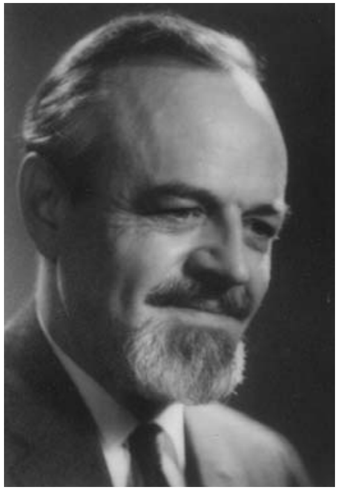
Soft Active Materials

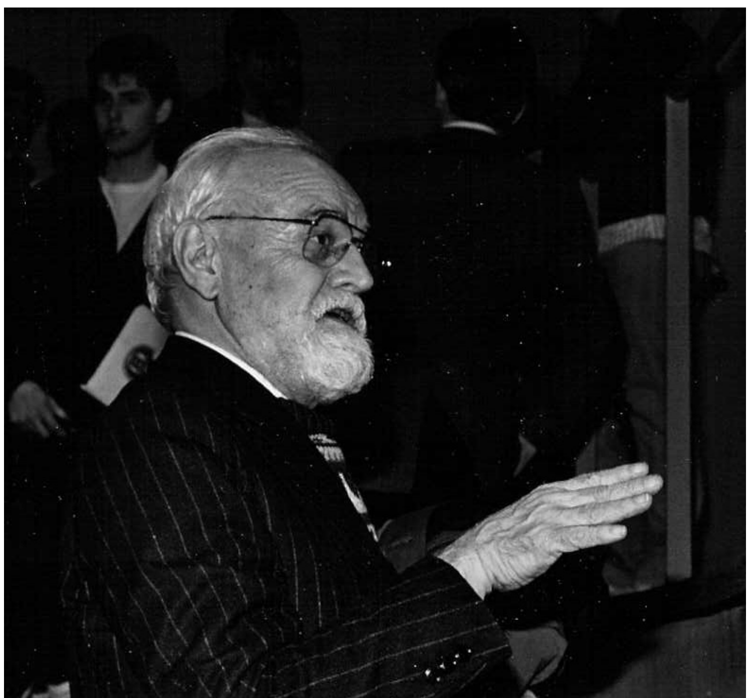
Zhigang Suo
Harvard University



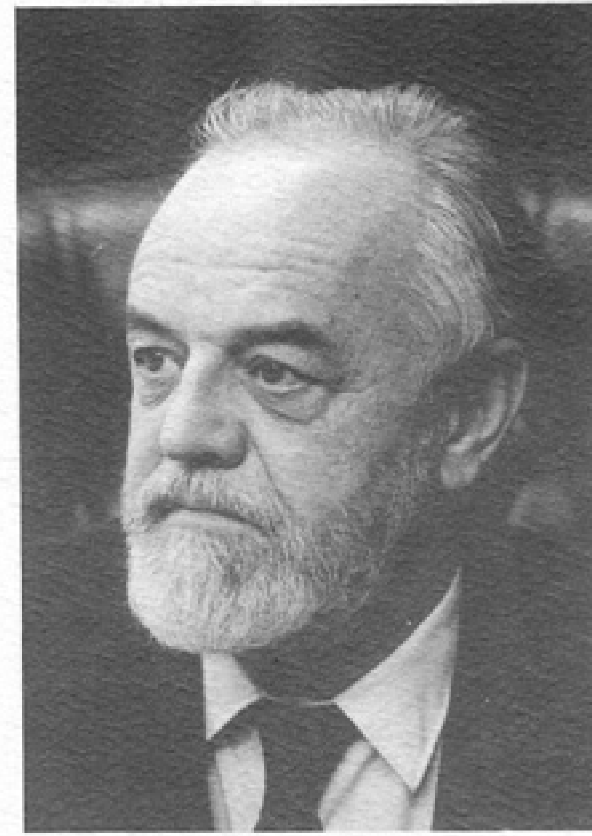
James F. Bell
1914-1995

The James F. Bell Memorial Lecture in Continuum Mechanics
Department of Mechanical Engineering, Johns Hopkins University
5 November 2009

Professors Sharp and Bell (1983)

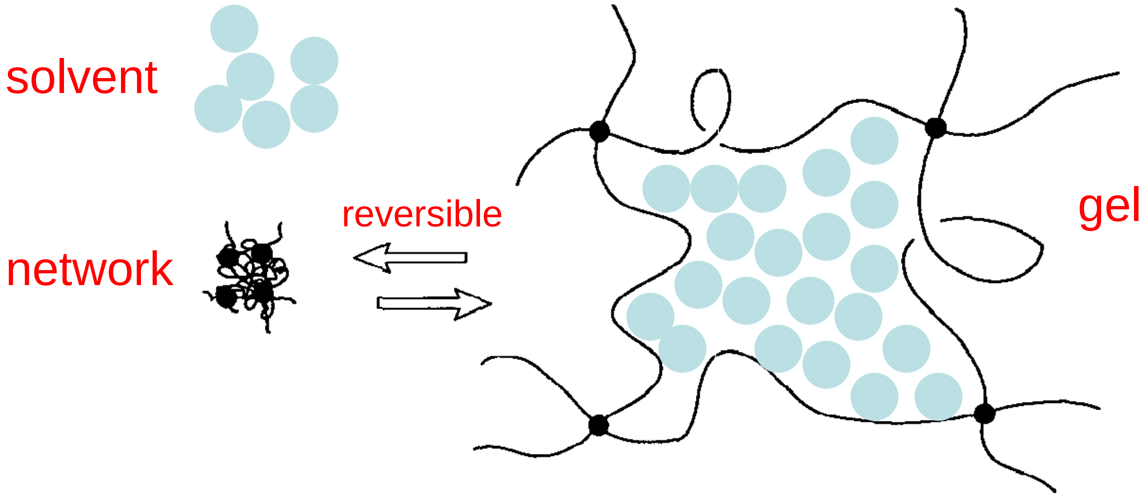


Professor Bell (about 1990)



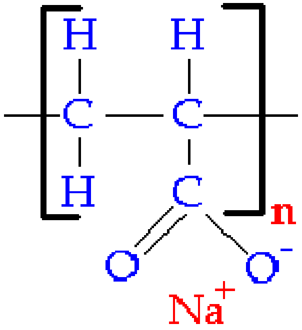
The late Professor Emeritus James Bell, spent his entire academic career—which spanned half a century—at Johns Hopkins.

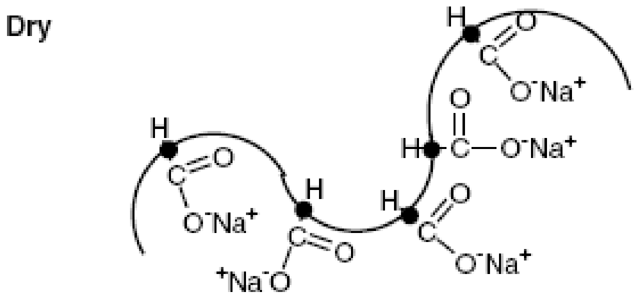
elastomer = network
gel = network + solvent

