A generalized Johnson parameter for pull-off decay in the adhesion of rough surfaces

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The authors would like to dedicate this note to the memory of Prof. K.L. Johnson (1925–2015).

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There is no simple theory at present to predict accurately the decay of pull-off in the adhesion of randomly rough surfaces. The asperity model of Fuller and Tabor has shown significant error in recent numerical investigations by Pastewka and Robbins of self-affine random roughness from micrometer to atomic scale which corresponds to low values of Tabor parameter. For sinusoidal contact, the Johnson parameter, originally introduced for the JKR regime (from Johnson–Kendall–Roberts) is the dominant parameter ruling the pull-off at intermediate Tabor values. Hence, we define a generalized Johnson parameter as the ratio between the adhesive energy to the elastic strain energy to flatten the surface in the case of multiscale roughness and find that it correlates very well with the data of Pastewka and Robbins spanning almost five orders of magnitude of reduction from theoretical strength, improving significantly with respect to other possible single parameter criteria. For the most important case in practice, that of low fractal dimensions, this suggests the product of amplitude and slope of the largest wavelength components of roughness dominate pull-off decay, and not small scales features like slopes and curvatures, as suggested by Pastewka and Robbins.

Keywords: roughness, adhesion, Johnson–Kendall–Roberts adhesion, Derjaguin–Muller–Toporov adhesion, Pastewka and Robbins's theory, Persson and Tosatti's theory, Fuller and Tabor's theory

Обобщенный параметр Джонсона для порога резкого уменьшения силы отрыва при адгезии шероховатых поверхностей

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В настоящее время отсутствует простая теория для точного предсказания порога резкого уменьшения силы отрыва в случае адгезии случайно шероховатых поверхностей. Модель дискретного контакта Фуллера и Тэйбора выявила значительную ошибку в численных исследованиях Пастевки и Роббинса самоаффинной случайной шероховатой поверхности с размерами шероховатостей от микрометра до атомного масштаба при низких значениях параметра Тэйбора. Для синусоидальной шероховатости режим адгезии определяется параметром Джонсона, первоначально введенным для режима JKR (Johnson–Kendall–Roberts). Основываясь на этом, в данной работе обобщенный параметр Джонсона определ как отношение энергии адгезии к энергии упругой деформации при полном уплощении шероховатостей на всех масштабных уровнях. Обнаружено, что данный параметр хорошо коррелирует с результатами, полученными Пастевкой и Роббинсом в области параметров, охватывающей пять порядков величины в редукции силы адгезии по сравнению с теоретической прочностью. Этот параметр обеспечивает существенное улучшение корреляции по сравнению с любыми другими критериями, основанными на единственном критическом параметре. В случае малой фрактальной размерности, наиболее важном для приложений, этот параметр сводится в произведению амплитуды шероховатости и наклона, отвечающего длинноволновым компонентам шероховатости. Именно он определяет силу адгезии, а не параметры более мелкого масштабного уровня, такие как наклоны и кривизна, как было предположено Пастевкой и Роббинсом.

Ключевые слова: шероховатость, адгезия, Джонсон, Кендалл, Робертс, Дерягин, Мюллер, Торопов, теория Пастевки–Роббинса, теория Перссона–Тосатти, теория Фуллера–Тэйбора

1. Introduction

Contact in the presence of adhesion is a lot richer of phenomena and parametric dependences than its counterpart without adhesion. It is instructive to start from a simple case, to show already quite a number of effects and, more importantly, to introduce a parameter which we shall generalize in a more general contact condition. For a single sinusoid of wavelength and amplitude λ, h paper [1] discusses that the adhesive contact problem under the Johnson–Kendall–Roberts (JKR) regime [2] of short range adhesion is governed by a single parameter α, defined as (notice that this is the square of the original Johnson parameter)

$$\alpha = \frac{2}{\pi^2} l_a \frac{\lambda}{h^2}, \quad (1)$$

where $l_a = w/E^*$ is a characteristic length of adhesion, w is the work of adhesion, $E^* = E/(1-\nu^2)$ is the elastic modulus under plane strain, and ν is Poisson's ratio. The physical meaning of α is that it represents the ratio of the surface energy in one wavelength to the elastic strain energy when the wave is flattened, U_{el}. The precise behavior can be obtained only by solving the partial contact problem exactly and not from energy considerations alone as there are energy barriers between states, and it turns out that for $\alpha > 0.57^2 = 0.325$, there is a spontaneous snap into full contact, from which detachment should occur only at theoretical strength. It is important to remark that this parameter (and this alone) governs the entire behavior of the system, also in partial contact. In particular, pull-off depends on α in the regime where full contact is not obtained first. It has been shown in more recent extensions with cohesive models [3] that, in a more general regime, the pull-off strength depends also on another parameter, the so-called Tabor parameter [4]

