EVALUATION OF S.I.F FOR CRACK EMANATING AT 45^0 ORIENTATION FROM A HOLE IN PRESSURISED CYLINDER USING FEA

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Abstract- The use of fracture mechanics techniques combining with Finite Element Analysis [F.E.A] in assessment of performance and reliability of pressure vessel structure is on demand .The machine elements with cylindrical profile such as cylindrical shells, which are used extensively as the structural configuration in aerospace and shipping industries needs to be leak proof. But during their service life, a crack may initiate on internal/external boundary of circular cylinder which influences on stress distribution in the structure. All flaws in a physical structure may not always have the same orientation relative to boundaries and material interfaces. In fracture mechanics, Stress Intensity Factor (SIF) is an important criterion to evaluate the impact of crack as the magnitude of SIF determines the propagation of crack. This paper reviews on investigation of S.I.F for 45^0 deg arbitrarily oriented flaw emanating from a hole in pressurised cylinder using Displacement Extrapolation Method (DEM) in F.E.M that would aid in the determination of the critical nature of such flaws.

Keywords- *(Finite Element Analysis, Fracture Mechanics, Stress Intensity Factor, Displacement Extrapolation Method, Linear Elastic Fracture Mechanics, Stress Concentration, Curvature Parameter, Plane Strain, Crack Propogation,Mixed Mode Fracture, Residual Strength, Crack growth life)*

I. INTRODUCTION

Pressure vessel and cylindrical shells have been used extensively as the structural configuration in aerospace and shipping industries needs to be leak proof. However new advancement in computers has made The use of fracture mechanics techniques combining with F.E.A a practical tool in the assessment of pressure vessels structures[1], especially in determining stresses in local areas such as penetrations and service holes. The most likely places for crack initiating and development of cracks are the service holes. Due to the high stress concentration in this area cracks may grow in time, leading to a loss of strength and the reduction of the lifetime of the product as shown in Figure1. If the structure is concerned with different loading, the crack behaviour must be assessed in order to avoid catastrophic failures. For this, the knowledge of the crack size, service stress, material properties and SIF is required [2].Hence the structural assessments of hallow cylinders /shell ranging from thick walled pressure vessel to thin walled pipes has to be done, that in-turn relies on availability of stress intensity factor for fracture and fatigue analysis. Thus it has been recognized that the stress intensity factor is an important parameter to determine the safety of a cracked component, but the basic practical problem a designer faces, is to make a decision to opt the method for determining stress intensity factors. It is not easy to strike a balance between the accuracy of the method, time required to get a solution, and cost. Numerous equations for stress intensity factors are available in the literature [1 – 6]. These factors represent various geometries and loading conditions of fundamental importance in the prediction of structural failure of cracked cylindrical bodies.

The objective of the present work is to determine S.I.F for 45^0 deg arbitrarily oriented flaw emanating from a hole in pressurised cylinder using Displacement Extrapolation Method (DEM) in F.E.M. The proposed procedure is been validated with the available procedure in literature for a standard practical problem of determining SIF of longitudinal cracks in a pressurized cylinder. The validated procedure further forms the basis for a detailed research on assessing the impact of SIF in a pressurized vessel.

Fig.1: Larger crack formed in thin pressurized cylind

II. FRACTURE MECHANICS

Fracture mechanics involves a study of the presence of the cracks on overall properties and behaviour of the engineering component. The process of fracture may be initiated at defect locations like micro-cracks, voids, and the cavities at the grain boundaries. These defects can lead to the formation of a crack due to the rupture and disentanglement of molecules, rupture of atomic bonds or dislocation slip [3]. Cracked body can be subjected to three modes of loads as shown in Figure 2. In some cases, body may experience combination of the three modes:

1. Opening mode: The principal load is applied normal to the crack surfaces, which tends to open the crack. This is also referred as Mode I loading (Figure 2a).
2. In-plane shear mode: This mode corresponds to in-plane shear loading which tends to slide One crack surface with respect to the other. This is also referred as Mode II loading (Figure2b).
3. Out-of-plane shear mode: This is the tearing and anti-plane shear mode where the crack surfaces move relative to one another and parallel to the leading edge of the crack (Figure 2c).