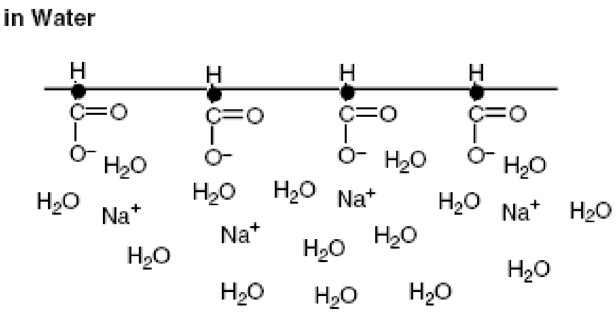


Super absorbent diaper

Sodium polyacrylate: polyelectrolyte

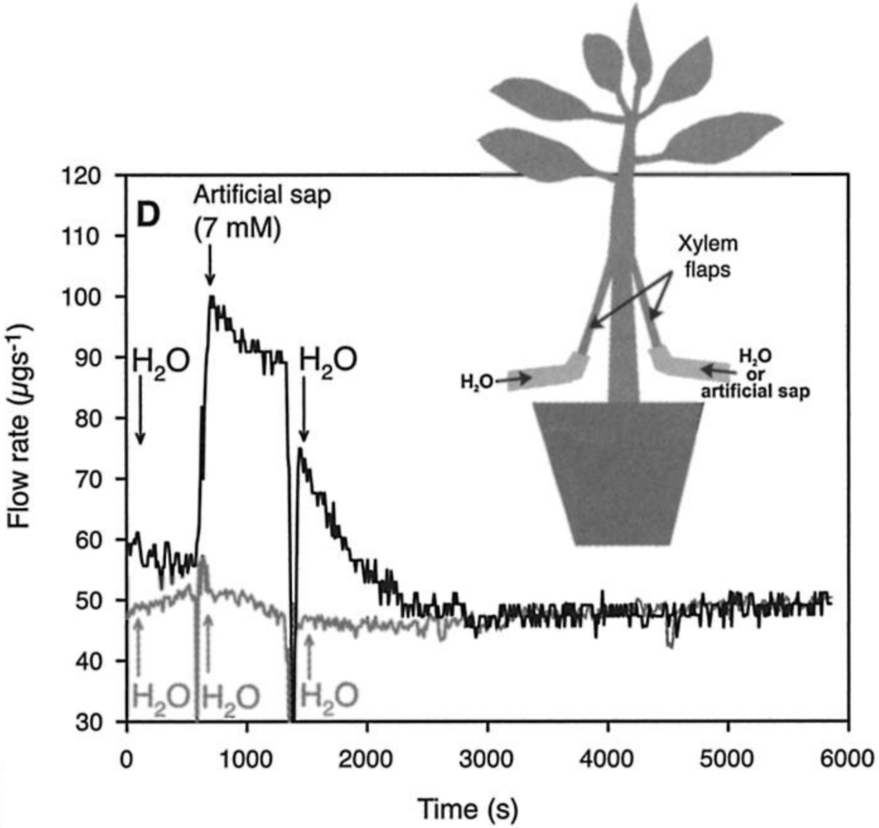


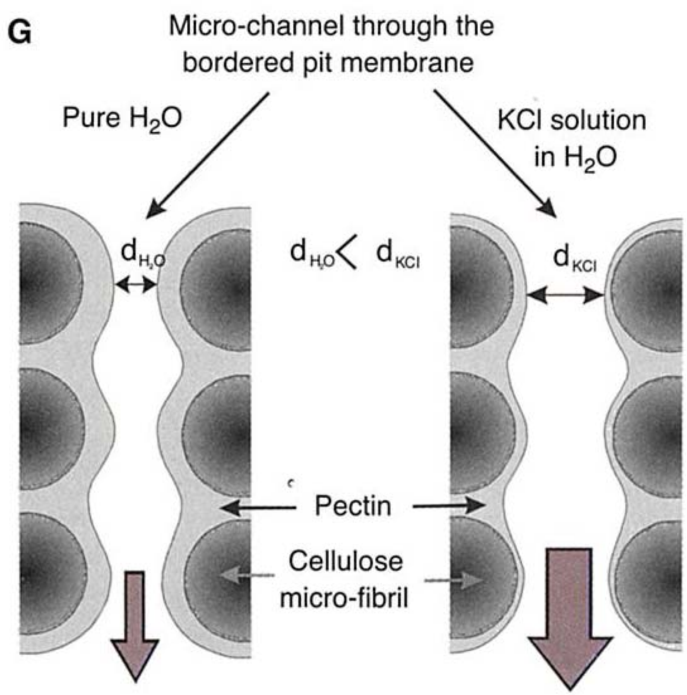




Gels regulate flow in plants

Missy Holbrook

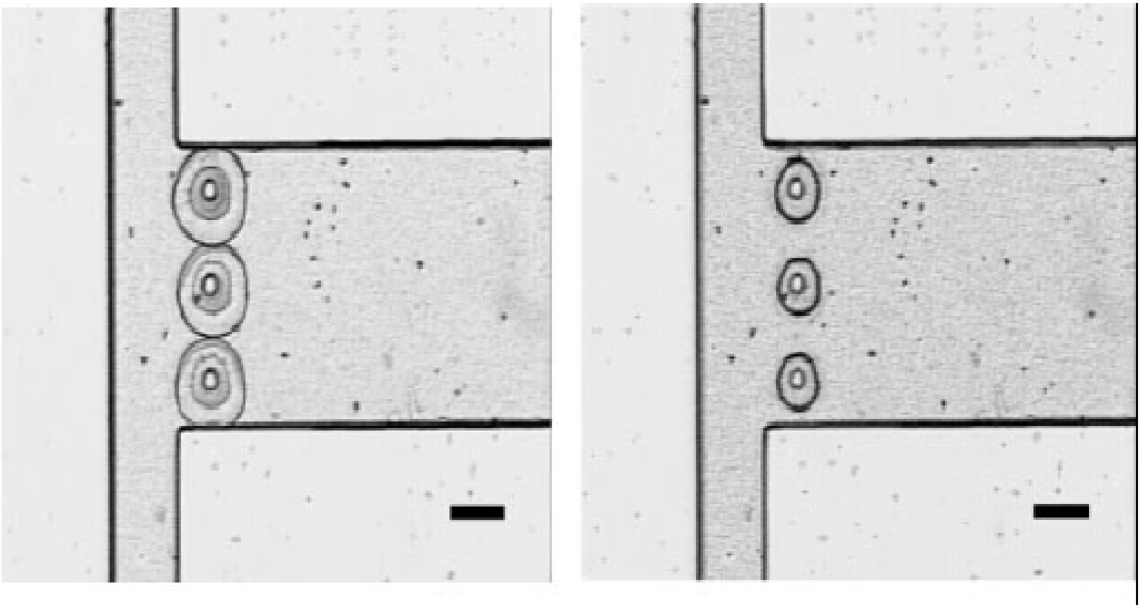




Zwieniecki, Melcher, Holbrook,
Hydrogel control of xylem hydraulic resistance in plants
Science 291, 1095 (2001)

Self-regulating fluidics

David Beebe

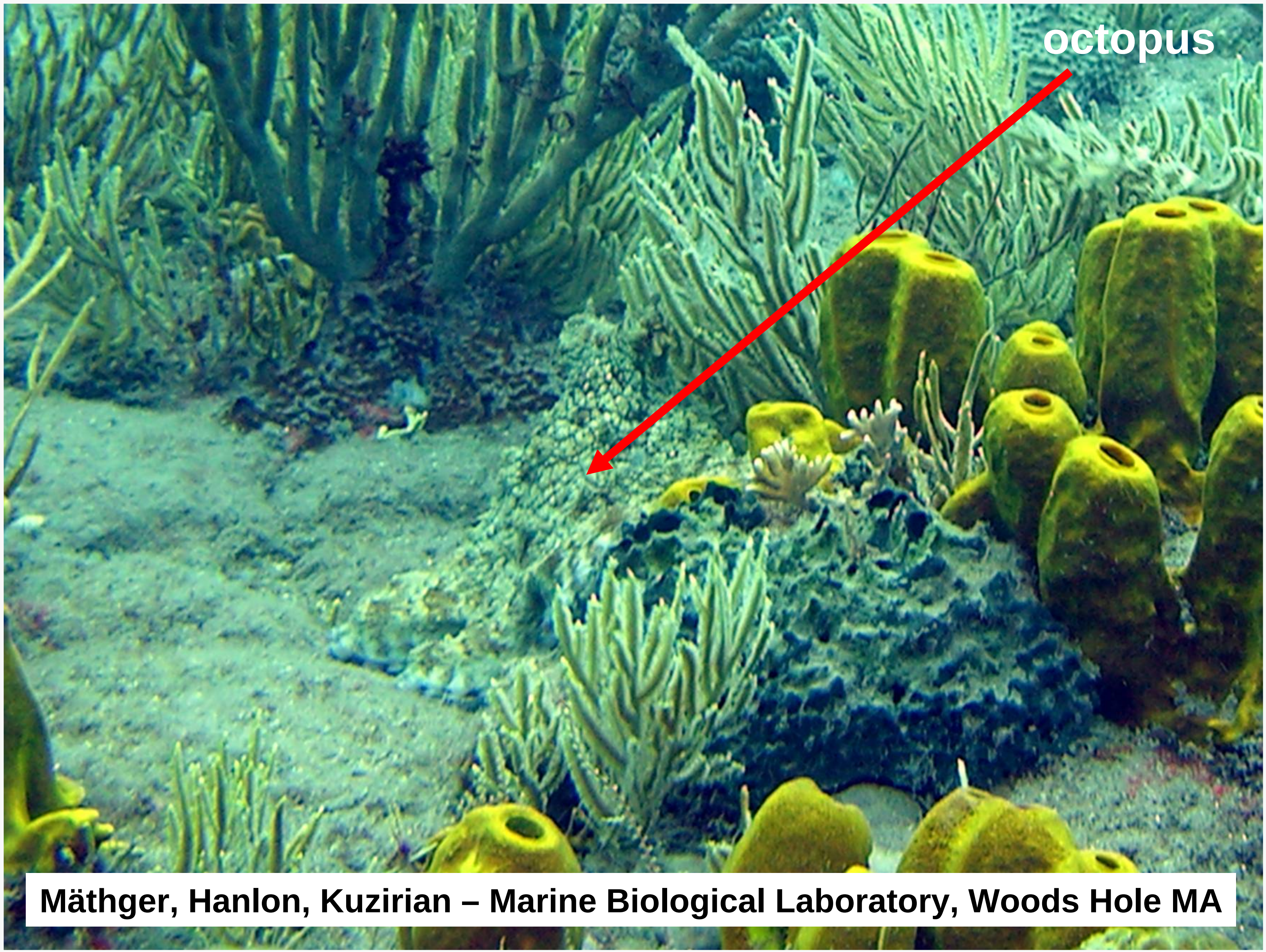


Responsive to Physiological variables:

- •pH
- •Salt
- •Temperature
- •light

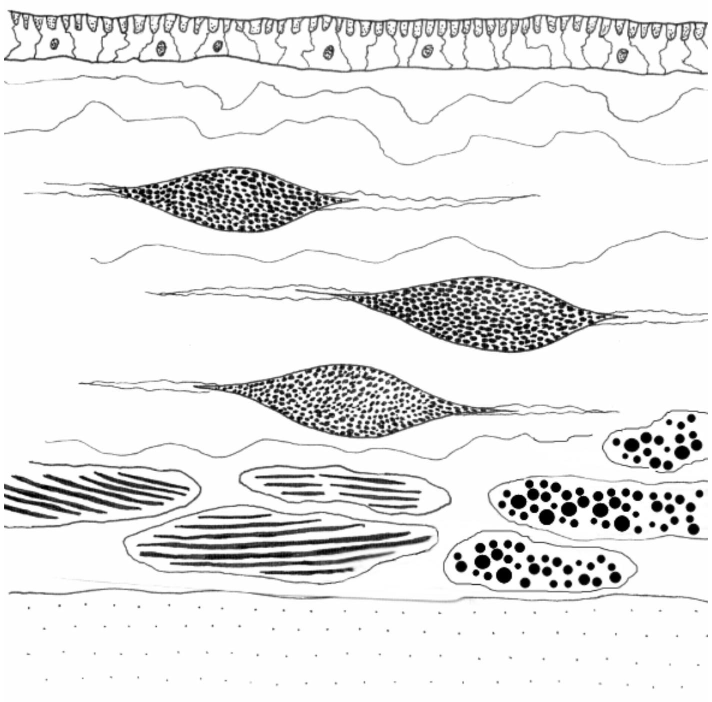
- •Many stimuli cause deformation.
- •Deformation regulates flow.