$$\mu = \frac{p_{th}}{E^*}\left(\frac{R}{l_a}\right)^{1/3}, \quad (2)$$

where p_{th} is theoretical strength, and R is the radius of the tip of the sinusoid. There is an extensive range of intermediate values of this second parameter, when μ is lower than 1, in which the pull-off depends exclusively on α. Indeed, it is very close to the value expected from the JKR regime pull-off at asperities (see [3] for further details). This motivated the present investigation, since for a large class of problems, including the case we are going to consider for the comparison with simulations [5], pull-off occurs at asperities in this range. An important remark here is that we refer, as many do, to this as shortly the DMT regime (Derjaguin–Muller–Toporov), but we do not imply we refer to the DMT solution for the sphere. In fact more than one DMT solution exists [6–8], all of which are instead quite controversial (see [9, 10]), since they assume compressive forces only deform the surfaces, and not tensile forces, despite the latter are of the same order of magnitude, if not larger.

For a rough surface, an estimate of μ when $p_{th} = 0.05E^*$ and $l_a/a_0 = 0.05$ (as in Lennard-Jones potential, where a_0 is the interatomic distance), gives $\mu < 1$ for $R/a_0 < 20^3 \times 0.05 = 400$ and hence for asperities not larger than 400 times the range of attractive forces, we expect that the main parameter for pull-off will be α according to the full solution of [3]—which does not assume a DMT method of solution.

Naturally, if both μ and α are high, there is a tendency towards the theoretical strength and to hysteretic behavior (pressure-dependent pull-off) as predicted by the simple JKR model of Johnson [1], see again [3]—which poses some restriction on the generality of our results—as we shall discuss later. But since no simple theory exists for considering all the possible ranges, we shall be satisfied to make some progress in the more complicated cases.

1.1. Attempts of quantitative theories of rough adhesion

The only simple theory which gives the roughness-reduction of stickiness seems still to be that given already 40 years ago, with the classical Fuller–Tabor asperity model [11], but unfortunately it contains a number of strong approximations. It defines an adhesion stickiness parameter (Δ_c in the original paper) which depends on the ratio between the separation at pull-off in the JKR model, δ_c, and the rms amplitude of asperities heights h_{rms}

$$\theta = \frac{\delta_c}{h_{rms}} = (\pi l_a)^{2/3} \frac{R^{1/3}}{h_{rms}}, \quad (3)$$

where R the radius of asperities. The main parameter therefore reducing roughness is rms amplitude, although there is a weak sensitivity to the short wavelengths content of the roughness, due to the cubic root of the radius. Although Fuller–Tabor theory was corroborated by some experiments, the correlation was not investigated in fine details as it is now possible to do with numerical investigations.

Authors [12, 13] develop a theory of adhesion of rough surfaces in the JKR regime. In its first part, it assumes full contact, and argues with an energy balance between the state of full contact and that of complete loss of contact that the effective energy available at pull-off is

$$w_{eff} = \frac{A}{A_0} w - \frac{U_{el}}{A_0}, \quad (4)$$

where, for full contact, they intended to add an effect of roughness-induced increase of contact area, $A/A_0 > 1$. This latter element is controversial as gradients are usually very small [14] and was disproved also in Persson's own later experiments [15].

In other words, Persson and Tosatti write that the elastic strain energy stored at the interface is available during the peeling process and reduces the nominal work of adhesion linearly, and this can be used at macroscopic scale to estimate pull-off knowing the nominal contact area. With this simple assumption, they can use the theory also for a sphere, and indeed attempted to estimate the pull-off values of Fuller–Tabor classical experiments [11], but found only very qualitative agreement, although better agreement was found in a new set of experiments [15]. Persson [12] continues with very elegant extensions of the theory for cases involving partial contact, and another theory [16] attempts to solve the problem using the DMT approximations. None of these theories however led unfortunately to simple analytic predictions or has been tested with extensive numerical simulations of adhesive contact.

