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Article · December 2017

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УДК 539.612

On the sensitivity of adhesion between rough surfaces to asperity height distribution

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There has been a long debate about the validity of asperity models in the contact between rough surfaces, much of it concentrated on relatively minor aspects of the solution for the special case of Gaussian random processes for roughness, like the exact value of the area–load slope or the extent of the linear regime. It is shown here that in the case of adhesion, the behavior is extremely sensitive to the shape of the height distribution. We show for example results for Weibull distributions, which has been suggested in a number of practical cases from macroscopic to nanoscopic roughness. Pull-off force is found to vary by several orders of magnitude both lower and higher than in the Gaussian case, whereas the "stickiness" criterion on the adhesion parameter changes by an order of magnitude. Additionally, in some operations like chemical-mechanical polishing, tails are almost completely removed and a sharp peak develops instead of a tail: modeling this with contact on the bounded side of the Weibull distribution, stickiness seems to occur for any level of roughness. Some qualitative comparison with recent numerical experiments is attempted.

Keywords: adhesion, Greenwood–Williamson's theory, rough surfaces

О зависимости адгезии шероховатых поверхностей от распределения высот шероховатостей

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Адекватность моделей шероховатостей для контактирующих шероховатых поверхностей является предметом широкого обсуждения в литературе. Большинство работ концентрируется на деталях решения для частного случая шероховатых поверхностей со случайным гауссовским распределением высот и на обсуждении точного значения наклона зависимости площади контакта от нагрузки или длины линейного участка этой зависимости. В настоящей статье показано, что адгезия сильно зависит от формы функции распределения шероховатостей по высоте. Приведены результаты для распределения Вейбулла, которое было предложено для описания шероховатости на микро- и наноуровне. Показано, что сила срыва может увеличиваться или уменьшаться на несколько порядков по сравнению с гауссовским распределением. При этом при использовании критерия «сцепления» параметр адгезии изменяется только на порядок. Кроме того, в ряде процессов, например при химико-механической полировке, хвосты функции распределения почти полностью отсутствуют, а вместо хвоста наблюдается резкий пик. При моделировании контакта с шероховатостью, описываемой обрезанным распределением Вейбулла, сцепление имеет место для любого уровня шероховатости. Проведено качественное сравнение с ранее полученными результатами численных экспериментов.

Ключевые слова: адгезия, теория Гринвуда–Уильямсона, шероховатая поверхность

1. Introduction

Adhesion is very hard to measure at the macroscale between solid bodies, and experiments inevitably used very soft materials with extremely small roughness. The classical explanation is that surface roughness even of atomic amplitude practically destroy adhesion as shown by Fuller and Tabor [1] (FT in the following) both with a model generalizing the GW (Greenwood and Williamson, [2]) asperity model to the case with adhesion, as well as with experiments with rubber spheres. In the asperity model, the single asperity was assumed to behave according to the then recently developed JKR theory (Johnson, Kendall and Roberts, [3]) contact. This seemed to put an end to the story, even though the model is extremely crude, involving the many assumptions of GW theory (identical asperities, non-interaction, the simplified geometry of asperity summits instead of the full geometry), which could be in principle even weaker in the case of adhesion when bodies can snap into full contact.

Surprisingly, advances in asperity models have concentrated especially on the case with no adhesion, in the implicit assumption that this aspect could be neglected—and Fuller and Tabor's paper may have confirmed this impression. Also, asperity models have almost uniquely been further studied in the assumption of Gaussian height distributions. This was not due much to evidence that real surfaces should be Gaussian: clear deviations from the Gaussian height distribution are shown in the original Greenwood and Williamson paper: Fig. 6 shows a surface of mild steel which had been abraded and then slid against copper, which resembles a "truncated Gaussian". Using the Archard law for wear and the GW model, it is clear that the original Gaussian distribution is continuously changing.

In general, as Nayak [4] puts it, "it is clear that many surfaces are non-Gaussian; but it is equally clear that many surfaces are Gaussian". Adler and Firman [5] developed non-Gaussian models of rough surfaces: in particular, they notice that GW recognized that the exponential distribution (Weibull with $a = 1$ in the following) is "a fair approximation to the upper 25% of the asperities of most surfaces", and remark that the model they propose yields asperity distributions approximating negative exponential even better than does a Gaussian model. It should be mentioned that even for ideally Gaussian random surfaces, the distribution of "extremes" (the summits) tends to Weibull distributions (in particular, at low bandwidths).