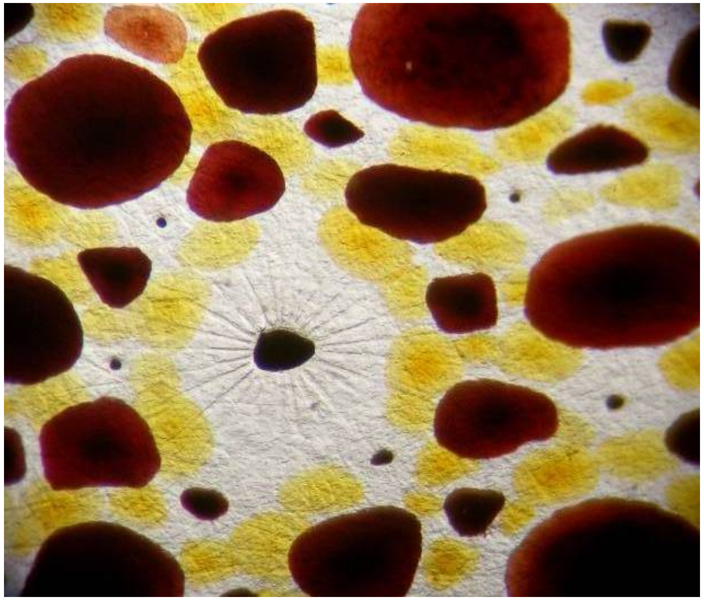
Beebe, Moore, Bauer, Yu, Liu, Devadoss, Jo, Nature 404, 588 (2000)


octopus
Mäthger, Hanlon, Kuzirian – Marine Biological Laboratory, Woods Hole MA

Squid changes color

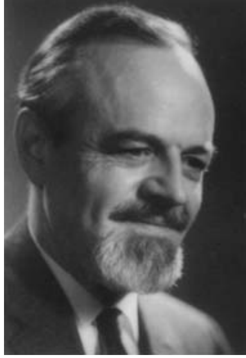
Expand pigmented sacs by contracting muscles



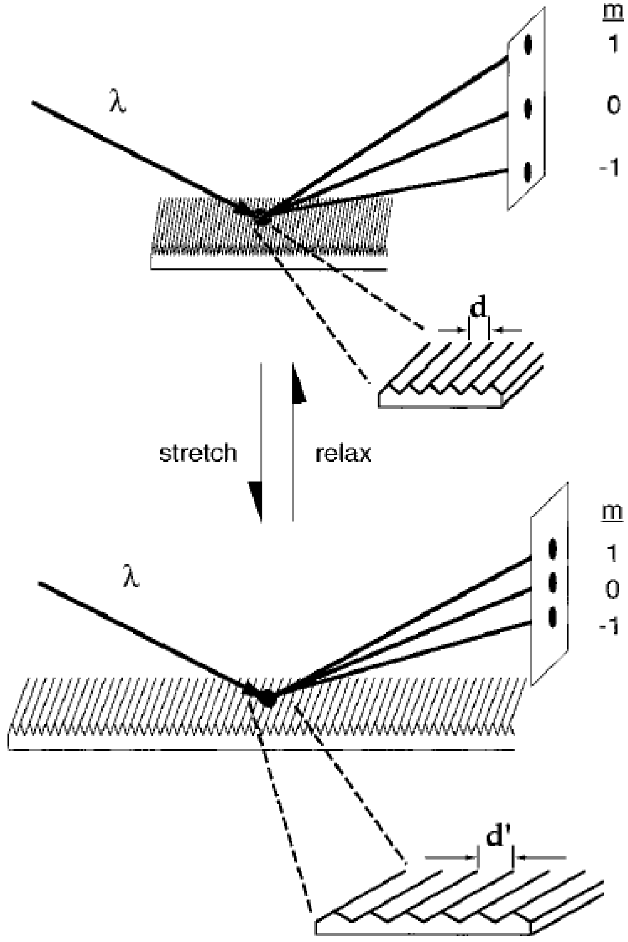


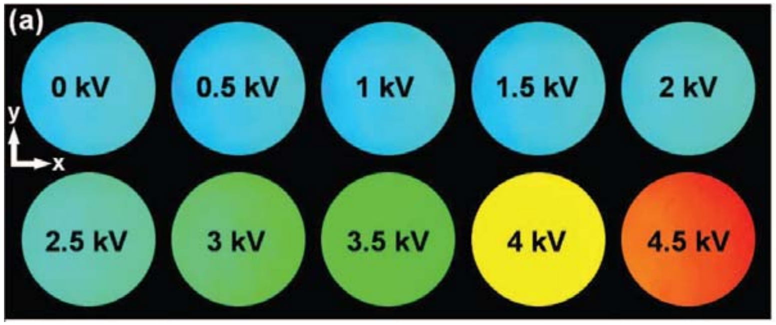
Mathger, Denton, Marshall, Hanlon, J. R. Soc. Interface 6, S149 (2009)

Adaptive Optics



Ah, optics!





- Many stimuli cause deformation.
- Deformation affects optics.

Wilbur, Jackman, Whitesides, Cheung, Lee, Prentiss
Elastomeric optics
Chem. Mater. 1996, 8, 1380-1385

Aschwanden, Stemmer
Optics letters 31, 2610 (2006)

Soft Active Materials (SAM)

Soft: large deformation in response to small forces
(rubbers, gels,…)

Active: large deformation in response to diverse stimuli
(electric field, temperature, pH, salt,…)

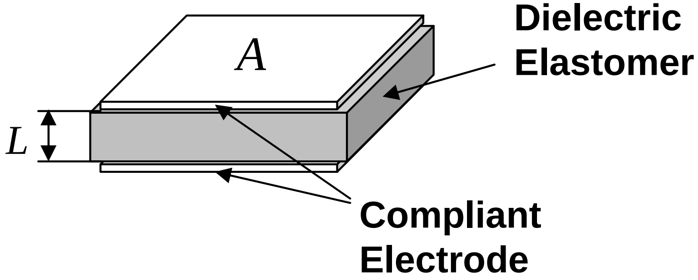
A stimulus causes deformation.

Deformation provides a function.

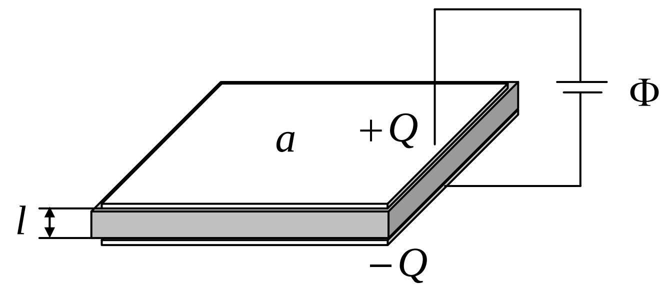
Very well, but how does stimulus X cause deformation?

Dielectric elastomer

Reference State

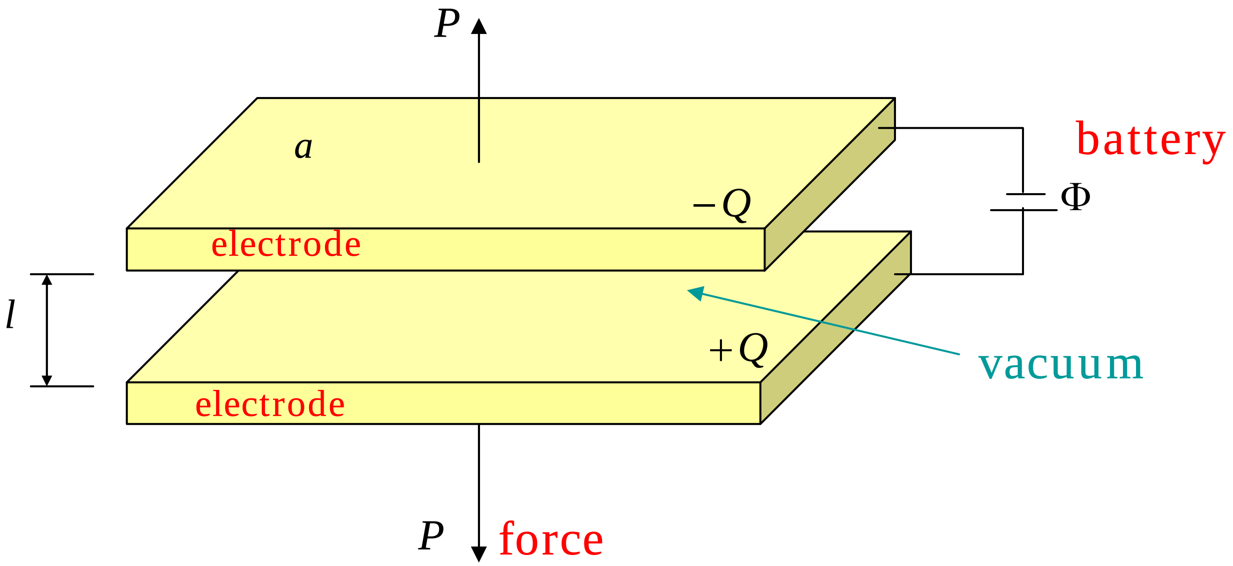


Current State



Pelrine, Kornbluh, Pei, Joseph
High-speed electrically actuated elastomers with strain greater than 100%.
Science 287, 836 (2000).