1.2. Pastewka and Robbins' simulations

Pastewka and Robbins ([5], PR in the following) have a quite extensive set of numerical results for pull-off of (nominally flat) rough surfaces, and found that the pull-off data decay [5] not only did not correlate well with the prediction of the classical Fuller–Tabor asperity model [11], but that orders of magnitude variations could be observed with the different set of parameters tested. They obtain a set of scaling equations to define the slope of the area-load equation making some assumptions similar to the DMT model (no deformation due to the cohesive stresses, see [17]) but working with the "fractal" geometry of the contact area, which introduces a dependence on a range of attraction Δr (of the order of atomic spacing), absent in Fuller–Tabor theory. They obtain a stickiness parameter which depends uniquely on small scale features of the roughness, like rms slopes and curvatures. However, they are not able to obtain a prediction of the pull-off stress. It should be remarked that they suggest that there is no hysteresis until the tangent becomes vertical and surfaces become sticky. However, more recent appraisal of their results have shown that their stickiness criterion (which we assume was tuned for the slope of the area-load, presumably during loading), is not an indication of stickiness in terms of pull-off [14], although their parameter can be used as an empirical parameter to correlate pull-off, as it has been attempted in [18]. We shall show this correlation here, as one of the possible alternative to the present proposal.

1.3. Outline of the present work

In the present paper, we will not make an attempt to derive a quantitative method of solution for adhesive contact, since we do not believe at this stage that very simple one can be found, but rather introduce a new "stickiness" single parameter, which is a natural extension of the Johnson parameter for a single contact, being the ratio between the adhesion energy and the elastic energy to flatten the contact. With this new parameter, we hope to find a good single parameter correlation with adhesion in the "intermediate" range of Tabor parameters—what is commonly called the DMT regime, although we would in principle prefer to avoid this denomination as it generates the confusion with the DMT theory of spheres which assumes no effect of adhesive forces on deformations.

Notice that although we use one ingredient of the simple theory of adhesion of [12, 13] under full contact, namely the elastic energy to flatten the roughness, we shall not assume "full contact" and we shall not assume that the pull-off reduction can be obtained from [12]. Actually, the prediction of the latter assumption will be shown for comparison, and will result in a very poor estimate, at least in our regime.

2. The new stickiness parameter

We start by the postulate that, as in sinusoidal case the pull-off value depends mainly on α (at intermediate range of Tabor parameters), the multiscale problem will depend mostly on a generalized Johnson α parameter—this intrinsically assumes that the contact is either nonhysteretic, or if it is, that we take the smallest pull-off value, that obtained after just gentle approach of the surfaces. A generalized Johnson parameter is readily defined based in general on the entire power spectrum density (PSD) of the (isotropic) rough surface $C(q)$, up to the magnification $\zeta = q_1/q_0$, where q_0, q_1 are the low cutoff and high wave vector cutoff, as

$$\alpha(\zeta) = \frac{w}{U_{\mathrm{el}}(\zeta)} = \frac{l_{\mathrm{a}}}{\pi/2 \int_{q_0}^{q_1} q^2 C(q) \mathrm{d}q} = \frac{l_{\mathrm{a}}}{l(\zeta)}, \tag{5}$$

where we have introduced an effective length of adhesion $l(\zeta)$, and $U_{\mathrm{el}}(\zeta)$ has been derived in [12].

Notice that we are not looking at the problem at different "magnifications", like Persson's theories do, but we are only interested in the actual problem with the entire spectrum content. The parameter α depends therefore on the second moment of the surface PSD as line integral, not to be confused with the usual second moment of the PSD as area integral, which assuming an isotropic roughness, is

$$m_n = m_{n0} = \int_{q_0}^{q_1} \int_0^{2\pi} [q \cos\theta]^n C(q) q \mathrm{d}q \mathrm{d}\theta = \\ = \int_0^{2\pi} \cos^n \theta \mathrm{d}\theta \int_{q_0}^{q_1} C(q) q^{n+1} \mathrm{d}q \tag{6}$$

for $n = 0, 2, 4$. Therefore, the moment giving the stored elastic energy has dimensions of a length, and is neither the actual 0th moment (rms amplitude squared, h_{rms}^2) which depends mostly on the low wavevectors content, nor the second moment (rms slopes squared $h_{\mathrm{rms}}'^2$) which depends mostly on the high wave vectors content. The weight of the different part of the spectrum highly depends on fractal dimension. In fact, if for illustrative purposes we take a typical power law PSD $Zq^{-2(1+H)}$ for $q > q_0$, where H is the Hurst exponent (equal to $3 - D$ where D is the fractal dimension of the surface), the integral depends on whether $H > 0.5$ or not. Specifically, as

$$Z = \frac{H}{2\pi} \left(\frac{h_0}{q_0} \right)^2 \left(\frac{1}{q_0} \right)^{-2(H+1)},$$

where $h_0^2 = 2h_{\mathrm{rms}}^2$ (see [12]), for $H \neq 0.5$

$$l(\zeta) = \frac{\pi}{2} \int_{q_0}^{q_1} q^2 C(q) \mathrm{d}q = \\ = \frac{\pi Z}{2} \int_{q_0}^{q_1} q^{-2H} \mathrm{d}q = \frac{\pi}{2} \frac{h_0^2}{\lambda_0} f(H, \zeta), \tag{7}$$