Many authors assume Gaussian height distribution, perhaps motivated by the central limit theorem (CLT), which should guarantee that a process defined by the sum of many independent components should slowly tend to Gaussian, usually no test is conducted on this assumption. Even numerical realization intended to be Gaussian in principle, sometimes fail to do so. Indeed, in the surfaces which today we call fractal or self-affine in a broad range of wavelength perhaps from macroscale down to atomic scale, the longest wavelengths will dominate the height distribution, and there will be very few of them. Then, they appear to deviate from Gaussian, or at least fluctuate widely from specimen to specimen (be not ergodic).

Non-Gaussian models have not attracted much attention in contact mechanics however this may well be that they do not lead to significantly different results in the purely repulsive contact. Gaussian processes have been studied instead in great details and connected to asperity models. Improved asperity theories show a contact area reduction for high Nayak bandwidth parameters α which is largely erroneous [6], probably because the bandwidth dependence is compensated by interaction effects [7].

A paper from Pastewka and Robbins [8] involving extensive numerical simulations of atomistic solids with (Gaussian) self-affine roughness suggested claims strong contrast with FT asperity theory possibly of several orders of magnitude in pull-off. In view of trying to explain the potential large deviations from FT theory (and not necessarily of all asperity theories), we were motivated to investigate more the results obtained by Johnson already in 1974[1] which is quoted in Fuller and Tabor's [1] model as "unphysical": that assuming an exponential distribution of asperity heights leads to either always tensile or compressive force: they then noticed that the decay of pull-off force was similar for the two distribution, if the total number of asperities in the exponential model was substituted by the number of asperities in contact. While GW were happy with the results from a negative exponential instead of the Gaussian, here this change passed completely unobserved and obscures a very strong effect, which we will therefore investigate here.

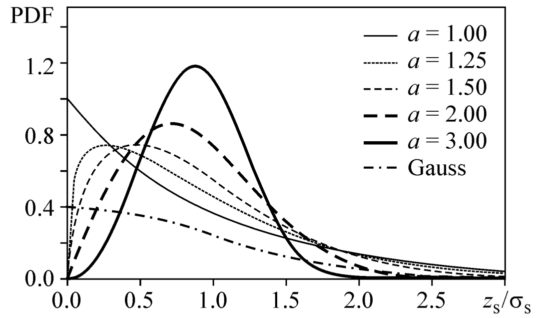

Fig. 1. Distributions of asperity heights studied in this paper. Weibull PDF from a = 1.00, 1.25, 1.50, 2.00, 3.00. The asperity distributions are approached with rigid counter-surfaces from the right. For special cases, like to model surfaces which have undergone CMP, we will also consider the case of approaching the surface from the bounded side, i.e. from the left

2. Weibull distribution of asperities

We consider the two-parameter Weibull distribution with scale parameter $\sigma_s > 0$ and shape parameter $a > 0$. Weibull's PDF is defined for $z_s > 0$ as (Fig. 1)

$$W(z_s; a, \sigma_s) = \frac{a}{\sigma_s}\left(\frac{z_s}{\sigma_s}\right)^{a-1} \exp\left(-\left(\frac{z_s}{\sigma_s}\right)^a\right) \quad (1)$$

and for $a = 0$ we obtain the special case of the exponential distribution $\varphi = 1/\sigma_s \exp(-z_s/\sigma_s)\ (z_s > 0)$, which is used by Johnson [9] in his classical textbook to explain the main findings of the GW theory. The main features of the Weibull distribution are mean $\sigma_s\Gamma(1+1/a)$, where Γ is the gamma function; variance $\sigma_s^2[\Gamma(1+2/a)-\Gamma(1+1/a)^2]$.

The asymmetric form of Weibull distribution is represented by a parameter called skewness, whereas kurtosis designates its flatness (for Gaussian, they are zero, and three, respectively). In many cases, either wear or special finishing operations eliminate the taller asperities. For example, Yu and Polycarpou [10] report a thin film magnetic disk used in hard disk drives in magnetic storage after it has been burnished after manufacturing to avoid contact with

[1] There is no proper reference, as FT quote "to be published".