Parallel-plate capacitor



Electric field $E = \frac{\Phi}{l}$

Electric displacement field $D = \frac{Q}{a}$

stress field $\sigma = \frac{P}{a}$

$D = \varepsilon_0 E$

ε_0, permittivity of vacuum

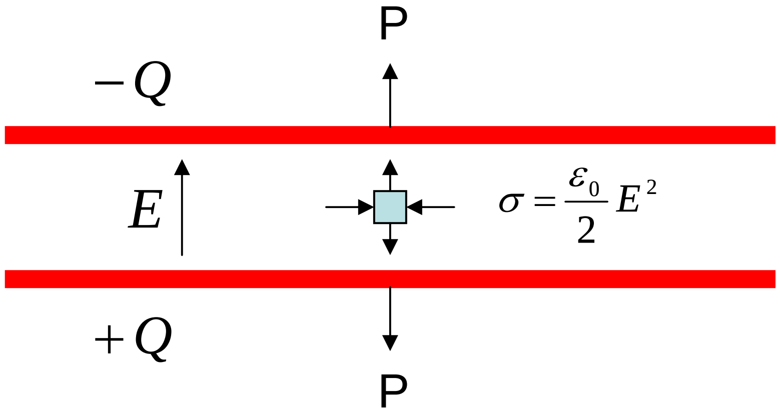
$\sigma = \frac{1}{2}\varepsilon_0 E^2$ Maxwell stress

Field equations in vacuum, Maxwell (1873)

Electrostatic field $$E_i = -\frac{\partial \Phi}{\partial x_i} \qquad \frac{\partial E_i}{\partial x_i} = \frac{q}{\varepsilon_0}$$

A field of forces maintain equilibrium of a field of charges $$F_i = qE_i$$

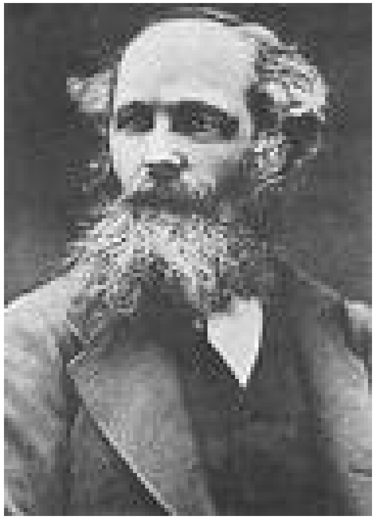
$$F_i = \frac{\partial}{\partial x_j}\left(\varepsilon_0 E_j E_i - \frac{\varepsilon_0}{2} E_k E_k \delta_{ij}\right)$$



$$\sigma_{ij} = \varepsilon_0 E_j E_i - \frac{\varepsilon_0}{2} E_k E_k \delta_{ij}$$

Maxwell stress

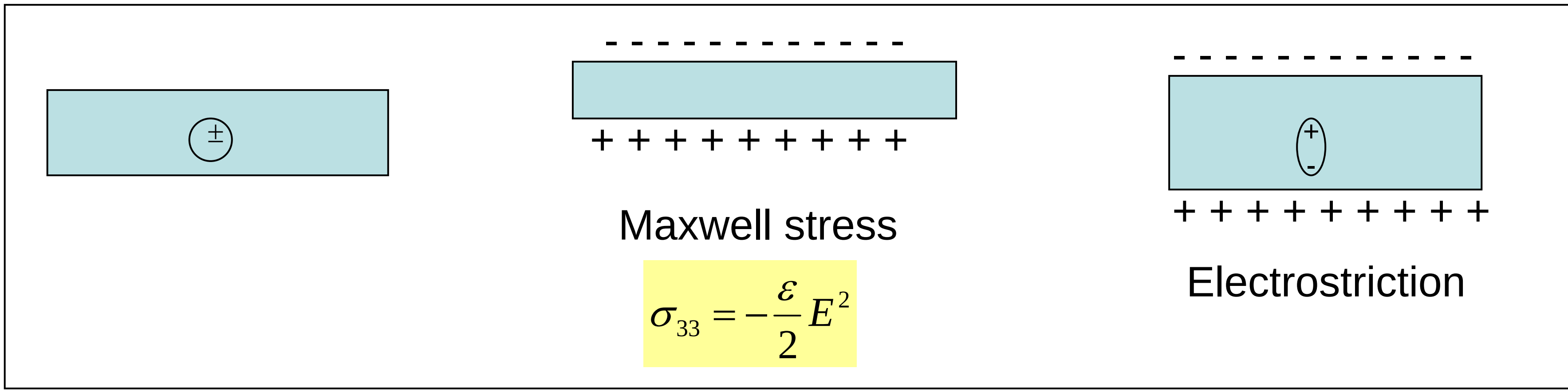
James Clerk Maxwell (1831-1879)



"I have not been able to make the next step, namely, to account by mechanical considerations for these stresses in the dielectric. I therefore leave the theory at this point…"

A Treatise on Electricity & Magnetism (1873), Article 111

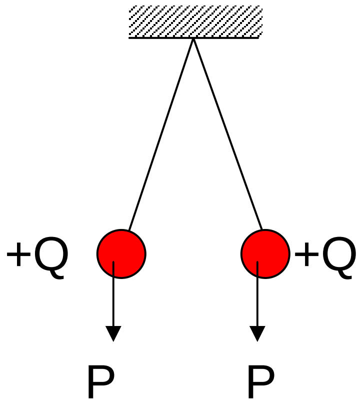
Trouble with Maxwell stress in dielectrics



Our complaints:

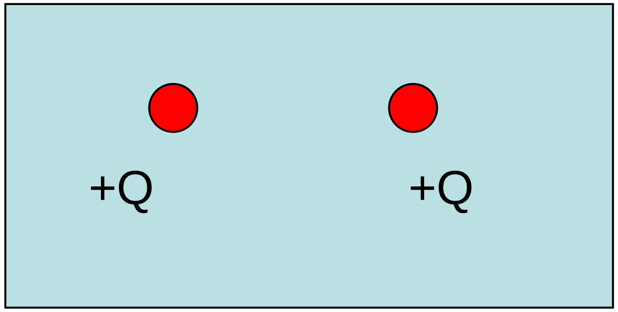
- In general, ε varies with deformation.
- In general, E^2 dependence has no special significance.
- Wrong sign?

Trouble with electric force in dielectrics



In a vacuum,
external force is needed to maintain equilibrium of charges

$$F_i = qE_i$$



In a solid dielectric,
force between charges is NOT an operational concept

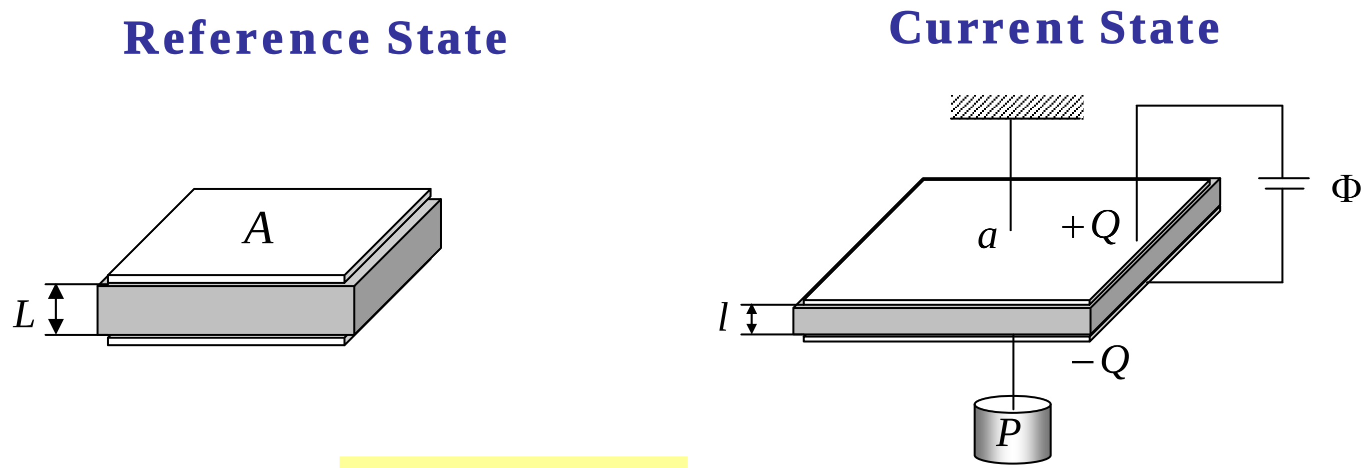
$$F_i = qE_i$$

The Feynman Lectures on Physics

Volume II, p.10-8 (1964)

"It is a difficult matter, generally speaking, to make a unique distinction between the electrical forces and mechanical forces due to solid material itself. Fortunately, no one ever really needs to know the answer to the question proposed. He may sometimes want to know how much strain there is going to be in a solid, and that can be worked out. But it is much more complicated than the simple result we got for liquids."

All troubles are gone if we use measurable quantities



equilibrate elastomer and loads $\delta F = P\delta l + \Phi \delta Q$

divide by volume $$\frac{\delta F}{AL} = \frac{P\delta l}{AL} + \frac{\Phi \delta Q}{LA}$$

name quantities $$\delta W = s\delta\lambda + \tilde{E}\delta\tilde{D}$$

equations of state $$s = \frac{\partial W(\lambda, \tilde{D})}{\partial \lambda} \qquad \tilde{E} = \frac{\partial W(\lambda, \tilde{D})}{\partial \tilde{D}}$$

