Parameters to from macro to micro coeffient

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1 Macroscopic parameters

From macroscopic level, take poroelastic matrix and fluid system as a "lumped" model [2], the solid skeleton is completely determined by either of the two parameters E, v, K, G, λ and Biot effective coefficient α.

$$\sigma = \sigma' - \alpha p \tag{1}$$

The introduction of α allows to get the value of effictive stress which is a combination of both the externally applied stress and the internal pressure of fluid phases and enables the conversion of multiphase porous medium into mechanically equivalent single-phase continuum[7]. As long as we can get the effective stress from total stress and pore pressure, the problem will be a effective stress added on the **solid skeleton material**.

According to mass conservation of fluid phase:

$$\frac{d\zeta}{dt} + \nabla q = 0 \tag{2}$$

The slow rate flow fluid is governed by Darcy's law,

$$q = -\frac{\kappa}{\mu}(\nabla(p - \rho_f g)) \tag{3}$$

Above two equations gives the equation govens the fluid phase in the poroelastic material,

$$S_\varepsilon \frac{\partial p}{\partial t} + \vec{\nabla}\cdot(-\frac{\kappa}{\mu}\vec{\nabla}(p + \rho_f g z)) = -\alpha\frac{\partial}{\partial t}\epsilon_{kk} + Qs \tag{4}$$

If a fluid-saturated porous rock undergoes undrained compression, the confining pressure causes the pores to contract, thereby pressurizing the trapped pore fluid. The ratio of pore pressure increment and confing pressure is defined and Skempton coefficient B, strictly speaking, B is not a propertyof the rock, but rather of the rock/fluid system [5].

$$B = \frac{\partial p_p}{dp_c}$$

In the linearized theoryof poroelasticity, the constitutive parameters are assumed to be independent of stress, fluid content increment ζ is the fluid mass increment which is defined as,

$$\zeta = \alpha\epsilon_b + \frac{\alpha(1 - \alpha\beta)}{K\beta}p \tag{5}$$

This equation gives the physical intepration of coefficient α [3, 10]. For constant pressure p, a dilation change results in a fluid content change $\triangle\zeta = \alpha\triangle\epsilon_b$ so that α represents the ratio of fluid content change to total volume change at constant pressure. For Skempton coefficient, under undrained condition,

2 Microscopic parameters [5].

From above macroscopic level, Biot effective coefficient α and Skempton coefficient B defines the whole solid-fluid system as a lumped model, which takes the a infinistimal point but still large enough to contains pores and solid grains, which reflects the macroscopic response of the system. However, the shortcomings of this approach are that the bulk material constants are tied to a specific solid-pore-fluid system. It is for example not known how these bulk constants are influenced by change in the compressibility of the fluid or in the porosity of the rock. It is thus desirable to look into the "micromechanics" of the solid-pore-fluid system to elicit the dependence of the bulk material coefficients to the micromechanical ones [2].

If the porous media can be taken as isotropic and homogeneous also elastic material, the following equation holds,

$$\alpha = 1 - \frac{K}{K_s} \tag{6}$$

$$B = \frac{\frac{1}{K} - \frac{1}{K_s}}{\phi(\frac{1}{K_f} - \frac{1}{K_s}) + (\frac{1}{K} - \frac{1}{K_s})}$$

$$S_\epsilon = \frac{\alpha(1 - \alpha B)}{KB}$$

Which can be also written in the following form:

$$B = \frac{\alpha K_f}{(\alpha - \phi(1 - \alpha)K_f) + \phi K}$$

$$S_\epsilon = \frac{\phi}{K_f} + \frac{\alpha - \phi}{K_s}$$

3 Parameters in the literature.

$$\zeta = \alpha\epsilon_b + \frac{\alpha(1 - \alpha\beta)}{K\beta}p \tag{7}$$

α represents the ratio of change in fluid content to change in parenchymal volume when pore pressure does not change shows that S_ϵ is a type of capacitance, the amount of interstitial fluid that can be forced into an unchanging volume of parenchyma per unit increase of pore pressure[3].

The specific storage term can be written in either macroscopic form,

$$S_\epsilon = \frac{\alpha(1 - \alpha B)}{KB}$$

Or be written in the equivalant microscopic form:

$$S_\epsilon = \frac{\phi}{K_f} + \frac{\alpha - \phi}{K_s}$$

In the literature regarding the parameters of α and B, there are some papers argues that since brain tissue is taken as completely saturated in CSF, and take the brain cells as completely incompressible $1/K_s = 0$, which will results $\alpha = B = 1$ which give value of $S_\epsilon = 0$.

For steady state problem, there is no mass exchange between phases, and solid velocity equal to zero, in this case, S_ϵ disappeas in the equations, and $\alpha = 1$ according to the equations used [6]. Different values of α is used when considering the blood volume effect, the values are $\alpha = 0, \alpha = 0.83, \alpha = 1$ [3]. $\alpha = 1, B = 0.99$ is used [1] considering brain is almost but not perfectly saturated with fluid which has been referenced [9]. In th later study, $\alpha = 0.83, B = 0.99$ [10] is used refered to [3]. $\alpha = 0.8$ to $\alpha = 1$ and $B = 1$ in the companion paper[4]. In the latest study which is estimated from the brain compliance $\alpha = 0.9995, B = 1$ [11].

4 Parameters in the model.

The steps are:

Given values of α and B , K and v which defines the poro system using macro parameters,

- Calculate S_ϵ according to equation $S_\epsilon = \frac{\alpha(1-\alpha B)}{KB}$.
- Calculate K_u according to $K_u = \frac{K}{(1-\alpha B)}$.
- Calculate $K_s = \frac{K}{1-\alpha}$ according to $\alpha = 1 - \frac{K}{K_s}$.
- Calculate $v_u = \frac{3K_u - 2G}{6K_u + 2G}$.
- Calculate $K_f = \frac{\phi}{S_\epsilon((\alpha-\phi)/K_s)_u}$ according to $S_\epsilon = \frac{\phi}{K_f} + \frac{\alpha-\phi}{K_s}$.