Table 1

Example measured surfaces as classified by Panda et al. [11]

Surfaces	Skewness	Kurtosis	Weibull shape a
Class A (leptokurtic)	<–0.5	>3.25	$\approx 1 < a < 1.5$
Class B (platykurtic)	>–0.5	<3.25	$2.4 < a < 6.7$

the recording slider, or worn-in surfaces from air conditioning compressors. They suggest in most practical cases, skewness is limited in the range [–1, 1]. For positive skewness and near 1, Weibull a is near 1, which is the well known negative exponential, and at skewness near –1, a tends to increase to higher values, with extremely high values being probably unrealistic and due to errors in fitting. Panda et al. [11] report a number of other measurements but loosely similar results (see summary in Table 1), showing that the majority of surfaces with kurtosis below 3.25 (Class B, or platykurtic) have skewness greater than –0.5, and Weibull $2.4 < a < 6.7$; another class (Class A, leptokurtic) shows a very close to 1 (the negative exponential distribution). Whitehouse [12] has also an interesting figure representing typical skewness and kurtosis values for various manufacturing processes (honing, milling, grinding, turning), which shows approximately the same trend of Table 1, i.e. high kurtosis only for low skewness, and more or less constant kurtosis (not far from 3) at higher skewness. In other words, the Weibull distribution seems quite adequate for most cases.

In other words, Weibull distribution has a slower decay than Gaussian distribution when it is positively skewed, and a much stronger decay when it is negatively so. For illustrative purposes, we will also consider the case of approaching the surface from the bounded side, i.e. from the left, in order to show how insensitive this choice is in the adhesionless model, and how very sensitive it becomes for the adhesion model.

We shall investigate the influence of this distribution on the contact quantities, as compared to the classical normal distribution $\Phi = 1/(\sigma\sqrt{2\pi})\exp(-z_s^2/(2\sigma^2))$ used in Fuller and Tabor (1975). For each asperity, we shall use the DMT (Derjaguin–Muller–Toporov) solution [13] instead of JKR, as it has been suggested to be more appropriate at small scales [14]

$$A = \pi Rd,\ P = \frac{4}{3}\frac{E}{R}(Rd)^{3/2} - 2\pi Rw, \qquad (2)$$

where d is the compression of the asperity, and R is its radius. For a rough surface, $d = z_s - d_0$, where d_0 is the separation. Notice that while separation $d_0 > 0$ for the Weibull distributions, it can also go into negative for Gauss. Notice that, as the distribution sensitivity results from the superposition of different order integrals, it should also remain present for adhesion models other than DMT, although the DMT theory seems to be the most appropriate in the studied problem.

According to the standard GW integration process as a function of asperity height (see [9]), but using (DMT) instead of the Hertz equations (or instead to the JKR equation in the Fuller and Tabor model), and (Weibull) for the PDF distribution, we obtain number of asperities in contact, area and load, integrating for the distribution of asperity heights

$$\begin{aligned}
n &= N\frac{a}{\sigma_s}\int_{d_0}^{\infty}\left(\frac{z_s}{\sigma_s}\right)^{a-1}\exp\left(-\left(\frac{z_s}{\sigma_s}\right)^{a}\right)\mathrm{d}z_s,\\
A &= N\pi R\frac{a}{\sigma_s}\int_{d_0}^{\infty}(z_s - d_0)\left(\frac{z_s}{\sigma_s}\right)^{a-1}\exp\left(-\left(\frac{z_s}{\sigma_s}\right)^{a}\right)\mathrm{d}z_s,\\
P &= N\frac{a}{\sigma_s}\int_{d_0}^{\infty}\left[\frac{4}{3}\frac{E}{R}(Rd)^{3/2} - 2\pi Rw\right]\left(\frac{z_s}{\sigma_s}\right)^{a-1}\times\\
&\times\exp\left(-\left(\frac{z_s}{\sigma_s}\right)^{a}\right)\mathrm{d}z_s,
\end{aligned} \qquad (3)$$

where N is number of asperities, σ_s the amplitude parameter, R the radius of asperities, w is surface energy of adhesion.

Then, changing variable to $d + d_0 = z_s$, normalizing the heights by σ_s, the lower extreme of integration is 0 and defining

$$I_n(d_0^*) = \int_0^{\infty}(d^*)^n(d^* + d_0^*)^{a-1}\exp(-(d^* + d_0^*)^a)\,\mathrm{d}d^* \qquad (4)$$

we obtain

$$\begin{aligned}
&n(d_0^*) = NaI_0(d_0^*),\ A(d_0^*) = (NR\sigma_s)\pi aI_1(d_0^*),\\
&P(d_0^*) = \frac{4}{3}NaER^{1/2}\sigma_s^{3/2}I_{3/2}(d_0^*) - 2\pi Rwn(d_0^*).
\end{aligned} \qquad (5)$$

In the special case of exponential distribution $a = 1$, $I_n(d_0^*) = \exp(-d_0^*)\Gamma(1+n)$, with $\Gamma(1+n) = 1, 1, 3/4\sqrt{\pi}$ if $n = 0, 1, 3/2$. Hence, for $a = 1$ we obtain exactly that area and load are each proportional to number of asperities in contact, and their ratio is constant

$$\begin{aligned}
&n = N\exp\left(-\frac{d_0}{\sigma_s}\right),\ A = \pi R\sigma_s n,\\
&P = n(E(\sigma_s^3 R)^{1/2}\sqrt{\pi} - 2\pi Rw).
\end{aligned} \qquad (6)$$