Nominal

$W = F/(AL)$

$\lambda = l/L$

$s = P/A$

$\tilde{E} = \Phi/L$

$\tilde{D} = Q/A$

True

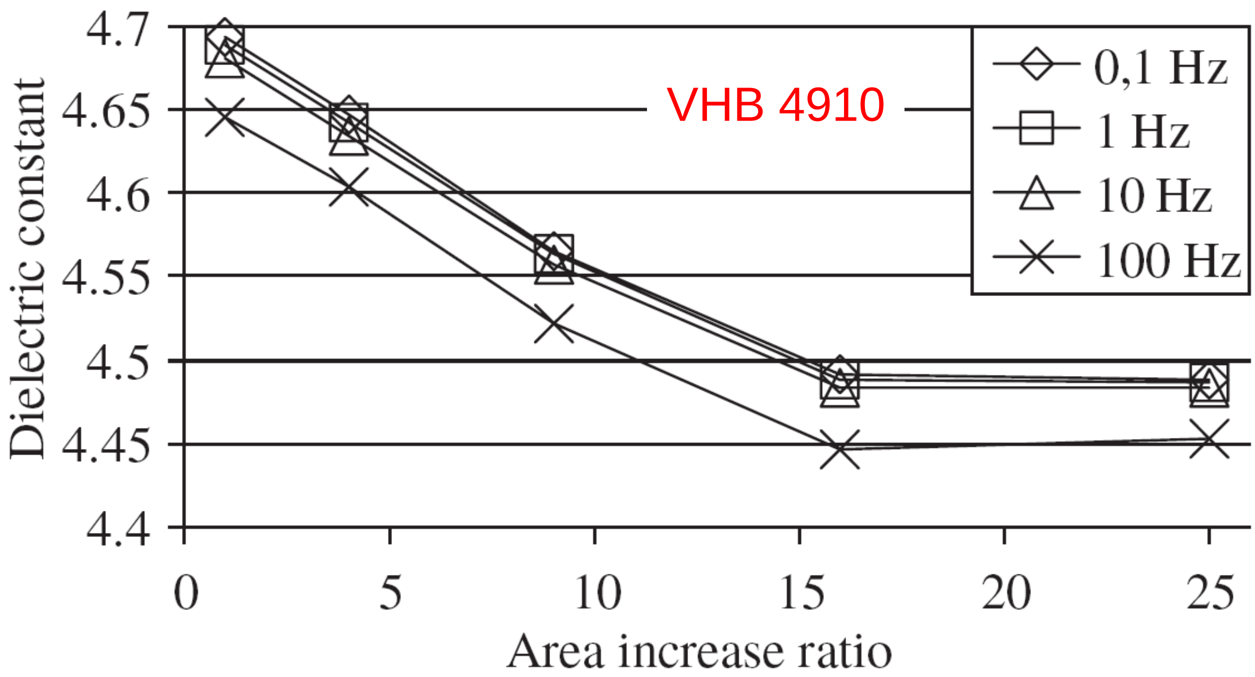
$\sigma = P/a$

$E = \Phi/l$

$D = Q/a$

Fine, Zhigang, but what is $W\left(\lambda, \tilde{D}\right)$ *?*

Dielectric constant is insensitive to stretch

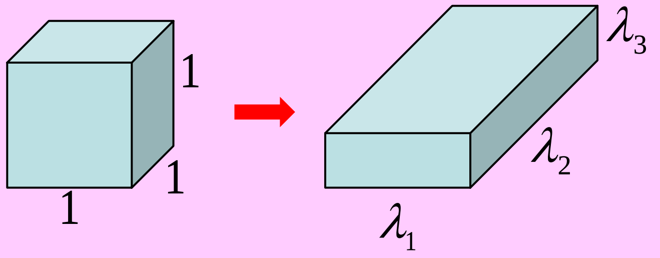


Kofod, Sommer-Larsen, Kornbluh, Pelrine
Journal of Intelligent Material Systems and Structures **14**, 787 (2003).

Ideal dielectric elastomer

incompressibility

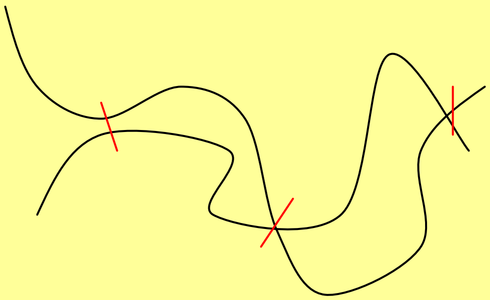
$$\lambda_1 \lambda_2 \lambda_3 = 1$$



Dielectric behavior is liquid-like, unaffected by deformation.

$$W\left(\lambda_1, \lambda_2, \tilde{D}\right) = W_{stretch}\left(\lambda_1, \lambda_2\right) + \frac{D^2}{2\varepsilon}$$

Elasticity Polarization



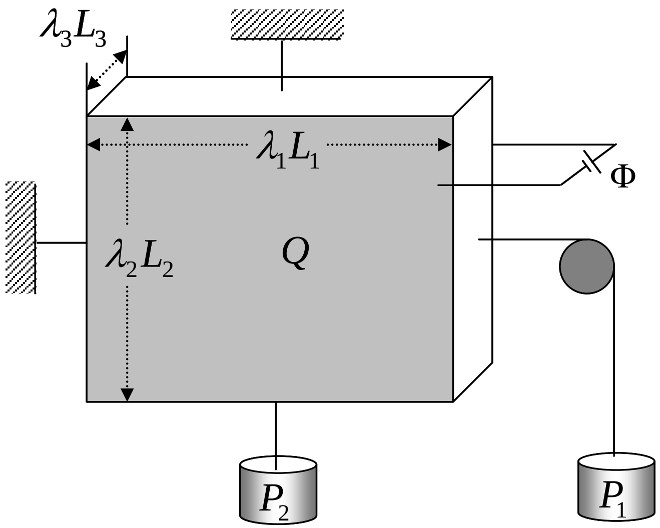
For an ideal dielectric elastomer, electromechanical coupling is purely a geometric effect:

$$D = \frac{Q}{a} \qquad \tilde{D} = \frac{Q}{A} \qquad D = \frac{\tilde{D}}{\lambda_1 \lambda_2}$$

Zhao, Hong, Suo, Physical Review B 76, 134113 (2007)

Ideal dielectric elastomer

$$W\left(\lambda_1,\lambda_2,\tilde{D}\right)=\frac{\mu}{2}\left(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3\right)+\frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}$$

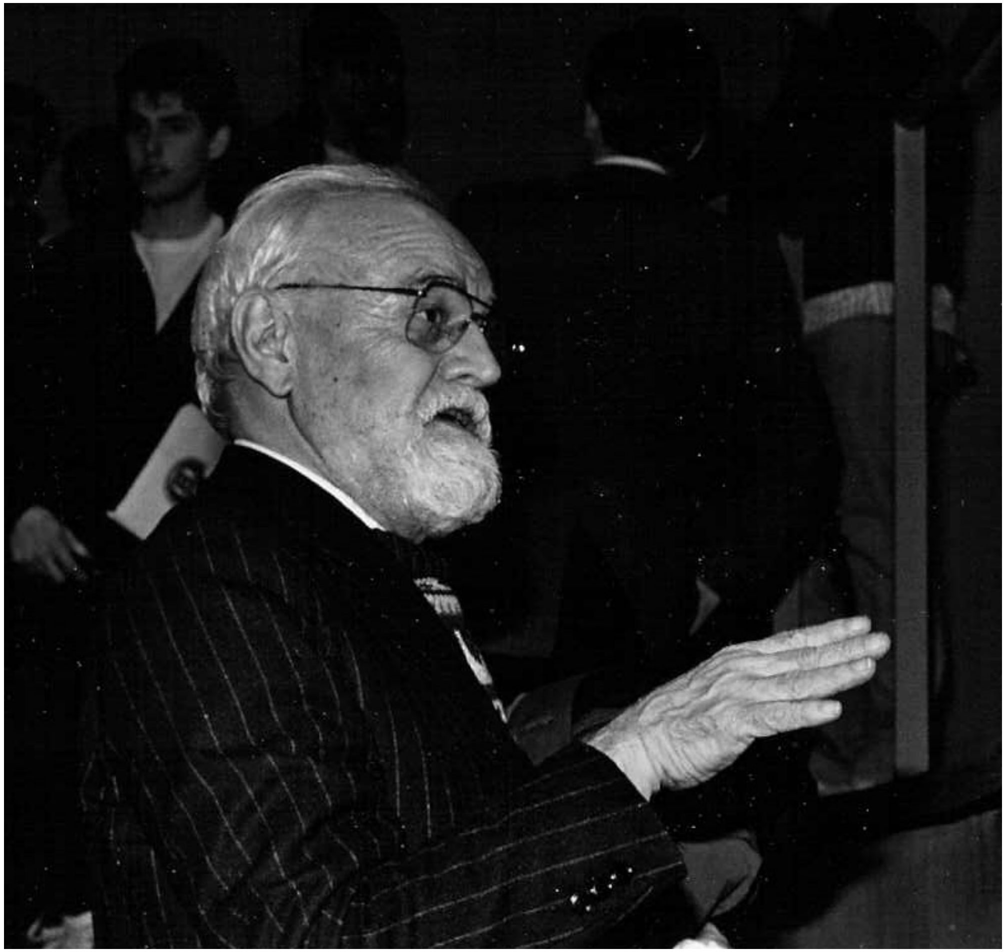


In terms of nominal quantities

$$s_1=\frac{\partial W\left(\lambda_1,\lambda_2,\tilde{D}\right)}{\partial \lambda_1} \qquad s_2=\frac{\partial W\left(\lambda_1,\lambda_2,\tilde{D}\right)}{\partial \lambda_2}$$

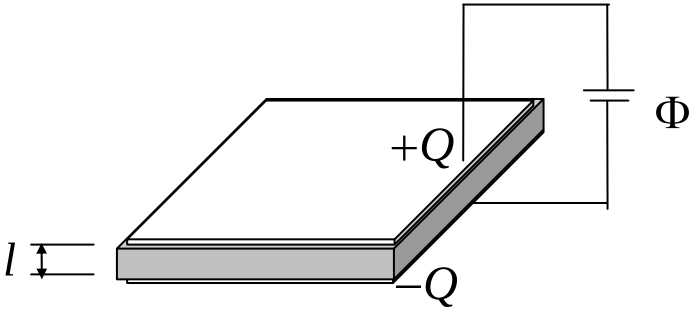
In terms of true quantities

$$\sigma_1=\mu\left(\lambda_1^2-\lambda_1^{-2}\lambda_2^{-2}\right)-\varepsilon E^2 \qquad \sigma_2=\mu\left(\lambda_2^2-\lambda_1^{-2}\lambda_2^{-2}\right)-\varepsilon E^2$$

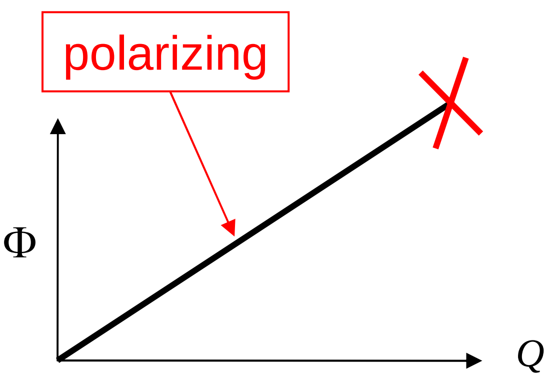


Young man, don't be too fast. The theory sounds fine, but can you relate the theory to experiments?

