Due in class, Thursday, 4 February 2010

1. Stress Concentration around a Circular Hole

Stress concentration at geometric discontinuities is the most important practical result in elasticity theory. For example, for a small circular hole in a large plate under uniaxial stress σ_{appl}, the elasticity solution gives the hoop stress around the hole:

$$\sigma_{\theta\theta} = \sigma_{appl}(1 - 2\cos 2\theta).$$

Here the polar angle θ is measured from the loading direction. The problem is solved in all textbooks of linear elasticity.

a) Under uniaxial tension, indicate the highest tensile stress around the hole.
b) Under uniaxial compression, indicate the highest tensile stress around the hole.
c) Use the above solution and linear superposition to calculate the stress concentration at the hole when the plate is under a shear stress.

2. Griffith's Model for Experimental Strength

Griffith found that the strength of a material relates to the surface energy, elastic modulus, and flaw size, namely,

$$S = \sqrt{\frac{2\gamma E}{\pi c}}\ .$$

Consequently, strength is not a material property, but depends on the flaw size. Consider the glass used by Griffith, with surface energy γ = 1.75 J/m^2 and Young's modules E = 62 GPa.

a) Calculate the tensile strength of a large glass panel with defect size c = 1 μ m.

b) The same glass panel now has a small circular hole of radius a = 1 mm, assuming that the defect size remains to be c = 1 μ m. How much load can the panel carry?

c) What if the circular hole is replaced by a crack-like notch with half-width a = 1 mm?

3. Griffith measured the surface energy of glass

To verify his theory, Griffith had to independently measure the surface energy of glass. He did so at a number of high temperatures and deduced the value of the surface energy at room temperature by extrapolation. The following is an excerpt from his paper (A.A. Griffith, The phenomena of rupture and flow in solids, *Philosophical Transactions of the Royal Society*, **221**, 163-198, 1920).

> Between 730° C and 900° C the method described below was found to be practicable. Fibers of glass about 2 inches long and from 0.002-inch to 0.01-inch diameter, with enlarged spherical ends, were prepared. These were supported horizontally in stout wire hooks and suitable weights were hung on their mid-points. The enlarged ends prevented any sagging except that due to extension of the fibers. The whole was placed in an electric resistance furnace maintained at the desired temperature. Under these conditions viscous stretching of the fiber occurred until the suspended weight was just balanced by the vertical components of the tension in the fiber. The latter was entirely due, in the steady state, to the surface tension of the glass, whose value could therefore be calculated from the observed sag of the fiber. In the experiments the angle of sag was observed through a window in the furnace by means of a telescope with a rotating cross wire. If w is the suspended weight, d the diameter of the fiber, T the surface

tension, and θ the angle at the point of suspension between the two halves of the fiber, then, evidently,

$$\pi dT \sin\frac{\theta}{2} = w.$$

For this method of determining the surface tension to be valid, it is evidently necessary that the angle of sag shall reach a steady value before the development of local contractions, arising from the instability of liquid cylinders, become appreciable.

Actions:
(a) Why he couldn't do this experiment much higher than 900° or much below 730°?
(b) Derive his formula.
(c) For a 0.1 mm diameter fiber to reach 20° steady angle of sag, which is easily observable, how much weight should be suspended? Assume a reasonable magnitude for the surface energy.

4. Griffith listed in his paper, for the glass he used, Young's modulus 62 GPa, tensile strength 170 MPa, and surface tension 1.75 J/m^2. Estimate the flaw size in the glass. Compare it with the typical molecular size and sample size.