In this particular case, the definition of "stickiness" as in Pastewka and Robbins [8] is very obvious and with no alternatives: the area–load relationship being linear, the exponential distribution model becomes "sticky" when the load carried in average per asperity becomes tensile, i.e. when

$$\theta_{\exp} = 2\sqrt{\pi}\frac{R^{1/2}}{\sigma_s^{3/2}}l_a > 1, \qquad (7)$$

where $l_a = w/E$ as a characteristic adhesion length.

Turning back to the general case of Weibull distribution, we can define a repulsive pressure on the contact

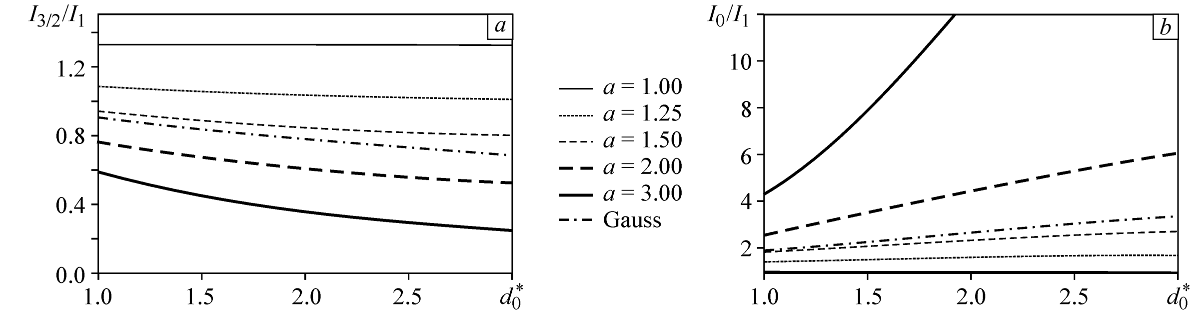

Fig. 2. The ratio between the integrals $I_{3/2}(d_0^*)/I_1(d_0^*)$ (*a*) and $I_0(d_0^*)/I_1(d_0^*)$ (*b*) proportional to mean pressure due to repulsive actions, and to adhesive actions, respectively. Contact from the tail is assumed

$$\bar{p}_{\mathrm{rep}} = \frac{P_{\mathrm{rep}}(d_0^*)}{A(d_0^*)} = \frac{4/3\, NaER^{1/2}\sigma_s^{3/2} I_{3/2}(d_0^*)}{(N_0 R\sigma_s)\pi a I_1(d_0^*)} = \frac{4}{3\pi} E \frac{\sigma_s^{1/2}}{R^{1/2}} \frac{I_{3/2}(d_0^*)}{I_1(d_0^*)}, \quad (8)$$

which is seen to be relatively constant (exactly so for the $a = 1$ case) in Fig. 2 where only the ratio $I_{3/2}(d_0^*)/I_1(d_0^*)$ is plotted for the distributions chosen in Fig. 1, in remarkable qualitative agreement with the numerical findings of Pastewka and Robbins, who found (in a condition very close to the DMT model we are using) that the mean repulsive pressure was almost constant even in the presence of adhesion. Notice however that the exact value of the pressure depends on the value of the Weibull shape parameter, and on the constants in (prep), and modestly on separation for high *a*. In particular, notice that the mean pressure decreases with separation (we are considering here only the case of surfaces in "direct" approach from the tail).

Further, we write nondimensional relationships,

$$\frac{A(d_0^*)}{A_0^g} = \frac{a\sqrt{2\pi}}{\sqrt{\Gamma(1+2/a) - \Gamma(1+1/a)^2}} I_1(d_0^*),$$

$$\frac{P(d_0^*)}{P_0^g} = \frac{a\sqrt{2\pi}}{(\Gamma(1+2/a) - \Gamma(1+1/a)^2)^{3/4}} \times \left[I_{3/2}(d_0^*) - \theta_0 I_0(d_0^*)\right]. \quad (9)$$