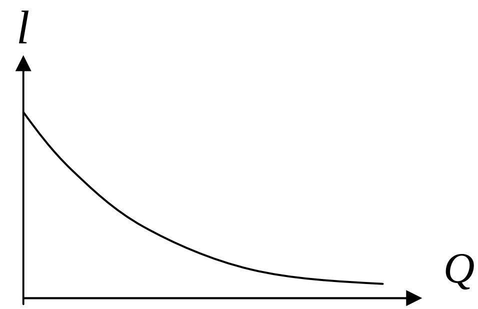
Two modes of failure

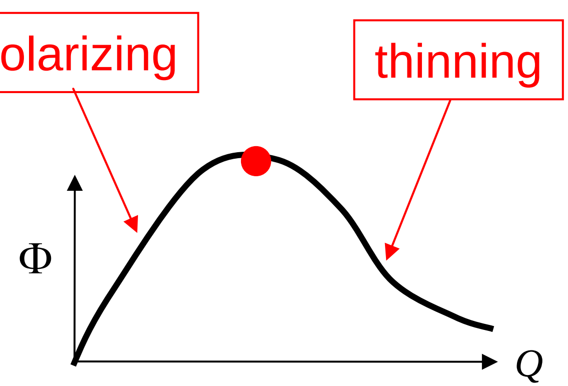


Electrical breakdown
Hard material



Electromechanical instability
Soft material





Stark & Garton, Nature 176, 1225 (1955)

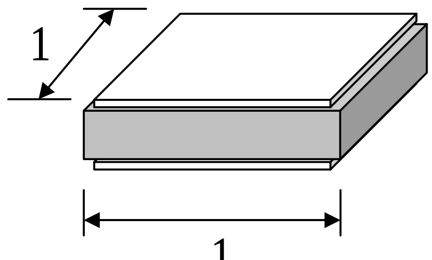
A dilemma

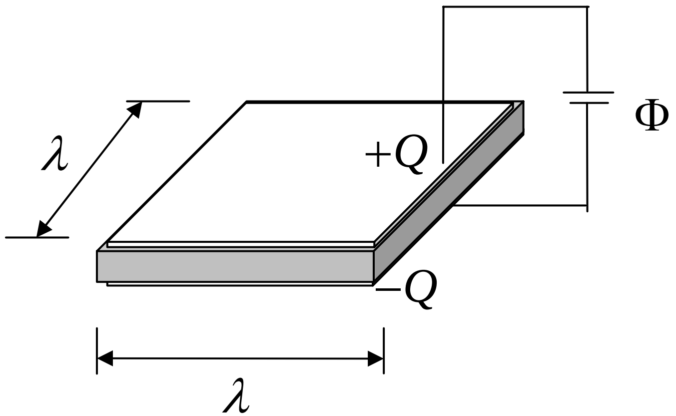
- To deform appreciably without electrical breakdown, the elastomer must be soft.

- But a soft elastomer is susceptible to electromechanical instability.

How large can deformation of actuation be?

Electromechanical instability limits deformation of actuation





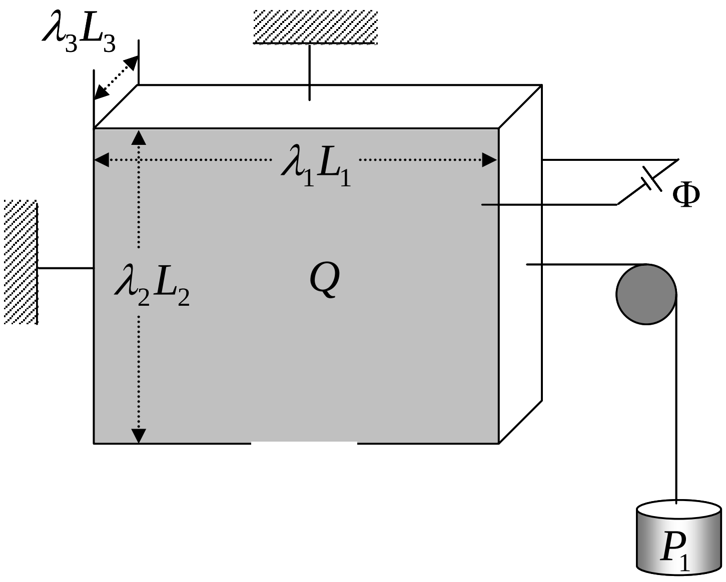
$$\lambda_c = 2^{1/3} \approx 1.26$$

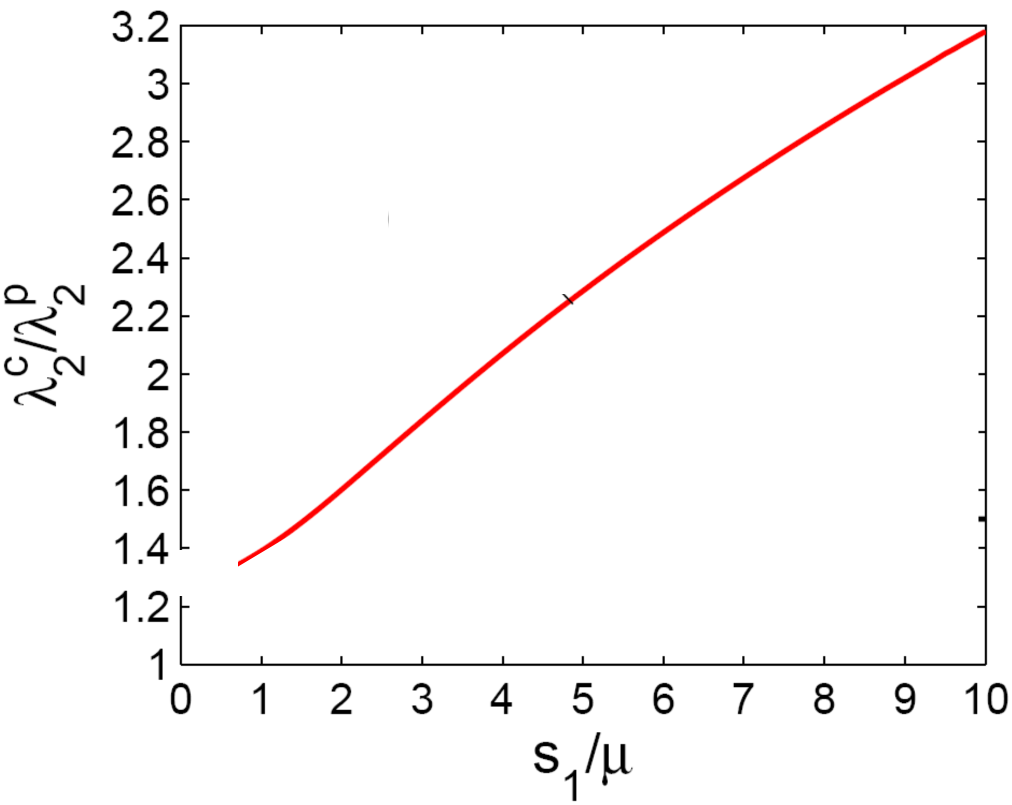
Experiment: Pelrine, Kornbluh, Joseph,
Sensors & Actuators A 64, 77 (1998)

Theory: Zhao & Suo
Appl. Phys Lett. 91, 061921 (2007)

Pre-stretch

increases deformation of actuation

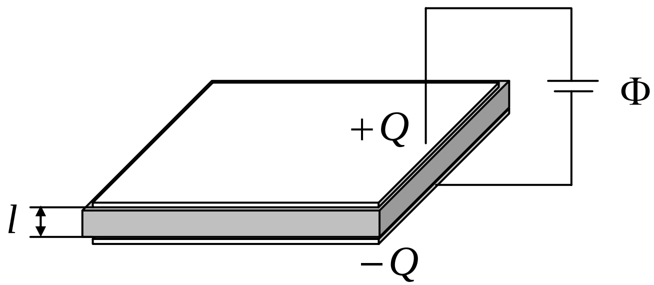


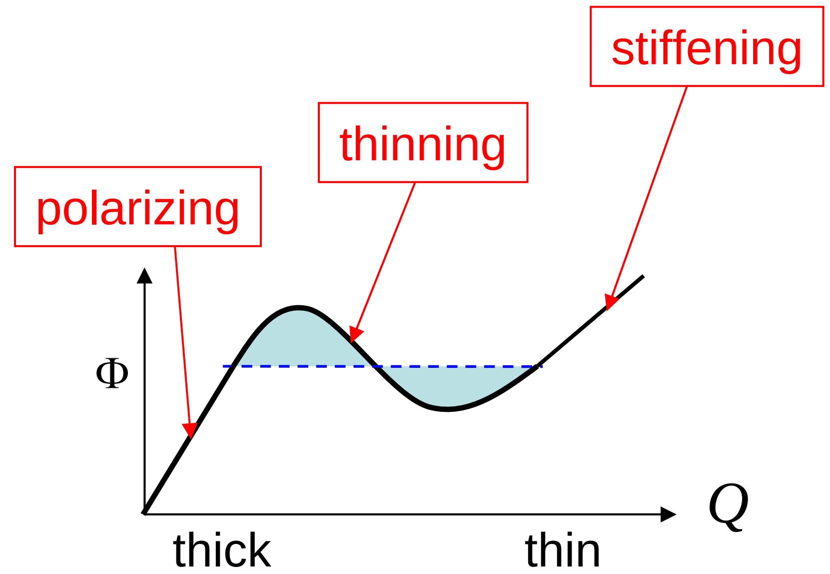


Experiment: Pelrine, Kornbluh, Pei, Joseph
Science 287, 836 (2000).