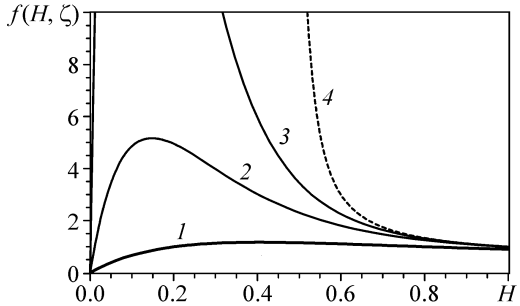

Fig. 1. The function $f(H, \zeta)$ versus H for magnifications $\zeta = 10$ (*1*), 100 (*2*), 1000 (*3*), ∞ (*4*)

where $f(H,\zeta) = H(\zeta^{-2H+1}-1)/(-2H+1)$ is a function identified also in [13], and is plotted in Fig. 1 as a function of H and of magnification $\zeta = 10, 100, 1000, \infty$. Notice that [13] only show the curve for $H > 0.4$ ($D < 2.6$) which, although is probably the range of most practical interest, is not necessarily the range used in simulations—indeed, PR have extensive data for $H = 0.3$.

For the usual case of $H > 0.5$ (low D) the integral converges quickly, is relatively insensitive to high wave vector truncation and indeed in most cases we can use even the limit value

$$l(\infty)_{\text{low}D} = \frac{\pi}{2}\frac{h_0^2}{\lambda_0}\frac{H}{2H-1}, \tag{8}$$

which gives a very gentle dependence on Hurst exponent: the energy is mainly stored in the long wavelength components, which depends on the factor h_0^2/λ_0 like for the single sinusoid of the longest wavelength, and notice that this can be seen as the product of the amplitude and the slope. Hence, if we have larger long wavelengths (for a given self-affine power law PSD) the energy will increase, $\alpha(\zeta)$ and hence the stickiness will be reduced[1]. This prediction for the fractal dimensions of practical interest (if of course the parameter is correctly correlated) is in contrast both with Fuller and Tabor parameter which gives a leading role to rms amplitude of roughness alone (the large wavelength slope is not present, although there is a mean radius of the asperities), but also in contrast for example with PR criterion, which seems to suggest that stickiness only depends on high wave vector content (rms slopes and curvature, which depend only on the tail of the spectrum), see Appendix. In [14] we already remarked that this prediction of the PR criterion to large size bodies was surprising and perhaps counterintuitive.

For $H < 0.5$ (high fractal dimensions), vice versa, the integral does not converge and is much more sensitive on the large wave vectors content

$$l(\zeta)_{\text{high}D} \approx \frac{\pi}{2}\frac{h_0^2}{\lambda_0}H\frac{\zeta^{1-2H}}{1-2H} = \frac{\pi}{2}\frac{h_0^2}{\lambda_0}\left(\frac{\lambda_0}{\lambda_1}\right)^{1-2H}\frac{H}{1-2H} \tag{9}$$

and therefore small wavelength cutoff implies larger $l(\zeta)$ and stored energy and smaller stickiness, and in the "fractal" limit, zero stickiness. Here, to compare with the PR criterion, one should observe also the variation in rms slopes and curvature, which strongly depend on the truncation. Appendix shows that qualitatively a similar conclusion is reached by PR in the range of high fractal dimensions, except perhaps for the threshold fractal dimension which is not $D = 2.5$, but $D = 2.4$ (as $H = 0.6$).

As a remark, one could conclude from this analysis that there is a sharp difference between fractal surfaces depending on the fractal dimension: this is certainly true, for a given amplitude and slope of the longest wavelength components, h_0^2/λ_0. However, if we compare cases of same rms slope, which depends essentially on the value of the spectrum at large wave vectors, as for example do PR (see below), then the difference in α will not be so large, since for large fractal dimension, h_0^2/λ_0 will be much smaller.

Finally, to compare to the stickiness criterion of the original Persson's theory assuming full contact, the condition $w_{\text{eff}} < 0$ leads to loss of stickiness when

$$\alpha(\zeta) < 1 \tag{10}$$

whereas here we are not assuming any threshold on $\alpha(\zeta)$.