Here, we have used the rms amplitude for a Weibull height distribution

$$h_{\mathrm{rms}} = \sigma_s \sqrt{\Gamma\left(1+\frac{2}{a}\right) - \Gamma\left(1+\frac{1}{a}\right)^2},$$

$$P_0^g = \frac{4}{3}\frac{N}{\sqrt{2\pi}} ER^{1/2} h_{\mathrm{rms}}^{3/2}, \quad A_0^g = \sqrt{\frac{\pi}{2}} (NRh_{\mathrm{rms}})$$

so as to make simple comparison with the Gauss distribution result, which is obtained replacing the integrals by

$$I_n^g(d_0^*) = \int_0^\infty (d^*)^n \exp(-1/2(d^* + d_0^*)^2)\, \mathrm{d}d^*.$$

In particular, in the Gaussian case, the number of asperities in contact is $n = N_0/\sqrt{2\pi}\, I_0^g(d_0^*)$ and

$$\frac{A}{A_0^g} = I_1^g(d_0^*), \quad \frac{P}{P_0^g} = I_{3/2}^g(d_0^*) - \theta_{0\mathrm{rms}} I_0^g(d_0^*), \quad (10)$$

where (notice that in Gaussian random process, the quantity (NRh_{rms}) in A_0^g is commonly described as nearly constant, although we shall return on this point later).

Also, we defined an adhesion parameter (independent on the distribution)

$$\theta_0 = \frac{3\pi}{2}\frac{R^{1/2} w}{E\sigma_s^{3/2}} = \frac{3\pi}{2}\frac{R^{1/2} w}{E h_{\mathrm{rms}}^{3/2}} \frac{h_{\mathrm{rms}}^{3/2}}{\sigma_s^{3/2}} = \theta_{0\mathrm{rms}} \left[\Gamma\left(1+\frac{2}{a}\right) - \Gamma\left(1+\frac{1}{a}\right)^2\right]^{3/4}, \quad (11)$$

i.e. $\theta_{0\mathrm{rms}}$ is the parameter obtained by measuring the rms amplitude of each Weibull distribution. Unfortunately, the condition for stickiness depends on *a*, as well as on the separation at which we decide to measure the condition, because the area–load relationship is no longer linear. Considering $d_0^* = 2$ as a good estimate, stickiness is obtained from $I_{3/2}(2) - \theta_0 I_0(2) = 0$ which permits to define a more "universal" adhesion parameter if one knows a priori the shape parameter of the distribution, as

$$\theta_a = \theta_0 \frac{I_0(2)}{I_{3/2}(2)} = \theta_{0\mathrm{rms}} \frac{I_0(2)}{I_{3/2}(2)} \left[\Gamma\left(1+\frac{2}{a}\right) - \Gamma\left(1+\frac{1}{a}\right)^2\right]^{3/4}. \quad (12)$$

Therefore, while $\theta_0 = \theta_{0\mathrm{rms}}$ for $a = 1$, the correction for $a = 3$ is $\theta_0 = 0.185\theta_{0\mathrm{rms}}$. However, this correction being approximate, we shall not pursue it further.

In Fig. 2, *b* is shown the ratio of the integrals $I_0(d_0^*)/I_1(d_0^*)$ which in turn (when multiplied by the adhesion parameter) are proportional to the mean tensile stress due to adhesion, which tends to increase with separation, suggesting a clear picture of the competition between effect of "elbowing" of the higher asperities, with attraction from the lower ones.

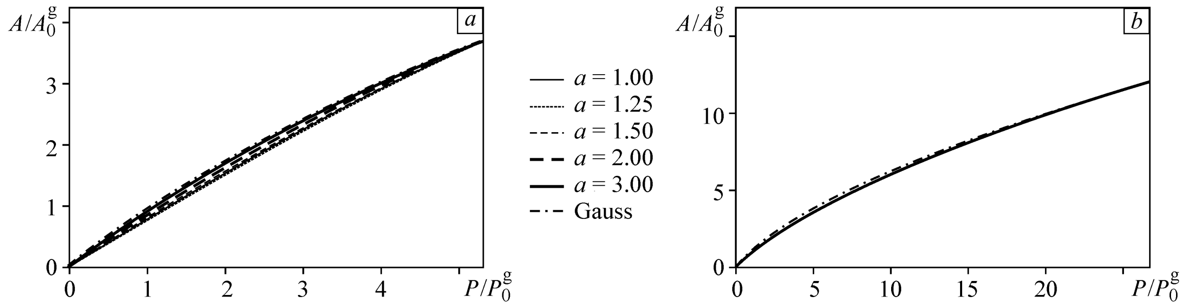

Fig. 3. Area–load curves for non-adhesive contact. Contact on the tail (*a*), contact from the left of the distribution (*b*). The curves are all indistinguishable. Dimensionless separation varies in the range $d_0^* = 0$–3 in all plots

