Mataga, Freund, Hutch (1987) J. Phys. Chem. Solids 48, 985





Relation of stress to plastic strain-rate at a given temperature showing the transition between high strain-rate regime and low to intermediate strain-rate regime.

Important fact: with $\dot{\gamma}^P \propto \tau^n$ for large stresses, elastic crack tip fields dominate at the crack tip for all $n < 3$.

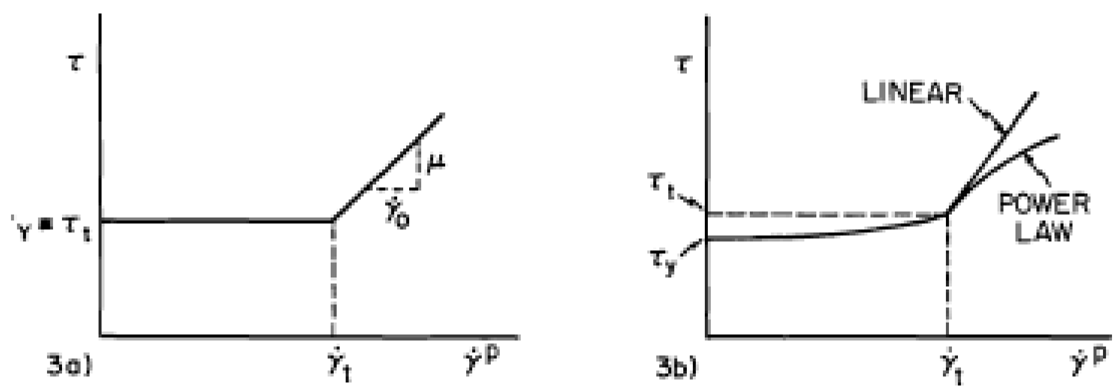
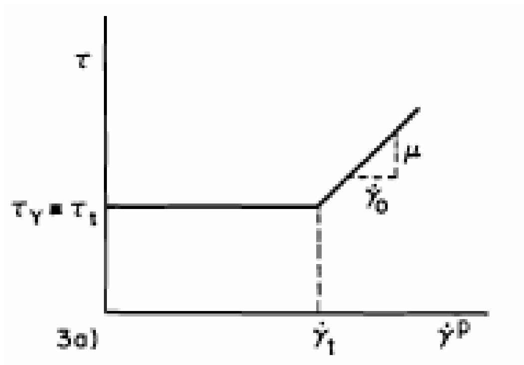


Fig. 3 Visco-plastic idealizations of material behavior. (a) Elastic/perfectly plastic at low strain rate, linear stress dependence at high strain rate. (b) Weak stress dependence at low strain rate, linear or power law stress dependence at high strain rate.

Basic plasticity aspects of dynamic toughness in small scale yielding
Mode I- steady-state growth with material rate-dependence--continued



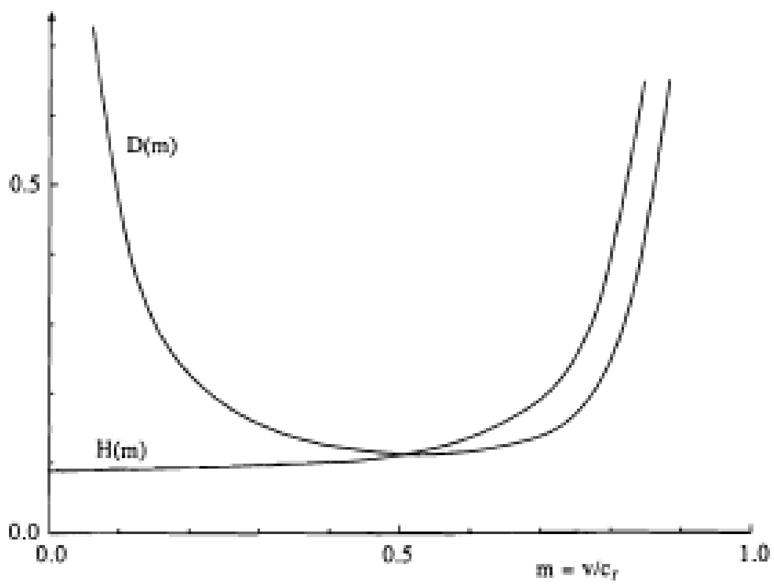
Idealized rate-dependent constitutive model:

$$\dot{\gamma}^P = \dot{\gamma}_t + \dot{\gamma}_0(\tau - \tau_Y)/\mu \quad \text{for } \tau > \tau_Y \;\&\; \dot{\gamma}^P > \dot{\gamma}_t$$

Analytical result for energy release rate at tip related to the "remote" energy release rate:

$$G_{tip} = G\left[1 + H(m)\left(\frac{\gamma^* \mu^2}{\tau_Y{}^3 V}\right)G\right]^{-1}$$

with $\gamma^* = \frac{1}{3}\dot{\gamma}_t + \frac{1}{3}\dot{\gamma}_0 \frac{\tau_Y}{\mu}$



Impose fracture criterion: $G_{tip} = G_{tip}^C(V)$

$$\Rightarrow \frac{G}{G_{tip}^C(V)} = \frac{1}{1 - D(m)P_C}$$

where

$$P_C = \frac{1}{3}\left(1 + \frac{2\dot{\gamma}_t \mu}{\dot{\gamma}_0 \tau_Y}\right)\frac{\dot{\gamma}_0 \mu G_{tip}^C(V)}{c_s \tau_Y{}^2}, \quad H(m) = \frac{1}{2}\frac{H(m)}{m}\frac{c_S}{c_R}$$