3. Application to Pastewka and Robbins data

PR study self-affine surfaces with Hurst exponent $H = 0.3, 0.5, 0.8$, i.e. a power law PSD $Z|\mathbf{q}|^{-2(1+H)}$ for wave vectors $q_r < |\mathbf{q}| < q_s$ ($q = 2\pi/\lambda$) with roll off to a constant for $q_L < |\mathbf{q}| < q_r$ (limited to $x = q_L/q_r = 1/2$, in the data presented) and zero otherwise. They introduce a truncated potential (cubic spline) which mimics the Lennard-Jones potential, and therefore have an atomic spacing a_0. Units of wavelengths are therefore introduced as $\lambda_s/a_0 = 4, 8, ..., 64$, whereas $\lambda_L = 4096\,a_0$ and $\lambda_r = 2048\,a_0$, which means magnification $\zeta = \lambda_r/\lambda_s = 512, 256, ..., 32$. They introduce two levels of rms slopes $h'_{\text{rms}} = 0.1, 0.3$ and two levels of range of attractive forces, $l_a/a_0 = 0.05, 0.005$, the former being closer to a true Lennard-Jones potential having range of attractive forces $\Delta r \approx a_0$, and the second having much shorter range[2], $\Delta r \approx 0.35\,a_0$. In [5], the pull-off decay is shown in terms of a scale which mimics the Fuller–Tabor scale of reduction of pull-off with respect to the theoretical pull-off of a set of aligned asperities [14, 18]. Hence, we have re-evaluated the actual pull-off values in terms of decay from theoretical strength. Indeed, mean pressure at pull-off is normalized by the factor

[1] Unless we enter into a JKR regime where perhaps stickiness shows a more complicated (and pressure-sensitive) dependence.

[2] This derives from their assumption of potential of fixed stiffness at $z = a_0$, equal to $k = E^* a_0/2$, so that the adhesion energy $w = k\Delta r^2/(12a_0^2)$ was changed by varying the range of attraction. One immediately obtains therefore $\Delta r/a_0 = (24 l_a/a_0)^{1/2}$.

Table 1

Estimated values of h_{rms}/a_0 for $h'_{rms}=0.1$. For $h'_{rms}=0.3$, obviously multiply by 3

λ_s/a_0	H		
	0.3	0.5	0.8
4	0.70	1.68	5.92
8	1.13	2.39	6.79
16	1.85	3.37	7.81
32	3.00	4.78	8.98
64	4.86	6.75	10.32

$$\frac{w}{4h_{rms}} = \frac{w}{4a_0} \Big/ \frac{h_{rms}}{a_0}.$$

Considering the truncated potentials PR define for the case $l_a/a_0=0.05$, the peak of their truncated potential is at about $p_{th}=0.07E^*$. For $l_a/a_0=0.005$, $p_{th}=0.025E^*$, and therefore, to estimate the actual pull-off values, we only need to estimate h_{rms}/a_0, from the PSD. Writing $T(n)=2\pi, \pi, 3/4\pi$ (n = 0, 2, 4), we get from the general definition (moment-general), that the moments of order n (variance of amplitude, slopes and curvature for n = 0, 2, 4) in the case of roll-off are

$$m_n = ZT(n)k_r^{n-2H}\left(\frac{1-x^{n+2}}{n+2}+\frac{\zeta^{n-2H}-1}{n-2H}\right). \quad (11)$$

For example, from the ratio of m_0 and m_2 we obtain from (moments) for $x=1/2$, and considering $h'_{rms}=\sqrt{2m_2}$

$$h_{rms} \approx \frac{h'_{rms}}{\sqrt{2}}\sqrt{(2-2H)\left(\frac{3}{4}+\frac{1}{H}\right)\frac{\lambda_s}{2\pi}\zeta^H}, \quad (12)$$

which permits to estimate the rms amplitudes in Table 1. Estimates of the mean radius of asperities based on the m_4 value from (11) leads to consistent values of h_{rms}/δ_c,which confirms our estimates are correct[1].

In [5] $h'_{rms}=0.1, 0.3$ are indicated with closed (open) symbols, $l_a/a_0=0.05, 0.005$ (red, blue), and shape depends on H (triangles are H = 0.3 in the original plot, squares are H = 0.5 and circles are H = 0.8, before the likely inversion of H = 0.3, 0.8) we shall use the same symbols in our own plot. We have discussed these results in [14] in particular elucidating more in details the difference with respect to asperity models in the stickiness criterion, and in [18], remarking that the stickiness criterion for the slope does not seem to correspond to the stickiness in terms of pull-off values, as it will also appear evident from the plots we are about to show.

In order to compute $\alpha(\zeta)$ from α, we therefore start from a given value of slope, for example, $h'_{rms}=0.1$, use the moments equation (11) to extract the PSD Z value for a given H and λ_s/a_0, and then compute α including the presence of roll-off, although small.