3. Results

3.1. Non-adhesive case

When mechanical load is considered, the normalization in (area, load, and area Gauss, load Gauss) with P_0^g, A_0^g is appropriate. Figure 3 shows some results for zero adhesion, and the various distributions. In Fig. 1, *a*, in particular, we have the distributions approaching contact from the tail, whereas in Fig. 3, *b* we have the hypothetical case of approach from the left side ("reverse"). We can see that the effect of the tail is really small, with the curves almost indistinguishable in both cases. Notice that the contact from the left of the distribution has larger loads and areas, as a result of the fact that the majority of the asperities are very close to the point of first contact, whereas in the "direct" case, many asperities will only have a small compression. It is difficult to trace where each curve ends, as the curves are so close. Notice however that, given the Gaussian distribution is specular and hence the curve is the same for either direct or reverse approach, the curve in Fig. 3, *a* is essentially the beginning of the curve in Fig. 3, *b*, and there is a small deviation from linearity which is exactly obtained for the exponential. Notice that in the "reverse" case, the exponential distribution obviously does not obtain linearity, and it behaves similarly to the other distributions. It is clear that, with respect to what we shall see in the next paragraph for adhesive contact, there is very little to argue about the robustness of this result, and the very long discussion about asperity models in mechanical contact (the original GW paper counts more than 4500 citations in Google Scholar) revolved around results that, in large parts, were already obtained with the simplest GW model. Not only could GW fit their tails of the distribution with exponential ones, they could equally fit any Weibull, and yet the results would not change significantly!

3.2. Adhesive case

In the adhesive case, it is more convenient to move to the ratio of the load with the pull-off of the entire set of asperities as if they were aligned, P_cN, where $P_c = 2\pi Rw$ according to DMT theory: for Weibull distributions

$$\frac{P(d_0^*)}{P_cN} = a\left[\frac{1}{\theta_0}I_{3/2}(d_0^*) - I_0(d_0^*)\right], \tag{13}$$

whereas for Gaussian distributions

$$\frac{P}{P_cN} = \frac{1}{\sqrt{2\pi}}\left[\frac{I_{3/2}^g(d_0^*)}{\theta_{0\mathrm{rms}}} - I_0^g(d_0^*)\right]. \tag{14}$$

Figure 4 shows some area–load results for adhesive contacts, with adhesive parameter $\theta_a = 0.1$: the curves differ significantly although they remain almost linear because of the choice of the normalization of the load axis, and of the parameter θ_a, in an attempt to capture the change of "slope" from "nonsticky" to "sticky" (at moderately high separation $d_0 = 2$) similarly for all curves. If we used the previous choice of the axis, for this low value of the adhesion parameter, the curves would still be very close to each other. However, different alternative choices of θ, like $\theta_{0\mathrm{rms}}$, do not make a much better job at collapsing the curves, as it will be clear when we will discuss the pull-off results.

The differences between one curve and the other obviously come from the different behavior of the I_n integrals, as compared to the purely repulsive contact case. The "character" of the area–load curves remains more linear in the contact on the tail, than in contact from the left of the distribution (which is not shown for brevity), as expected.

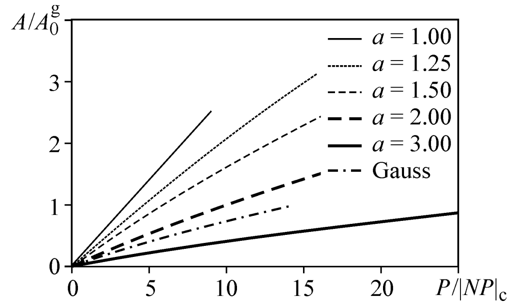

Fig. 4. Area–load curves for adhesive contact, adhesive parameter $\theta_a = 0.1$. Contact on the tail. The curves are already quite different one from the other, indicating a large sensitivity to the exact height distribution function

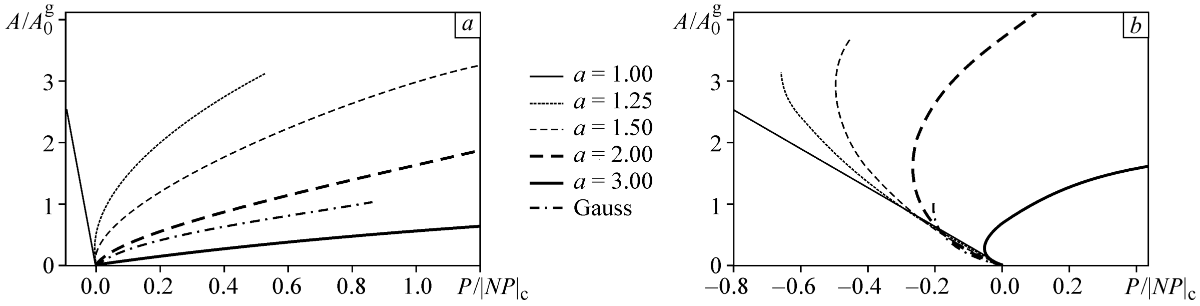

Fig. 5. Area–load curves for adhesive contact. Contact on the tail. Adhesive parameter θ_a = 1.1 (*a*), 5.0 (*b*)