The Stress Intensity Factor (SIF) is one the most important parameters in fracture mechanics analysis. It defines the stress field close to the crack tip and provides fundamental information of how the crack is going to propagate. In this study, a typical and practical point matching technique, called Displacement Extrapolation Method (DEM) is chosen for the numerical analysis method. Plane strain assumption is valid for very thin-walled structures; the evaluation of S.I.F (K_I) by Displacement Extrapolation Method (DEM) is as discussed bellow.

The stress intensity factors at a crack for a linear elastic fracture mechanics analysis may be computed using the KCALC command. The analysis uses a fit of the nodal displacements in the vicinity of the crack. The actual displacements at and near a crack for linear elastic materials are

$$u = + \frac{K_{11}}{2G}\sqrt{\frac{r}{2\pi}}(1+k) \quad \text{------------------------}(1)$$

$$v = + \frac{K_1}{4G}\sqrt{\frac{r}{2\pi}}(1+k) \quad \text{------------------------}(2)$$

$$w = + \frac{2K_{111}}{G}\sqrt{\frac{r}{2\pi}} \quad \text{------------------------}(3)$$

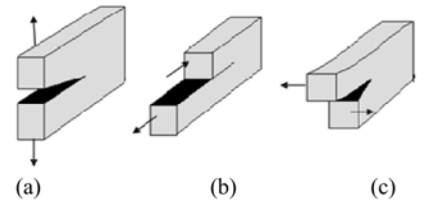

Fig. 2: Three modes of loading that can be applied to a crack

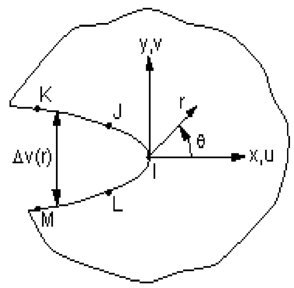

Fig. 3: Nodes Used for the Approximate Crack-Tip Displacements for Full crack Model

Where: u, v, w = displacements in a local Cartesian coordinate system as shown in figure 3r, θ = coordinates in a local cylindrical coordinate system as shown in figure 3.

G = shear modulus

$$K = \frac{v}{1+v} \quad \text{In Plane Stress} \ldots\ldots\ldots\ldots(4)$$

K=3-4ν In Plane strain………………………………….....(5)

v = Poisson's ratio

For Mode-1, SIF at crack tip is expressed as

$$K_1 = \sqrt{2\pi}\,\frac{4G}{1+k}\,\frac{|\Delta V|}{\sqrt{r}} \quad \ldots\ldots\ldots\ldots(6)$$

Where Δv, is the motion of one crack face with respect to the other.

At points J and K. if, let r approach zero

$$\left.\begin{aligned} \lim_{r\to 0}\frac{|V|}{\sqrt{r}} &= A \\ \lim_{r\to 0}\frac{|u|}{\sqrt{r}} &= B \\ \lim_{r\to 0}\frac{|w|}{\sqrt{r}} &= C \end{aligned}\right\} \quad \text{------------------------}(7)$$

Therefore

$$K_1 = \sqrt{2\pi}\frac{4GA}{1+k}MPa\sqrt{mm} \quad \ldots\ldots\ldots\ldots(8)$$

$$K_{11} = \sqrt{2\pi}\frac{2GB}{1+k}MPa\sqrt{mm} \quad \ldots\ldots\ldots\ldots(9)$$

$$K_{111} = \sqrt{2\pi}\frac{GC}{2(1+k)}MPa\sqrt{mm} \quad \ldots\ldots\ldots\ldots(10)$$

III. METHODOLOGY

The objective of this work is to determine SIF for a 45^0 deg arbitrarily oriented crack emanating from a hole in a pressurized cylinder as shown in figure 4. The objective is achieved by developing a model of a cylinder with hole and a through crack using CATIA V5 software. The CATIA model is imported to ANSYS.The FE model is meshed using 8-node quadrilateral doubly curved SHELL 93 elements in the pre-processor of the ANSYS software. A shell with a longitudinal crack was meshed using three different mesh densities. Mainly, the area around the imperfection was modeled with a finer mesh.