Basic plasticity aspects of dynamic toughness in small scale yielding
Mode I- steady-state growth with material rate-dependence--continued

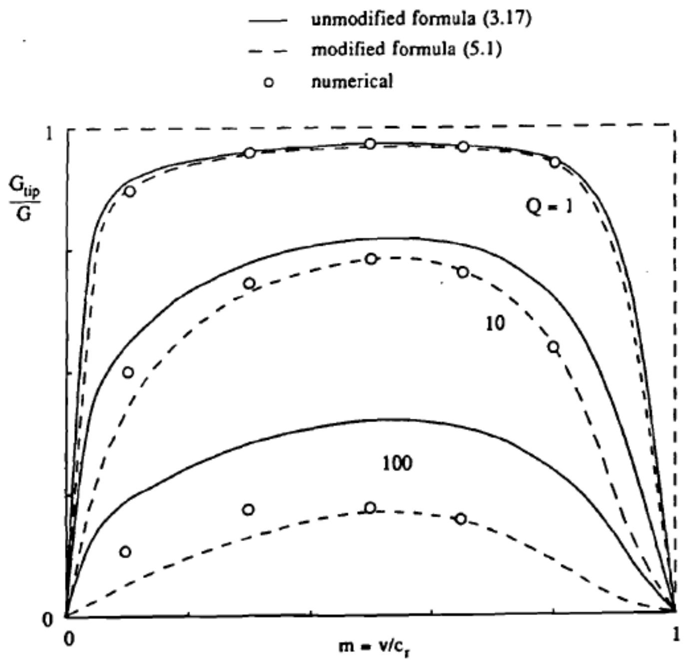
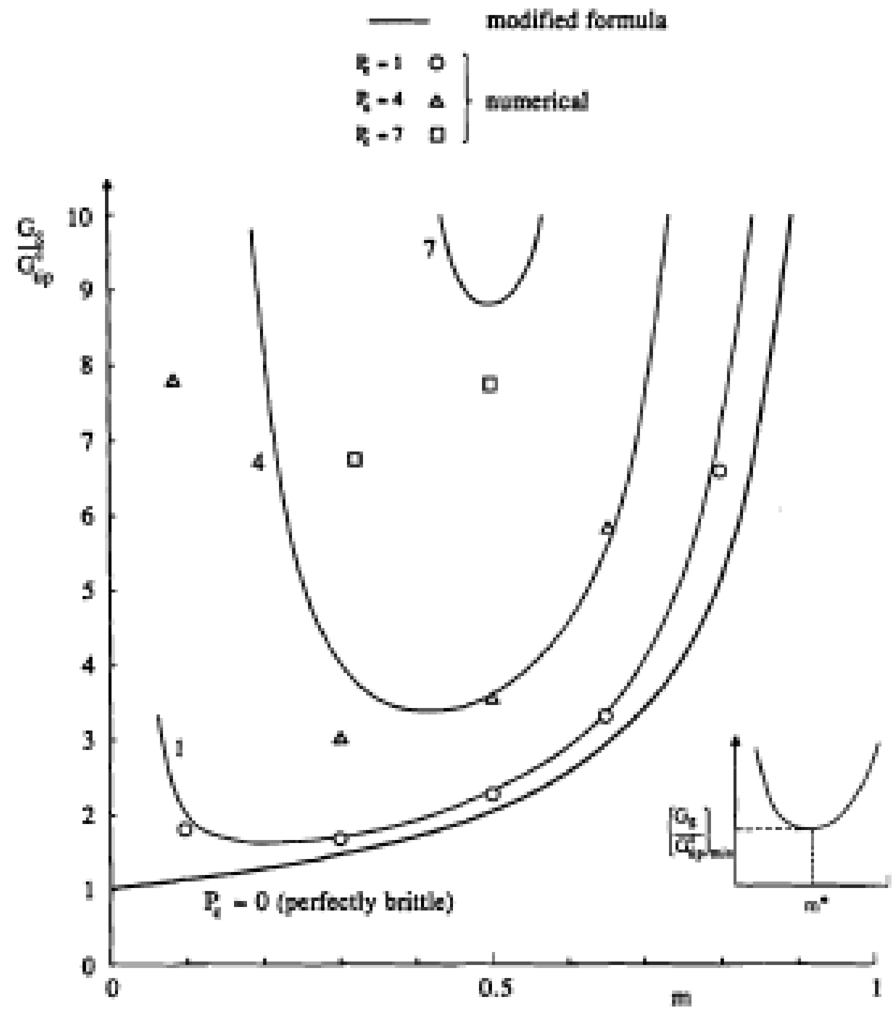


Fig. 13 Comparison of mode I plane strain dynamic numerical results to asymptotic form
G_{tip}/G as a function of m ($\zeta = 0.01$, $\nu = 0.29$).

$$\text{Using} \quad G = (1-m)G_S \quad \Rightarrow$$



Material rate-dependence gives rise to high toughness
at low velocities (or equivalently a dip in toughness in intermediate velocity range.
At high velocities, inertia again dominates.