We now consider the just about "sticky" cases, obtained for $\theta_a = 1.1$ (Fig. 5, *a*). As it is evident, the exponential distribution, which remains linear, now implies a natural tendency of the surfaces to snap into "full contact" in the limited sense of an asperity model, which is that all asperities jump into contact. Notice that the load in this condition is not NP_c, as there is still a compressive component to the load. This behavior was disregarded by Fuller and Tabor as nonrealistic, and clearly it is a limit solution for an asperity model. However, this large difference between the exponential and the Gaussian distribution (contrary to the case of mechanical contact), indicates a real phenomenon, simply that there is a completely different balance between repulsive and adhesive forces. If the asperity model becomes a little "stretched" in the limit case of the exponential as we are leading to the extreme case of all asperities in contact for all "sticky" cases, for any other shape parameter of the Weibull distribution, there is a wide range of behavior which is very similar in form to that of the Gaussian case which FT considered with success and found in agreement with experiments. For the case depicted in Fig. 5, *a*, for example, pull-off loads appear very small for all distributions except a = 1: hence, there is clearly a very high sensitivity to the shape of Weibull parameter, and even in the limit case of the exponential, the result is not too dissimilar from that obtained in the other cases for high adhesion parameters. Figure 5, *b* shows indeed similar results for higher adhesion parameters, showing that now the exponential case is much similar to the other cases, and the area–load slope behavior is also similar so that the parameter θ_a correctly capture the "stickiness". However, the different shape distributions result in quantitatively different pull-off, very sensitive to the shape parameter.

3.3. Pull-off

We can now move to present some comprehensive results for the pull-off obtained for the different models. In particular, Fig. 6 shows that this choice of normalization makes all the Weibull distributions with $a \le 3$ (except the exponential in the restricted range where it predicts "stickiness"), to have lower pull-off than the Gaussian one. Vice versa, Weibull distributions with $a > 3$ have higher pull-off forces. Remarkable the correspondence of the Gaussian distribution with the Weibull with $a = 3$, and the very clear exponential trend. In fact, a very good approximation to the results is

$$\lg\left(\frac{P}{P_c N}\right)_{\min} = -0.1 - \frac{0.82}{\theta_{0\mathrm{rms}}}. \qquad (15)$$

It is also apparent that defining stickiness as the point where the pull-off has dropped of a factor 10^{-4}, the increase of Weibull shape parameter from 1 to 7 makes the critical $\theta_{0\mathrm{rms}}$ parameter to increase of about an order of magnitude. Hence, from the perspective of the stickiness condition, increasing the a from 1 to 7 as it seems realistic for the so-called platykurtic surfaces, corresponds to an increase

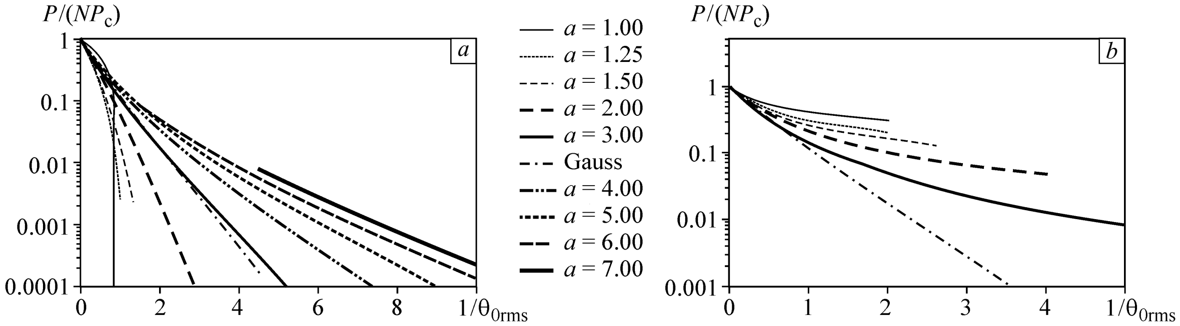

Fig. 6. Normalized pull-off values for the different distributions, as a function of the inverse of adhesive parameter θ_{rms}. Contact on the tail (*a*), contact from the left of the distribution (*b*)

of surface energy of a factor 10, or else a reduction on the rms amplitude of a similar factor (it can be shown that considering the remaining term involving $h_{\rm rms}$ and the radius of asperities really should be transformed into a $h'_{\rm rms}$ slope when considering random process theories).