As a part of the finite element work, a mesh sensitivity Study was conducted. Further, the crack tip singular elements were created using KSCON command. For this model there are 36 singular elements around the crack tip and the radius of the first row elements is Δa (Where Δa = a/100).The model is then solved (Static Analysis) by subjecting it to an internal pressure of 1MPa load with appropriate boundary conditions. Then the SIF is evaluated in general postprocessor by using KCALC command.

The geometry of the meshed test model with crack tip singular elements in ANSYS 12 is as shown in the Figure 5.The material considered is 304 steel (ASME). The material is assumed to be linear elastic with young's modulus of 2.5GPa and poisons ratio 0.3

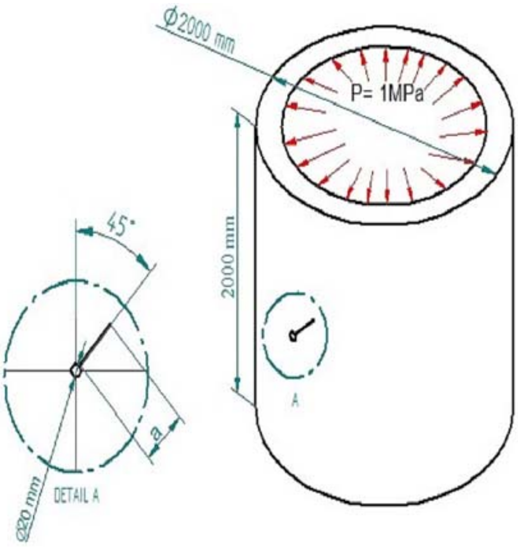

Fig.4: Geometry of model

Where,

D= Diameter of the hole (20mm),

a= Half Crack length mm

σ=applied hoop stress (Pr/t)

P= Internal pressure 1MPa

t=Thickness of the cylindrical shell 10mm

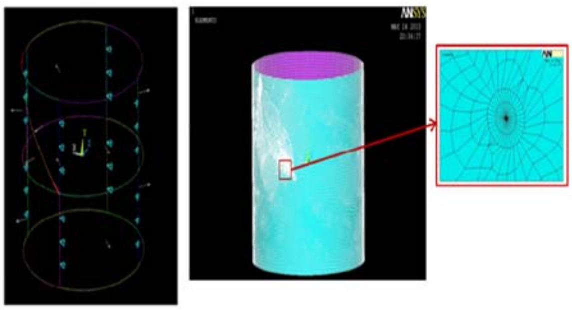

Fig. 5: (a) Finite Element Model with Boundary Condition (b) Meshed Finite Element Model (c) Zoomed View of Crack Tip Singular Elements

IV. RESULTS AND DISCUSSION

The Figure 6 shows a pressurized cylindrical shell with varying longitudinal crack length. The theoritical formulae involved in the determination of mode 1 SIF of the crack is given below [3]. Mode I S.I.F (K_I) is given by

$K_I(\text{Theo}) = \sigma\sqrt{\pi a}.\ f_1(\alpha)$ ……………………….(11)

Where

$$f_1(\alpha) = \sqrt{(1 + 0.52x + 1.29x^2 - 0.07x^3)}$$

$$x = \frac{a}{\sqrt{(Rt)}}$$

The half crack length was varied from 20mm to 439.53mm .The maximum crack length in a given dimensions of cylindrical shell was determined using curvature parameter β [11]

$$\beta = \frac{a}{\sqrt{Rt}}\sqrt[4]{12(1-\nu^2)} \quad \text{…………………………. (12)}$$

if β =8 for longitudinal cracks , thickness of the cylindrical shell is 10mm and radius of the cylindrical shell is 1000mm then the maximum crack length for given set of cylindrical shell dimension is 439.53mm.

The values of mode 1 SIF from the procedure explained in the previous section is also determined for the same problem using FEA in ANSYS. Table 1 shows the values of SIF obtained by the theoretical and FEM.