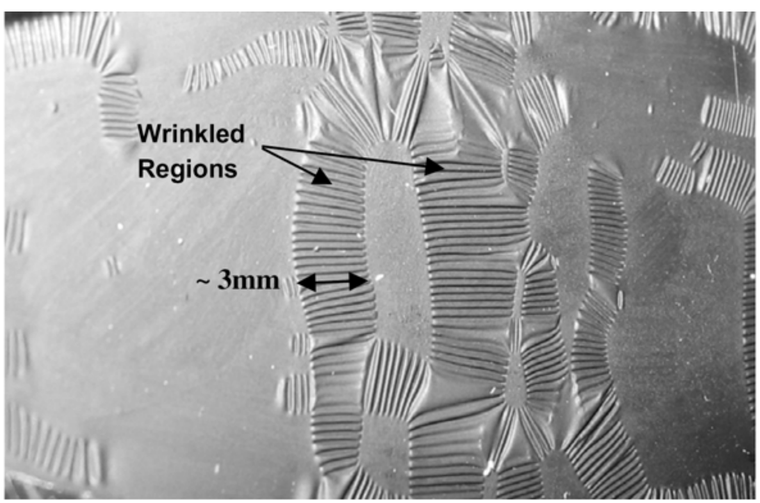
Theory: Zhao, Suo
APL 91, 061921 (2007)

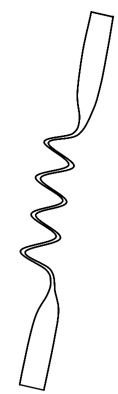
Coexistent states





Top view





Cross section

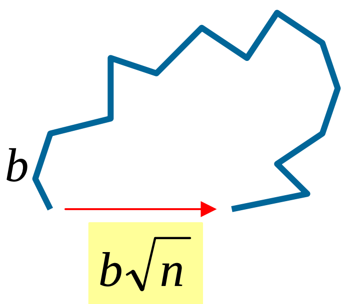
Coexistent states: flat and wrinkled

Observation: Plante, Dubowsky,
Int. J. Solids and Structures **43**, 7727 (2006)

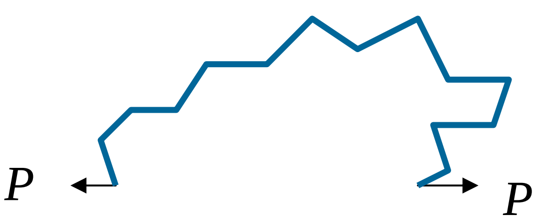
Interpretation: Zhao, Hong, Suo
Physical Review B 76, 134113 (2007)

When stretched, a polymer stiffens

n links

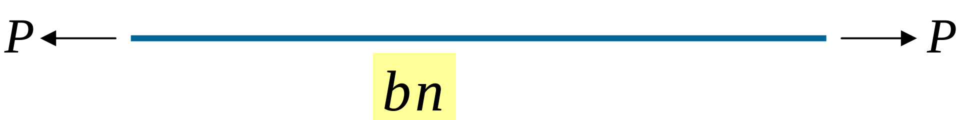


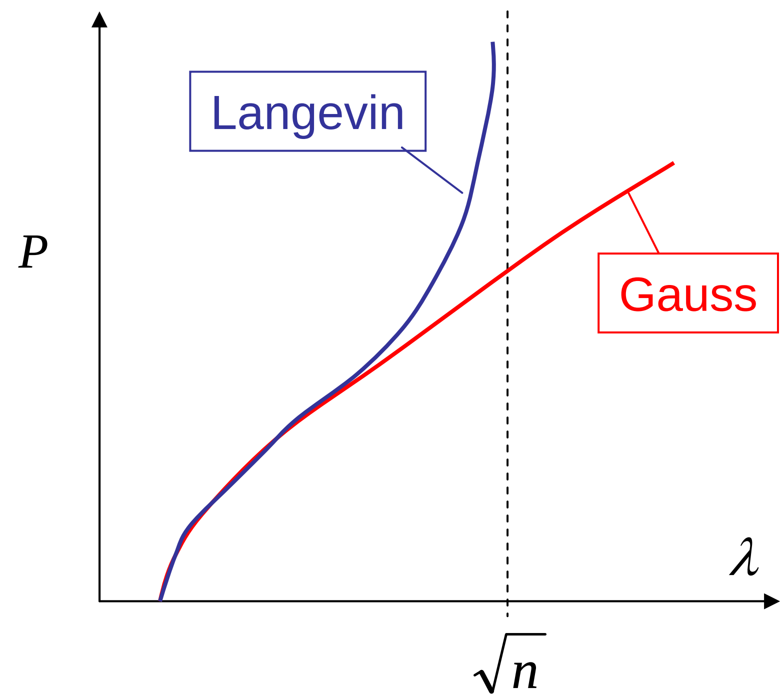
released



stretched

fully stretched



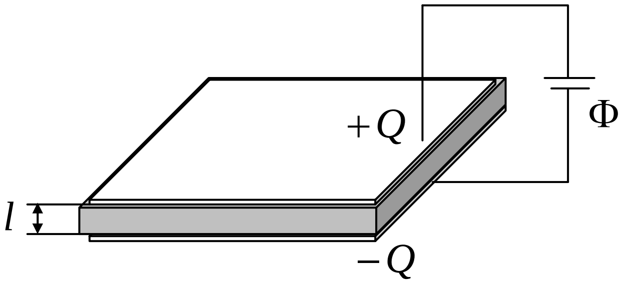


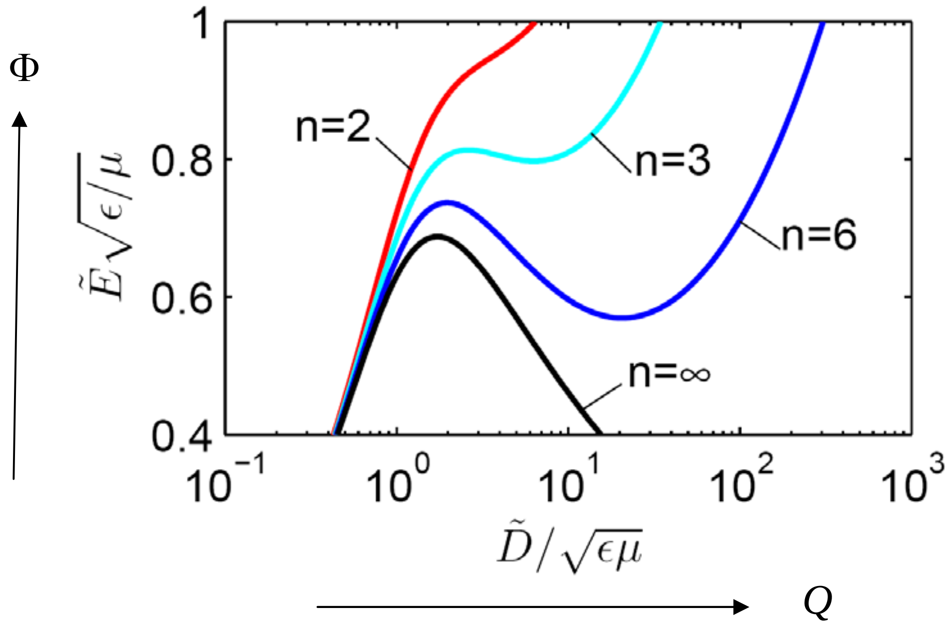
Stiffening stabilizes elastomer

$$\frac{\mu}{2}\left(\lambda_1^2+\lambda_2^2+\lambda_3^2-3\right) \quad \rightarrow \quad \mu\left[\frac{1}{2}\left(\lambda_1^2+\lambda_2^2+\lambda_3^2-3\right)+\frac{1}{20n}\left(\left(\lambda_1^2+\lambda_2^2+\lambda_3^2\right)^2-9\right)+\ldots\right]$$

Gauss Langevin

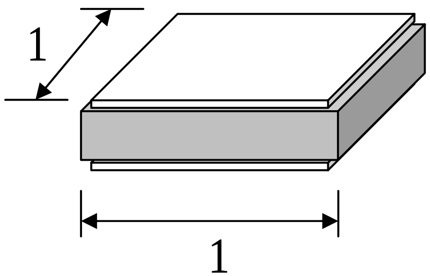
Arruda, Boyce, J. Mech. Phys. Solids 41, 389 (1993)

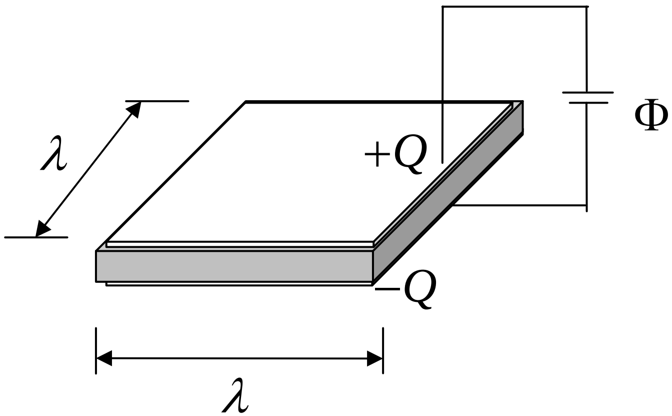


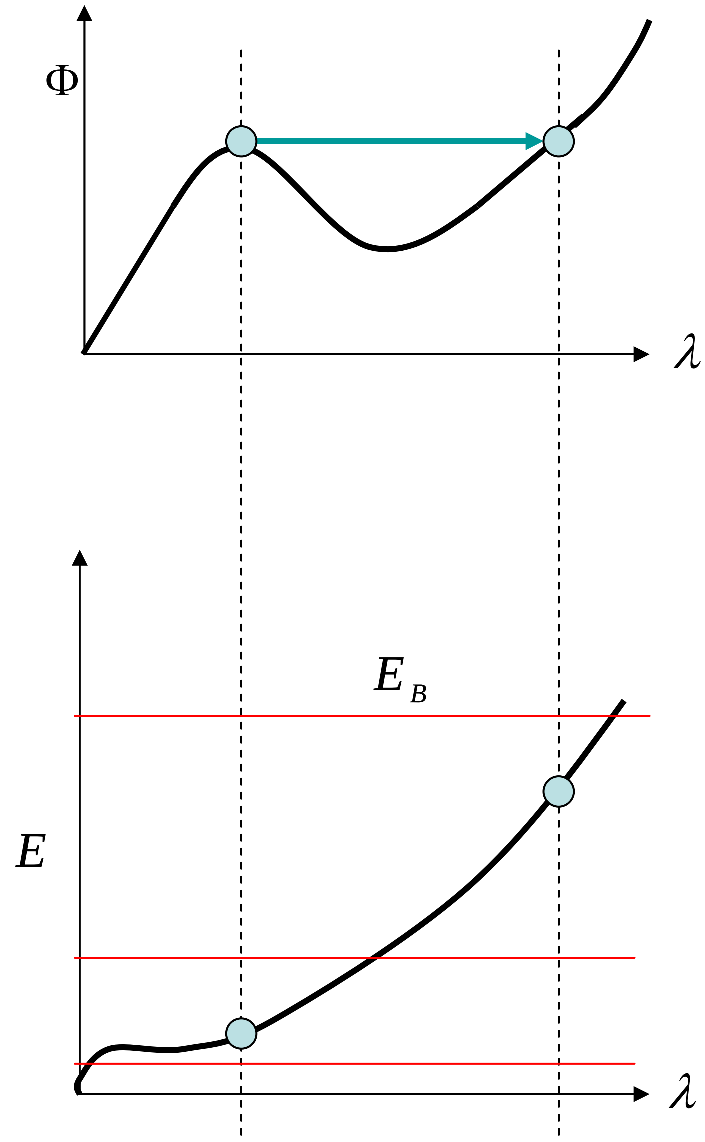


Zhao, Hong, Suo
Physical Review B 76, 134113 (2007)

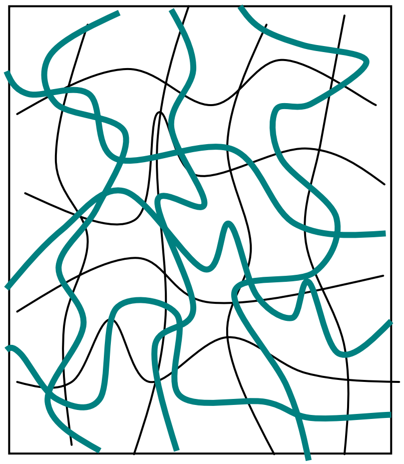
Giant deformation of actuation?