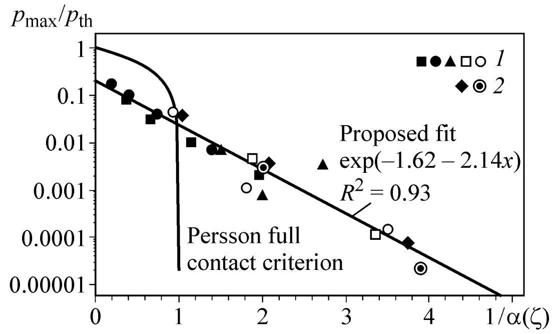

Fig. 2. Correlation of PR data for pull-off in terms of ratio with theoretical strength, with the the new stickiness parameter. Also shown, the Persson full contact parameter prediction. As in [12], $h'_{rms}=0.1, 0.3$ closed (open) symbols, l_a/a_0 = = 0.05 (*1*), 0.005 (*2*), and shape depends on H

Figure 2 shows the pull-off values of PR re-elaborated in terms of the new stickiness parameter, namely plotted against $1/\alpha(\zeta)$. Clearly, there is a very good correlation with all the data from [5], with an exponential decay—for example, using the line fit

$$\frac{p_{max}}{p_{th}} = \exp(-1.62-2.14/\alpha(\zeta)) \quad (13)$$

shows a very high coefficient of determination of R^2= = 0.93. It is obvious that this law does not include the limit case of extremely high stickiness, where there should be a transition towards the theoretical strength—for which there are no data either. Instead, the full contact Persson criterion [12] becomes in this plot a highly nonlinear curve which rapidly decays at $\alpha(\zeta)=1$ and clearly does not seem either quantitatively, nor qualitatively useful. It remains unclear under which conditions this criterion could be used, as in [15].

Notice that the dependence on the range of attraction is different from that in the PR stickiness criterion (see Ap-

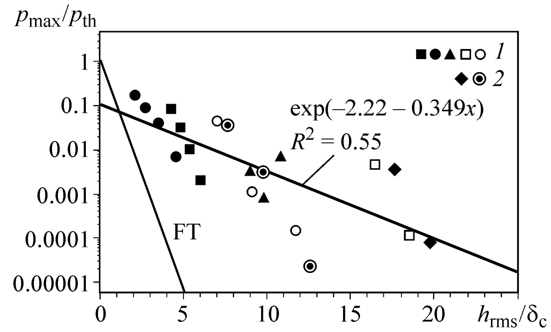

Fig. 3. Comparison of PR pull-off data with Fuller–Tabor stickiness parameter. Notice that we are plotting also approximately the original Fuller–Tabor asperity model curve, which would give a sharp loss of stickiness at $h_{rms}/\delta_c>5$. Details of symbols as Fig. 2

[1] Although there is clear indication that the H = 0.3, 0.8 symbols have been inverted.

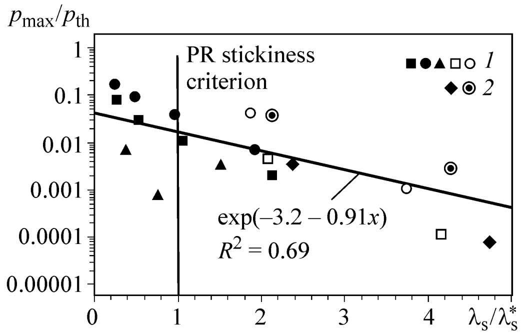

Fig. 4. Comparison of PR pull-off data with PR stickiness parameter. Notice that the original PR criterion suggests loss of stickiness for $\lambda_s/\lambda_s^* > 1$, which clearly is largely in error. Details of symbols as Fig. 2

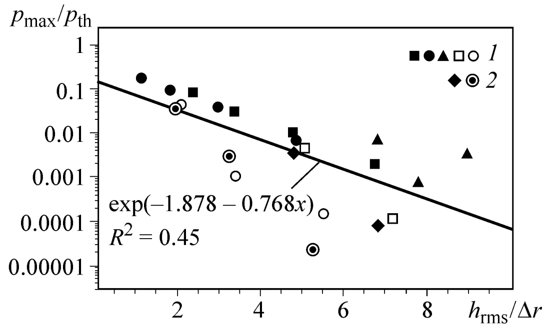

Fig. 5. Comparison of PR pull-off data with a criterion based on rms amplitude of roughness, normalized by range of attraction. Details as Fig. 2

pendix), since in the α equation, the elastic energy terms is proportional to a_0 and therefore α becomes proportional to l_a/a_0.

Considering the largely varying conditions on fractal dimension, cut-off wavelength, range of attractive forces, the correlation found is very good, and much better than correlation with other alternative stickiness parameters.