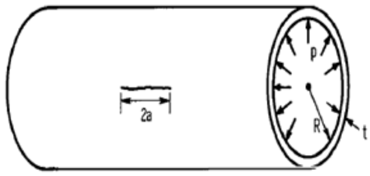

Fig.6: Longitudinal crack in internally pressurized cylinder

Table 1: Mode-1 S.I.F (K_I) Using FEA and theoretical K_I(Theo) for different Half Crack lengths (a)

Half Crack Length (a) mm	Mode -I SIF by FEA $MPa\sqrt{mm}$	Mode -I SIF by Analytical $MPa\sqrt{mm}$	% error
20	821.49	851.322	3.50
40	1282.6	1330.62	3.61
60	1749.6	1828.751	4.33
80	2265.4	2352.63	3.71
100	2831.4	2931.63	3.42
120	3448.9	3555.117	2.99
140	4114.8	4221.655	3.42
180	5570.8	5669.381	1.74
220	7168.4	7247.292	1.09
260	8886.5	8933.523	0.53
300	10711	10710.546	0.00
340	12634	12561.166	-0.58
380	14608	14471.544	-0.94
400	15685	15446.58	-1.54
439.532	17798	17398.911	-2.29

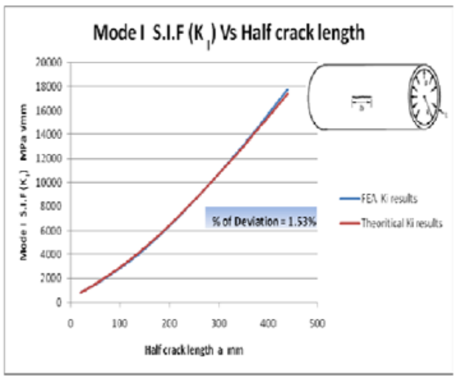

Fig.7: variation of theoretical and FEA values of S.I.F Vs crack length in Pressurized cylinder

Figure 7 shows the variation of theoretical and simulated FEA SIF values for various longitudinal crack length in pressurized cylinder. From the Figure it is indicated that the results which were obtained by using FEM are in good agreement with theoretical equation for a longitudinal through crack emanating in internally pressurized cylindrical shell

The average percentage of error between the FEA and theoretical SIF values is 1.53%, which is negligible. Thus the proposed methodology to determine the Mode-1 S.I.F for longitudinal cracks in pressurized cylindrical shell is validated against a standard procedure elucidated in literature [3].

The above procedure is used for "Determination of SIF for a Crack Emanating at 45^0 orientation from a Hole in a Pressurized cylinder" the test models containing a through 45^0 crack emanating from a hole is meshed under plane strain condition it is internally pressurized and respective SIF's are calculated by varying half crack length(a).

$$K_0(Theo) = \sigma\sqrt{\pi a_{eff}}\,MPa\sqrt{mm}$$

The specimen graphs are plotted to variation of normalized Stress Intensity Factor's (K_I/K_O), (K_{II}/K_O), (K_{III}/K_O) with respect to (a/D) ratio is as shown in Figure 8(a),8(b),8(c) respectively.

The values of K_I, K_{II} & K_{III} with respect to (a/D) is as shown in table 2. It is observed that K_I & K_{II} (opening mode and in-plane shearing mode) have equal contribution for crack propagation. The specimen graphs 8(a) & 8(b) shows that the Mode-I S.I.F (K_I) & Mode-II S.I.F (K_{II}) increases steadily as crack grows. The value of Mode-III S.I.F (K_{III}) is very small compared to Mode-I (K_I) & Mode-II (K_{II}).The Mode-III S.I.F (K_{III}) has negligible contribution for crack propagation compared to Mode-I & Mode-II.

Therefore for 45^0 oriented crack in internally pressurized cylinder the Mode-I S.I.F (K_I) & Mode-II S.I.F (K_{II}) are dominant and have equal contribution for crack propagation.

Table 2: The normalized Stress Intensity Factor (K_I/K_O) with respect to a/D ratio

Half Crack Length a mm	a/D	Mode -I SIF (K_I) by FEA $MPa\sqrt{mm}$	Mode -II SIF (K_{II}) by FEA $MPa\sqrt{mm}$	Mode -III SIF (K_{III}) by FEA $MPa\sqrt{mm}$	Theoretical SIF K_0 $MPa\sqrt{mm}$
60	3.00	553.49	527.41	36.462	1120.71
80	4.00	644.64	595.16	51.005	1253.00
120	6.00	828.91	732.21	79.618	1482.57
160	8.00	1042.2	881.71	126.45	1681.07
180	9.00	1134.7	942.8	149.21	1772.00
200	10.00	1260.4	1034.8	180.3	1858.49
260	13.00	1623.8	1287.8	263.74	2096.66
300	15.00	1879.4	1463.3	420.24	2241.43
340	17.00	2146.5	1.64E+03	568.88	2377.39

