



Energy-based versus stress-based material failure criteria: The experimental assessment

Suhil Abu-Qbeith^{a,b,*}, Konstantin Y. Volokh^c, Stephan Rudykh^a

^a School of Mathematical & Statistical Sciences, University of Galway, Galway H91 TK33, Ireland

^b School of Mechanical Engineering, Tel Aviv University, Tel Aviv, Israel

^c Faculty of Civil and Environmental Engineering, Technion - Israel Institute of Technology, Haifa 32000, Israel

ARTICLE INFO

Keywords:
Failure criteria
Energy
Strength
Experiments

ABSTRACT

Previous studies have reported fracture localization within the inclusions of 3D-printed staggered composites, despite their significantly higher strength compared to the matrix – a seemingly counterintuitive phenomenon. In this letter, we investigate whether material failure is governed by the volumetric energy of fracture rather than the maximum stress criterion. We perform experiments on the constituent phases of 3D-printed staggered composites to evaluate the validity of energy-based failure criteria. Our findings support the idea that the work of fracture, rather than strength, governs failure. Specifically, at relatively higher strain rates, the ability of the soft matrix to absorb more energy before failure suggests that fracture localization is driven by energy considerations rather than stress thresholds. This aligns with previous observations that inclusions may fail before the matrix despite their higher strength. More broadly, since engineering materials often exhibit a crystalline molecular structure where failure is dictated by the energy required to break atomic and molecular bonds, it naturally follows that the work of fracture – rather than strength – should serve as the primary failure criterion. Our results reinforce this perspective, offering a more physically grounded approach to predicting material failure.

1. Introduction

Criteria for failure serve as a crucial methodology in science and applications, enabling the evaluation of materials' ability to withstand loads, a fundamental aspect that has been integral to engineering practices for centuries. Thus, achieving engineering excellence is fundamentally dependent on a thorough understanding of material failure mechanisms [1].

In materials science, failure is defined as the loss of a material's ability to carry load [2]. Several failure criteria have been established in the literature [3–12], such as von Mises [13], Tresca [14] — both widely used in yield analysis [15], strength criterion [16–19], maximum strain criterion [20], Hill's criterion [21], Tsai–Wu criterion [22], ellipse criterion [23,24], and energy-limiter [25–32]. Where these failure criteria can be broadly categorized into two main types: (1) stress- or strain-based criteria, and (2) energy-based criteria [33,34].

Stress-based failure criteria are theoretical frameworks that predict material failure by evaluating the internal stress state. These models determine whether the applied stresses surpass the material's intrinsic strength, initiating failure through yielding or fracture. In contrast, energy-based criteria are grounded in the principle that failure occurs

due to the release of stored potential energy within atomic and molecular bonds [35]. This release is constrained by the finite number of such bonds in the material, ultimately limiting the material's energy absorption capacity before failure [36–39].

The objective of this study is to experimentally evaluate whether strength or energy should serve as the foundation for developing robust failure criteria. Through utilizing advanced additive manufacturing techniques, we fabricate and mechanically test samples until complete failure. Additionally, this investigation aims to elucidate the underlying mechanisms leading to the failure of hard inclusions, as observed in [40], despite their superior strength relative to the surrounding soft matrix.

The note is organized as follows: Section 2 presents the experimental setup, tested materials, methodology, and results, while Section 3 provides a discussion of the key findings and arguments.

2. Experiments

In the work of [40], nacre-like composite samples, as shown in Fig. 1 (A), were fabricated via additive manufacturing and tested under

* Corresponding author at: School of Mechanical Engineering, Tel Aviv University, Tel Aviv, Israel.

E-mail address: suhilabu@tauex.tau.ac.il (S. Abu-Qbeith).