4. Comparison with alternative parameters

We start with the adhesion parameter defined by Fuller–Tabor, and we remark that we use the parameter just to correlate the data, and not using the actual Fuller–Tabor asperity model curve of decay of pull-off with the parameter, which clearly would be extremely poor. In Fig. 3, in fact, Fuller–Tabor would predict practically negligible pull-off at $h_{rms}/\delta_c > 5$. However, in terms of pure correlation with the parameter, Fig. 3 shows that the best fit exponential line has a coefficient of determination of only $R^2 = 0.55$, which is certainly quite poor as it is evident also to the eye.

A similar result is obtained PR stickiness parameter λ_s/λ_s^* which we derived in [18] and briefly described in Appendix. Figure 4 shows that the best fit exponential line shows a coefficient of determination, improved with respect to the Fuller–Tabor case, but still of only $R^2 = 0.69$. Notice, in using the PR stickiness parameter, we are not using their "criterion", which prescribes (quite inaccurately) loss of stickiness for $\lambda_s/\lambda_s^* > 1$. We have already remarked in previous papers that this discrepancy between "slope" of area–load criterion, and pull-off criterion, was not remarked in the original PR paper, and it is unclear if it stems from an hysteretic behavior of the contacts, or from some imprecise determination of the slope of the area–load curve, which is probably very non-linear near the origin.

Finally, an interesting comparison can be given for a simple $h_{rms}/\Delta r$ parameter—as would make sense for a "rigid" limit model [19]. This is shown in Fig. 5, obtaining a coefficient again very low, of only $R^2 = 0.47$, but still of the same order of that of the Fuller–Tabor asperity model which apparently comes from a much more refined argument. In these respects, notice the deviation is distributed differently: for very low amplitudes, a simple dependence on rms amplitude is indeed obtained, as it is reasonable to obtain with a transition towards the theoretical strength.

Also, notice that the improvement from the Fuller–Tabor parameter to the PR one is only from $R^2 = 0.45$ to $R^2 = 0.69$, is not as significant as the improvement made with the present criterion, showing $R^2 = 0.93$.

5. Discussion

The PR data do not clearly show a clear threshold for stickiness, and accordingly, we have fitted the data with exponential lines. Naturally, even a decay of few orders of magnitude from the theoretical strength could still be significant, and the question remains relatively obscure if there should be a finite threshold.

An important remark concerns the Tabor parameter [4]. It is clear that we can define this quantity at different scales and our assumption was that our correlation with α should work when it does not grow very large even at macroscopic scales. PR already remarks that in their cases, the Tabor parameter defined at asperity scale was small. We estimate the parameter using $R = 2/h''_{rms}$ and the moments equation (11) at small scale, and at large scales we use for simplicity $R = 2/h''$ where for $h'' = 4h_{rms} \times \times \pi^2/\lambda_L^2$, and results are shown in the Table 2.

Table 2

Estimated values of Tabor parameter μ at small scale (and under parenthesis for large scale) for $l_a/a_0 = 0.05$ and $h'_{rms} = 0.1$, for $h'_{rms} = 0.3$ values are smaller, whereas for $l_a/a_0 = 0.005$, they are slightly larger

λ_s/a_0	H		
	0.3	0.5	0.8
4	0.26 (6.30)		0.3 (3.1)
64	0.67 (3.30)		0.78 (2.6)

It can be seen that the values do not grow very large even at large scales (obviously because of the cubic root dependence on radius). We expect that, as the Tabor number grows, the pull-off becomes pressure-sensitive, and the contact more and more hysteretic. Indeed, PR remark that they plot the pull-off force obtained as surfaces are "brought together", i.e. a lower bound for the pull-off force. In general, there is some uncertainty over which cases they find really hysteretic and which not, but most likely for the stickiest surfaces reported (those which satisfy also their criterion), the pull-off is most likely non-unique and depends on the peak loading pressure. In our correlation, we have not found any strong departure from the exponential decay at very high stickiness, suggesting perhaps that pull-off continues to be governed by the α parameter when the lower bound value is considered. Perhaps there is a region in between our exponential curve, and Persson's full contact estimate, where various higher values of pull-off could be found, depending on the peak loading pressure.