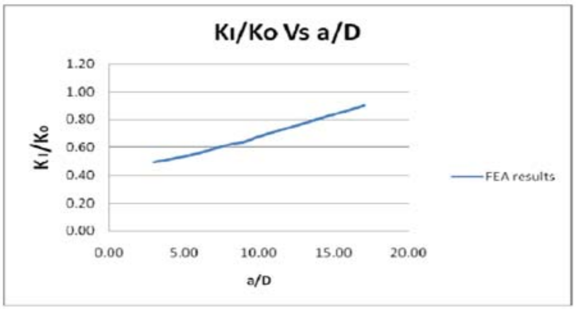

Fig.8(a): Variation of normalized S.I.F (K_I/K_0) for a crack emanating from circular hole in pressurized cylinder Vs a/D

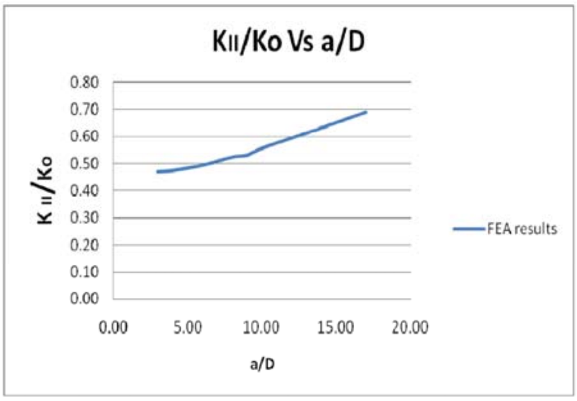

Fig.8 (b): Variation of normalized S.I.F (K_{II}/K_0) for a crack emanating from circular hole in pressurized cylinder Vs a/D

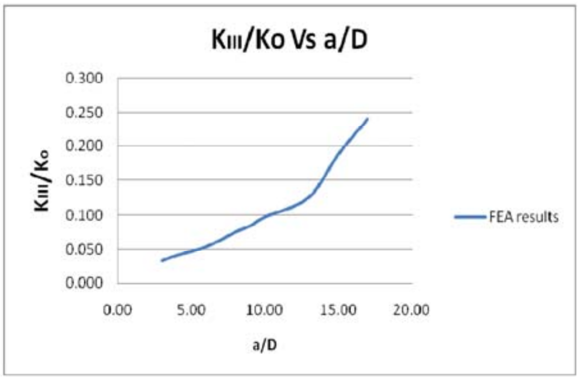

Fig.8(c): Variation of normalized S.I.F (K_{III}/K_0) for a crack emanating from circular hole in pressurized cylinder Vs a/D

CONCLUSION

The problem of determining SIF for a 45^0 deg arbitrarily oriented crack emanating from a hole in a pressurized cylinder is of prime importance in damage tolerance analysis. In the present study ANSYS12, unified FEA software is chosen. It has the required pre-processing capabilities for finite element modeling and analysis of cracked shell structures as Demonstrated in this paper. The proposed methodology to determine SIF by DEM in FEM is validated with the standard procedure available in literature.

The variation of normalized S.I.F (K_I/K_O),(K_{II}/K_O) ,(K_{III}/K_O) with respect to a/D ratio is used to obtain the characteristic curves of SIF which depends only on the geometrical factor and its variation within the given domain (a/D) are plotted separately. It is observed that for, 45^0 oriented crack in internally pressurized cylinder the Mode-I S.I.F (K_I) & Mode-II S.I.F (K_{II}) are dominant and have equal contribution for crack propagation.

The presented stress intensity factors in this paper are essential to predict
(1) Mixed mode fracture under static, dynamic and sustained loads
(2) Residual strength
(3) Crack growth life under cyclic loading conditions.

However there is a clear need to verify the predictions using experimental investigations, but the method used in this paper can be utilized for calculating the stress intensity factor for many other loading cases and many values of the crack length. This provides important information for subsequent studies, especially for fatigue loads, where stress intensity factor is necessary for the crack growth rate determination.

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