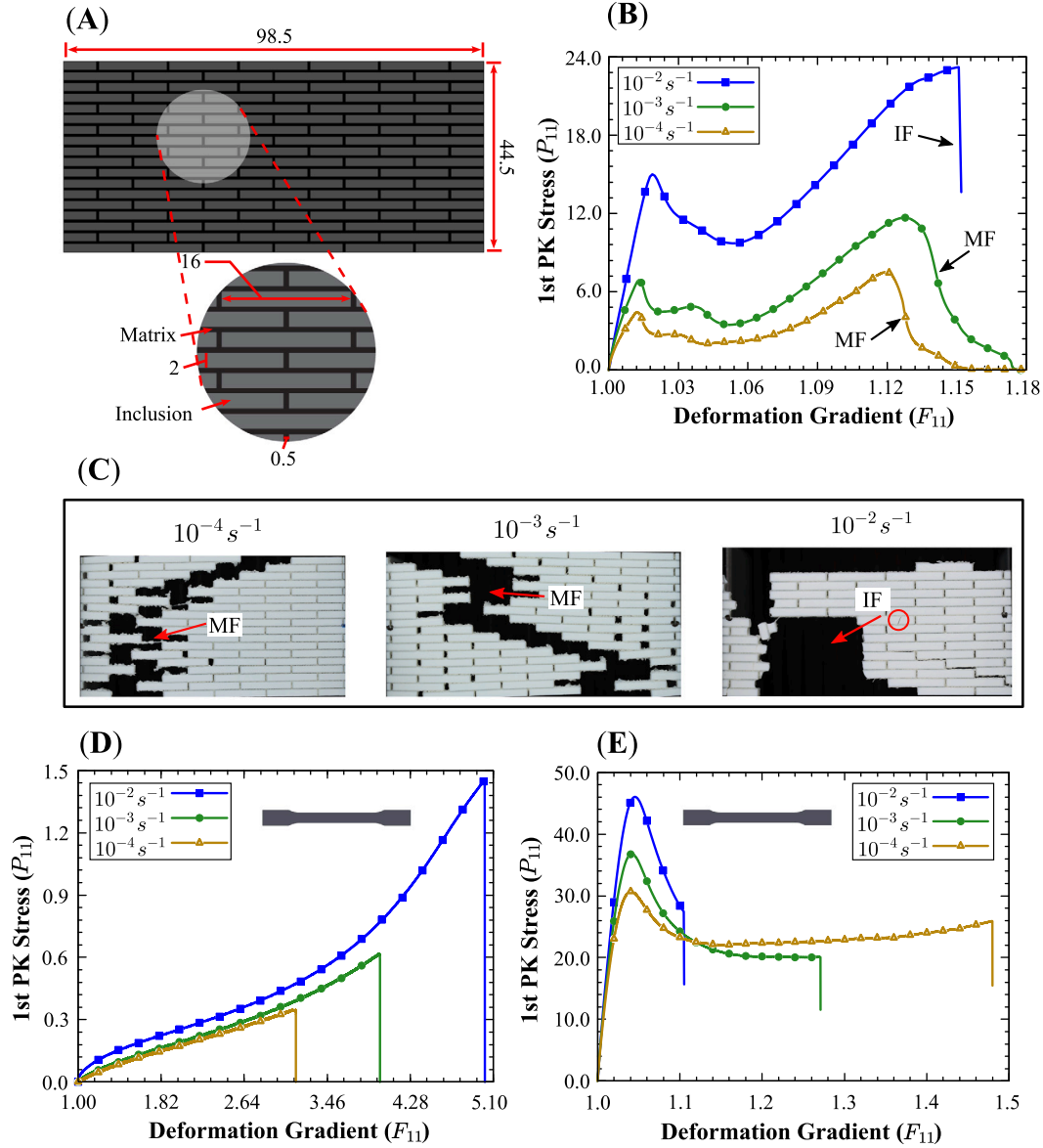


Fig. 1. Nacre-like experiments of [40]: (A) Geometric setup, (B) uniaxial tensile responses, (C) damage localization — IF and MF designate inclusion failure and matrix failure, accordingly. Uniaxial tensile responses of homogeneous samples: (D) ElasticoClear, and (E) VeroUltraWhite, respectively, for different values of applied speed.

uniaxial tension. The hard fillers were made from VeroUltraWhite (RGD824), and the soft interface from ElasticoClear (FLX934) [41]. The samples were tested under different strain rates, where they were under tension applied in the horizontal direction on the right edge, while the left edge is encastre.

Fig. 1 (B) shows the uniaxial tensile responses of the composite until the full sample's separation for different values of strain rates. MF refers to the case when the matrix of the composite ruptures leading to the full damage leaving the inclusions intact. On the other hand, IF refers to the case when the inclusions fracture leading to the full separation of the composite.

For small applied tensile speeds, the damage localized in the soft interface and the hard fillers remained undamaged. However, for relatively higher applied tensile speeds, the hard fillers failed abruptly leading to debris scattered from the specimen. Fig. 1 (C) demonstrates the localization of damage across all cases, with damage concentrated in the matrix at low strain rates and in the inclusions at elevated strain rates.

Failure of the inclusions rather than the matrix at relatively higher tensile speeds is quite unexpected, where the hard fillers strength is

much greater than soft interface strength. So, the question is why the inclusions fractured although they can sustain much higher stress than the matrix. To answer the latter question, it is of high interest to test the composite's constituents separately, which will be done in this work.

In this section, we independently evaluate the composite constituents utilized by [40] under identical experimental conditions. Thus, homogeneous dog bone samples are fabricated from ElasticoClear (FLX934) and VeroUltraWhite (RGD824) interfaces and tested for the same applied speeds. We fabricate the homogeneous samples via the same additive manufacturing procedures used by [40] with a Stratsys J35 Pro 3D printer. Afterwards, we test the samples until the complete damage via an Instron 34SC-5 universal testing machine. The samples are under tension applied on one side while the second side is fixed.

Three repeated experiments were conducted, with Figs. 1 (D) and (E) presenting the first Piola–Kirchhoff stress component in the tensile direction as a function of the deformation gradient in the same direction for a representative trial. The average strength of each constituent is shown in Fig. 2 (A). The significant strength disparity between the two materials is evident, rendering them essentially incomparable.