To illustrate the effect of completely removing the tail, as it may happen for some specific surfaces (like those treated with chemical-mechanical polishing) or even as an example of the possible effect of discrete realizations of random processes, we plot in Fig. 6, *b* the case of approaching the Weibull distribution from the left. This limit case clearly shows that "stickiness" becomes an universal feature, independent on amplitude of roughness, provided that additionally the Weibull shape parameter is less than 3.

These results clearly show that a high "concentration" of asperities near the surface boundary makes the compressive effect much smaller than the adhesive effect of farther asperities, resulting in possibly extremely increased adhesion, and certainly much increased "stickiness".

Finally, some considerations about the possible validity of results obtained with the simple asperity model. In adhesionless models, results of asperity models are considered realistic when obtained at large enough separations ($d_0^* > 1,$ for example), as otherwise interaction effects are important. Indeed, neglecting this effect does not permit the asperity model to converge to full contact, as, instead, does the Persson's solution which is actually obtained by a correction of the full contact regime. We cannot apply this rule in the case of adhesion, as the problem is much more complex. We appreciate that merging of the areas between asperities can occur even at large separations, and that we use the solution sometimes also at low separations. This is strictly not only true in the case of the exponential distribution, where we have seen that pull-off occurs when all asperities are in contact, but even in the Gaussian case, when pull-off occurs for dimensionless separation often in the range between 0–1. However, for these very same reasons we suppose that the asperity model predicts a sort of lower bound to the pull-off force, as when considering fine enough scales. When we say "lower bound", obviously we do not intend this in a strict sense, since, because of the sensitivity to the height distribution, we may not be surprised if a particular realization finds pull-off to be load dependent (hysteresis).

4. Discussion

We have mainly attempted to show that, in the adhesive behavior, the exact shape of the height distribution matters, contrary to the case of pure mechanical contact. In particular, easily orders of magnitude variations in pull-off force are expected changing the distribution from having a symmetrical Gaussian shape, to a different distribution with much stronger decay in the tail (or a bounded one). Even considering the practical ranges of Weibull parameters, as from Table 1 and the discussion therein, confined in between say, 1 and 6, this is still quite a wide range.

There are a number of processes which can cause a real surface to approach very unsymmetrical shapes, because of effects acting only or mostly, for the highest asperities, including mechanical (wear, manufacturing, the very important case of chemical mechanical polishing, or even just plastic deformations on surfaces), or chemical (exposure to environment), which were originally thought to be Gaussian. Tabor [15] in summarizing his findings on adhesion, wrote that "if the surfaces are ductile the junctions can accommodate the prizing action of the higher asperities and strong adhesions may be observed. The overall conclusion is that adhesion between solids depends not only on surface forces but also on surface roughness and the degree of ductility of the solids themselves".

Unfortunately, there is also limited literature on the effect of roughness on adhesion, and particularly on the form of the height distribution. So, while we have used some literature about macroscopic metallic surfaces and Weibull distributions, these models generally do not look for adhesion effects. Ideal experiments to compare to would be similar to the Fuller and Tabor ones on elastomers.

5. Conclusions

Gaussianity of random surfaces has become so much the standard assumption that very seldom any statistical test is conducted when measuring real surfaces: their "fractal dimension" is measured and the entire power spectrum density (PSD) function, although in the end for a Gaussian process all we need is the variance of heights, slopes and curvatures. Self-affinity and power law PSD do not mean Gaussian distributions of heights. While the shape of the asperity height distribution has not much effect in adhesionless contact, there is a large effect when adhesion is included. Here, a Weibull distribution is used to attempt to quantify this effect. It is found that the "stickiness" criterion (i.e. when the area load relationship turns into negative loads quadrant) increases with Weibull shape parameter changing the critical parameter of an order of magnitude. The pull-off forces can have values orders of magnitude lower or larger than what expected by the Gaussian distribution. Probably results would vary very much from one numerical experiment to the other, unless special care is taken to try to render the process more "ergodic". But it may be still important to quantify the fluctuations as in using the model for a real rough surface (assumed Gaussian), there may be a significant error. In particular, the present discussion is of practical importance for many applications in biology and medicine [15].

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Поступила в редакцию
06.04.2017 г.

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