A final remark about the general philosophy of searching for "semi-empirical" single parameters correlating with stickiness. Despite asperity models have shown clearly very strong limitations for adhesion, no alternative simple theory has so far been produced, that could be ultimately be collapsed in analytic estimates for pull-off. PR own attempt to produce a scaling theory for the slope of the area–load curve using concepts similar to the DMT model for the sphere, seem a lot less powerful in terms of actual pull-off values, showing it is difficult to estimate these from a simple quantitative theory. The van der Waals interactions produce very significant pressures, which may give rise to strong adhesion of small objects, such as gecko setae because they can accommodate roughness and be insensitive to shape. But as soon as bulk bodies have macroscopic dimensions, the large decay of adhesion due to roughness becomes evident, and has been called the "adhesion paradox" [20]. Having a model which accurately describes this interplay of strong attractions across different lengths scales is a very difficult task. PR data, despite showing about five orders of magnitude reduction of pull-off pressures, still are relative to amplitude of roughness of just more than one order of magnitude, from the atomic dimension to about 10–30 times more. This was probably due to limitation in the calculations which must have been already at the boundary of present computational possibilities, but it remains very possible that, if many more orders of magnitude of roughness are added, the behavior could be more complex. This is therefore a warning also with respect to extrapolating the present scaling results to wider range of conditions.

6. Conclusions

A new stickiness parameter has been introduced, which generalizes the parameter defined by Johnson for a single sinusoid. Indeed, the new parameter is simply the ratio of the adhesion energy to the elastic energy stored to flatten the rough surface. It is shown that this new parameter, for the extensive set of results by Pastewka and Robbins, correlates much better than Fuller–Tabor, Pastewka–Robbins, and Persson full contact stickiness parameters with the data. We suggest this parameter should work in the so-called DMT regime where, more precisely, we refer to low values of the Tabor parameter rather than to the DMT method of solution assuming the cohesive attractions do not deform the surfaces. Introduction of the new stickness parameter based purely on the macroscopic energetic balance is supported by recent numerical and experimental studies of adhesion of indenters with complicated contact shapes [21].

Appendix. Pastewka and Robbins criterion for stickiness

PR stickiness criterion is obtained in the original paper [5], in the form

$$\frac{h'_{\text{rms}}\Delta r}{\kappa_{\text{rep}} l_{\text{a}}}\left[\frac{h'_{\text{rms}} d_{\text{rep}}}{4\Delta r}\right]^{2/3} < 1, \tag{A.1}$$

where Δr is range of attractive forces, and d_{rep} is a characteristic diameter of repulsive contact areas, which they estimate as $d_{\text{rep}} = 4h'_{\text{rms}}/h''_{\text{rms}}$ and finally $\kappa_{\text{rep}} \approx 2$. Notice that PR define clearly $h'_{\text{rms}} = \sqrt{m_2}$ (see their Eq. (1))[1] and $h''_{\text{rms}} = \sqrt{m_4}$, and this is confirmed by the fact that our calculations correspond to the results they plot in Fig. 4 for the slope they measure with respect to the one they predict.

Depending only on small scale features, only the tail of the PSD matters—and roll-off can have a role only to achieve a more Gaussian and ergodic profile. Assuming therefore the tail is a power law $Z|\mathbf{q}|^{-2(1+H)}$, from the moments (moments), we estimate (with their definitions)

$$\frac{h''_{\text{rms}}}{h'_{\text{rms}}} \approx \sqrt{\frac{3}{4}\frac{1-H}{2-H}}\left(\frac{2\pi}{\lambda_{\text{s}}}\right) \tag{A.2}$$

and

$$h'_{\text{rms}} = (Z\pi)^{1/2}\left(\frac{1}{2-2H}\right)^{1/2}\left(\frac{2\pi}{\lambda_{\text{s}}}\right)^{1-H} \tag{A.3}$$

for $\zeta = k_{\text{s}}/k_{\text{r}} \gg 1$, and hence we obtain for stickiness

$$\left(\frac{\lambda_{\text{s}}}{2\pi}\right)^{1-5/3H} > \left(\frac{\lambda_{\text{s}}^*}{2\pi}\right)^{1-5/3H} = \frac{\Delta r^{1/3}}{2l_{\text{a}}}\left(\frac{Z\pi}{2-2H}\right)^{5/6}\left(\frac{4}{3}\frac{2-H}{1-H}\right)^{1/3}, \tag{A.4}$$

which permits to draw the plot in Fig. 4 of the main paper.

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[1] Confusing the rms surface slope with the rms profile slope, whereas normally for an isotropic surface, the sum of two orthogonal components of the gradient should be summed and being uncorrelated, this gives $h'_{\text{rms}} = \sqrt{2m_2}$. In the rest of the paper, for our own calculation, we have used the latter more correct result.

Since $1-5/3H>0$ when $3/5=0.6>H$, for the usual values of $H \gg 0.5$ (low fractal dimensions), in the fractal limit there is always stickiness (provided of course that the fractal limit has sense, i.e. that stickiness is reached before λ_s becomes too small to apply this condition). However, for $H<0.5$ (high fractal dimensions), stickiness should eventually be removed.

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