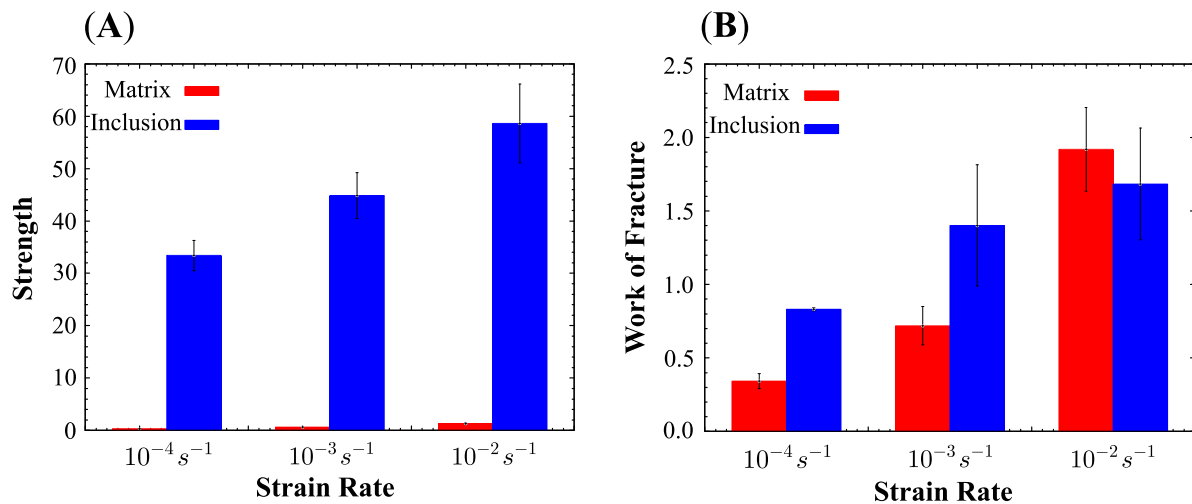


Fig. 2. (A) Tensile strength (measured in MPa) of ElasticoClear (red) and VeroUltraWhites (blue) under different strain rates. (B) Volumetric work of fracture (measured in mJ/mm³) for ElasticoClear (red) and VeroUltraWhites (blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The critical question arises: if the fillers possess significantly higher strength compared to the matrix, what mechanisms lead to fracture initiating or propagating within the fillers rather than in the matrix? To answer this question, we shall calculate the volumetric work of fracture that can be provided by each constituent. The volumetric work of fracture can be quantified by integrating the area under the response curves up to the critical limit points.

Fig. 2 (B) shows the average volumetric energy of fracture for both constituents. After comparing the results in Fig. 2 (B), it becomes clear why the damage localized in the inclusions rather than in the matrix despite their high strength. It is because the volumetric work of fracture inside inclusions is less than the volumetric work of fracture inside the matrix at relatively higher strain rates. This underscores a crucial observation: traditional strength criteria, such as maximum stress, are inadequate. Instead, energy-based criteria, such as the work of fracture or energy limiter approaches, should be employed when evaluating materials.

It is important to note from Fig. 2 (B) that at intermediate strain rates, a “boundary” case arises, enabling cracks to propagate through any phase.

3. Conclusions and discussion

Our experimental findings provide an explanation for the fracture localization within the inclusions, rather than the matrix, as reported in [40], under relatively higher strain rates. This behavior persists despite the inclusions exhibiting greater strength compared to the softer matrix.

The work of fracture, that is to say the energy, is the main reason behind such unexpected phenomenon. At relatively higher strain rates, the soft interface surpasses the hard material in its work of fracture leading to localization of fracture in the hard constituent explaining the reason of inclusions damage before matrix failure.

If we only rely on strength as failure criterion, then we would say that the matrix shall fail before the inclusions because of its low incomparable strength with respect to the hard fillers. However, this is not the case, where inclusions failed rather than the matrix. Thus, the results suggest that using strength as the failure criterion is not optimal; instead, energy should be considered.

Further, engineering materials often exhibit a crystalline molecular structure, where atoms and molecules are bonded together by atomic and molecular bonds. The latter implies that material failure depends

on their work of fracture, in other words, the energy required to break the bonds. Thus, it flows naturally to use the work (energy) of fracture as the criterion to predict material failure rather than strength.

It is also noteworthy that, while this study focuses on uniaxial tension, the anisotropic architecture of the material suggests that failure behavior may differ under other loading conditions, such as compression, shear, or loading in different directions. The insights presented here provide a robust foundation for understanding fracture mechanics under uniaxial tension. Building on this, further investigations under multi-axial or alternative loading scenarios would be valuable for broadening the understanding of mechanical performance and failure mechanisms.

CRedit authorship contribution statement

Suhil Abu-Qbeitah: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Konstantin Y. Volokh:** Writing – review & editing, Investigation, Conceptualization. **Stephan Rudykh:** Writing – review & editing, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The support from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (MAGIC –[852281]) is gratefully acknowledged. Also, K. Y. Volokh is supported by the Israel Science Foundation, Israel (ISF-394/20) as well as the Israeli Ministry of Science and Technology (MOST-0005173).

Data availability

Data will be made available on request.

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