4.1 Parameter set 1 $\alpha = 0.9995, B = 1$

$\alpha = 0.9995, B = 1$				
	Brain tissue $\phi = 0.2, v = 0.35$		SAS $\phi = 0.99, v = 0.499$	
	$E = 9010$	$E = 584$	$E = 9010$	$E = 584$
$S_\epsilon = \frac{\alpha(1-\alpha B)}{KB}$	4.992e-8	7.702e-7	3.3280e-10	5.134e-9
$K_u = \frac{K}{(1-\alpha B)}$	2.002e7	1.297e6	3.003e+9	1.947e8
$K_s = \frac{K}{1-\alpha}$	2.002e7	1.297e6	3.003e+9	1.947e8
$v_u = \frac{3K_u - 2G}{6K_u + 2G}$	0.499916	0.499916	0.499999499	0.4999994997
$K_f = \frac{\phi}{S_\epsilon((\alpha-\phi)/K_s)}$	2.002e7	1.297e6	3.003e+9	1.947e8
As long as α close to 1, v_u is almost same close to 0.5 no matter how small drained value v is, means that $v = 0.35$can be resonable since still results v_u close to 0.5. The calculation of $S_\epsilon = \frac{\alpha(1-\alpha B)}{KB}$ doesn't seem to include porosity, since even according to according to $S_\epsilon = \frac{\phi}{K_f} + \frac{\alpha-\phi}{K_s}$, since $B = 1$, results $K_s = K_f$, porosity not affect. However, the porosity effect is by taking $v = 0.499$ larger than brain tissue.				

4.2 Parameter set 2 $\alpha = 0.83, B = 0.99$

$\alpha = 0.83, B = 0.99$				
	Brain tissue $\phi = 0.2, v = 0.35$		SAS $\phi = 0.99, v = 0.499$	
	$E = 9010$	$E = 584$	$E = 9010$	$E = 584$
$S_\epsilon = \frac{\alpha(1-\alpha B)}{KB}$	1.493e-5	2.303e-004	9.954e-8	1.536e6
$K_u = \frac{K}{(1-\alpha B)}$	5.615e4	3.63e3	8.42213e6	5.459e5
$K_s = \frac{K}{1-\alpha}$	5.889e4	3.817e3	8.833333e6	5.725e5
$v_u = \frac{3K_u-2G}{6K_u+2G}$	0.470860	0.470860616	0.4998216	0.499821
$K_f = \frac{\phi}{S_\epsilon((\alpha-\phi)/K_s)}$	4.724e4	3.062e3	8.414e6	5.454e+005

4.3 Discussion

The set of parameters $\alpha = 0.9995, B = 1$ seems to be reasonable altough the resutant value of $K_u = 2.002e7, K_s = 2.002e7, v_u = 0.499916, K_f = 2.002e7$, doesn't seem to be good especially bulk modulus of fluid $K_f = 2.002e7$ which is quite far from water.

The set of parameters $\alpha = 0.83, B = 0.99$ gives completely unreasonable results.

For parameters, $\alpha = 0.9995, B = 1$,$v = 0.35$, when no fluid is allowed to leave the specimen, is practically zero even though it has a Poisson's ratio 0.35, the undraind value of Poisson ration $v_u = 0.499916$ with $E = 584Pa$. This should be a clarification of the fact that the v is based on the dry material and that both the solid and fluid parts of the poroelastic mixture are incompressible [8].

The other way around, if according to the above, both mixtures are incompressible, take $\alpha = 0.9995$ and $K_f = 2.2e9 = K_s$, with E=584, gives,

$S_{\epsilon_{brain}} = \frac{\phi}{K_f} + \frac{\alpha-\phi}{K_s} = 4.54e-10$, and $S_{\epsilon_{SAS}} = \frac{\phi}{K_f} + \frac{\alpha-\phi}{K_s} = 4.54e-10$, same for both brain tissue and SAS since $K_s = K_f$.

Using equation $S_\epsilon = \frac{\alpha(1-\alpha B)}{KB}$, calculate $B = \frac{\alpha}{S\epsilon K+\alpha^2} = 1.000499954880484 > 1$.

If $\alpha = 0.9995$ and $K_f = 2.2e9, K_s = 0.1K_f$, with E=584, gives,

$S_{\epsilon_{brain}} = \frac{\phi}{K_f} + \frac{\alpha-\phi}{K_s} = 3.725000000000000e-009$, and $S_{\epsilon_{SAS}} = \frac{\phi}{K_f} + \frac{\alpha-\phi}{K_s} = 4.931818181818184e-010$ due to porosity difference.

$B_{tissue} = \frac{\alpha}{S\epsilon K+\alpha^2} = 1.000497829390513 > 1$, $B_{sas} = 1.000499929624452 > 1$.

The above parameters for B doesn't seems to be reasonable sinc B>1 which according to the theory should be B<1.

This can only be obtained by choosing $K_s > K_f$ which doesn't seem to be reasonable again!

5 Questions.

1. According to my understanding, for ideal poroelastic material, the above equations holds, and it should be correct to directly derive microscopic parametersK_s, K_f from macroscopic parameters α and B, and it's true inversely to get alpha and B from K_s and K_f. Is this correct?

2. Should I use the parameters in the model, $\alpha = 0.9995, B = 1$ as shown in first table?

3. Resonable to set young's modulus of SAS same as brain tissue? Since the more fluid property is taken by porosity and poisson's ratio, since this represent the trabecula connecting in the solid phase.

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