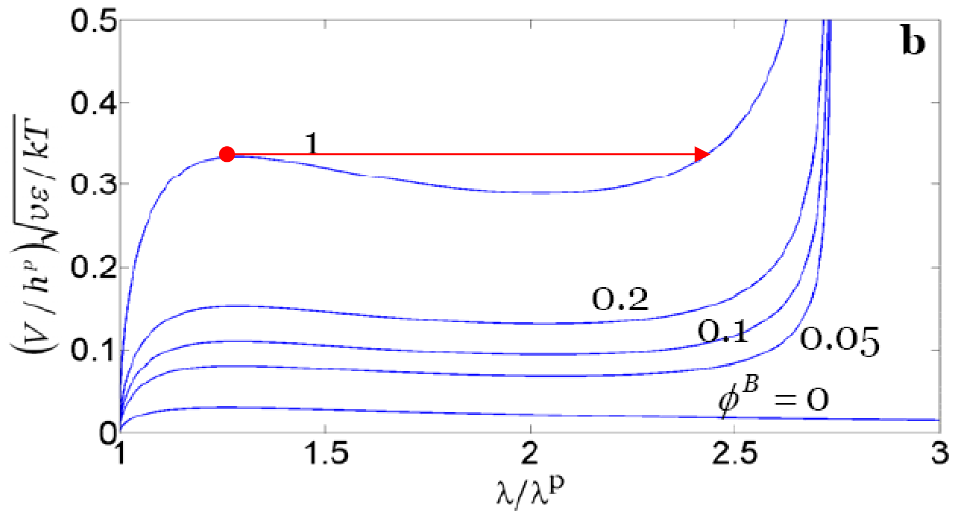


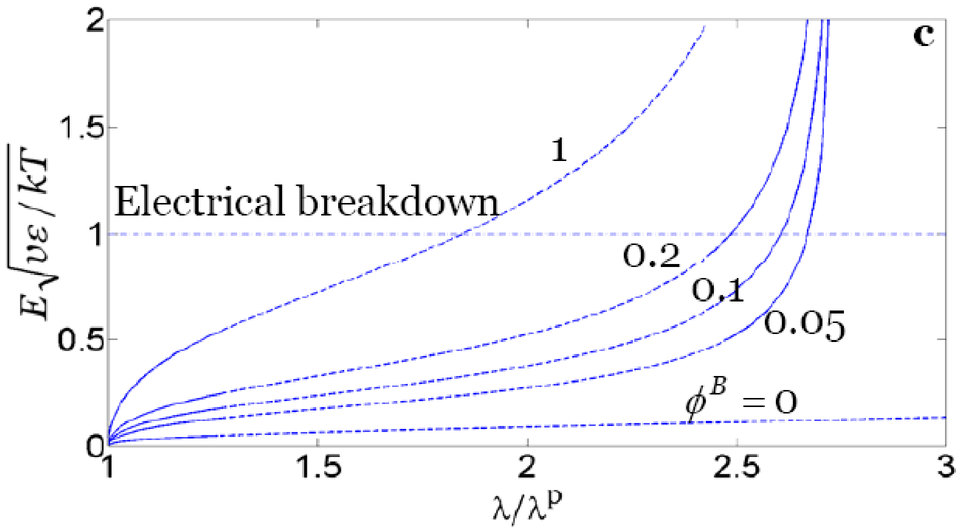




Interpenetrating networks







- Short chains provide a safety net.
- Long chains fill the space.

Experiment: Ha, Yuan, Pei, Pelrine
Adv. Mater. 18, 887 (2006).

Theory: Suo, Zhu. Unpublished

3D inhomogeneous field

Condition of equilibrium

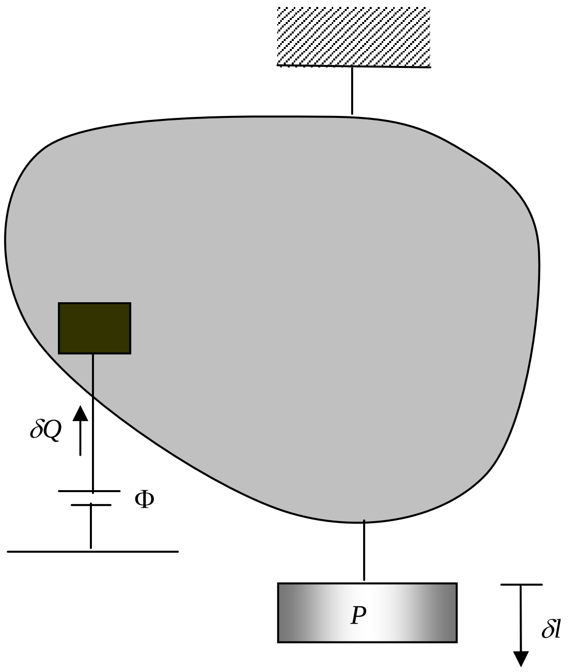
$$\int \delta W dV = \int B_i \delta x_i dV + \int T_i \delta x_i dA + \int \Phi \delta q dV + \int \Phi \delta \omega dA$$

Need to specify a material model

$$W\left(\lambda, \tilde{D}\right) \rightarrow W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)$$

$$\delta W\left(\mathbf{F}, \tilde{\mathbf{D}}\right) = s_{iK} \delta F_{iK} + \tilde{E}_K \delta \tilde{D}_K$$

$$s_{iK} = \frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial F_{iK}} \qquad \tilde{E}_K = \frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial \tilde{D}_K}$$



PDEs

$$F_{iK} = \frac{\partial x_i(\mathbf{X},t)}{\partial X_K}$$

$$\tilde{E}_K = -\frac{\partial \Phi(\mathbf{X},t)}{\partial X_K}$$

$$\frac{\partial s_{iK}(\mathbf{X},t)}{\partial X_K} + B_i(\mathbf{X},t) = 0$$

$$\frac{\partial \tilde{D}_K(\mathbf{X},t)}{\partial X_K} = q(\mathbf{X},t)$$

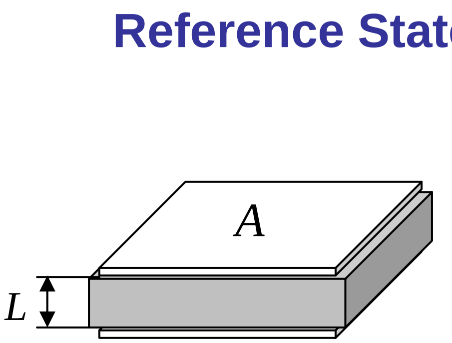
$$s_{iK} = \frac{\partial W(\mathbf{F},\tilde{\mathbf{D}})}{\partial F_{iK}}$$

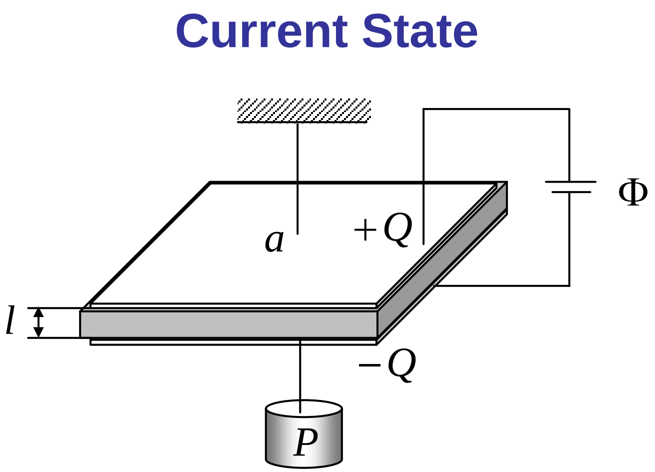
$$\tilde{E}_K = \frac{\partial W(\mathbf{F},\tilde{\mathbf{D}})}{\partial \tilde{D}_K}$$

Toupin (1956), Eringen (1963), Tiersten (1971),
Goulbourne, Mockensturm and Frecker (2005), Dorfmann & Ogden (2005), McMeeking & Landis (2005)…

Suo, Zhao, Greene,J. Mech. Phys. Solids 56, 467 (2008)

The nominal vs the true





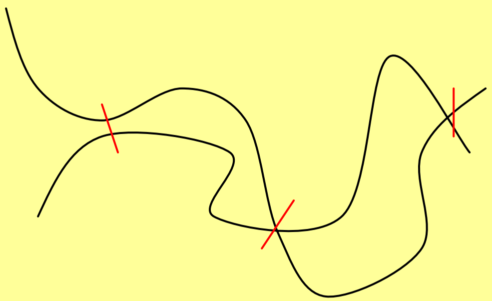
$s = P / A$	$\sigma = P / a$	$\sigma_{ij} = \frac{F_{jK}}{\det(\mathbf{F})} s_{iK}$
$\tilde{E} = \Phi / L$	$E = \Phi / l$	$F_{iK} E_i = \tilde{E}_K$
$\tilde{D} = Q / A$	$D = Q / a$	$D_i = \frac{F_{iK}}{\det(\mathbf{F})} \tilde{D}_K$

Ideal dielectric elastomers

Liquid-like dielectric behavior, unaffected by deformation

$$W\left(\mathbf{F},\tilde{\mathbf{D}}\right) = W_s\left(\mathbf{F}\right) + \frac{D^2}{2\varepsilon}$$

Stretch   Polarization



Ideal electromechanical coupling is purely a geometric effect:

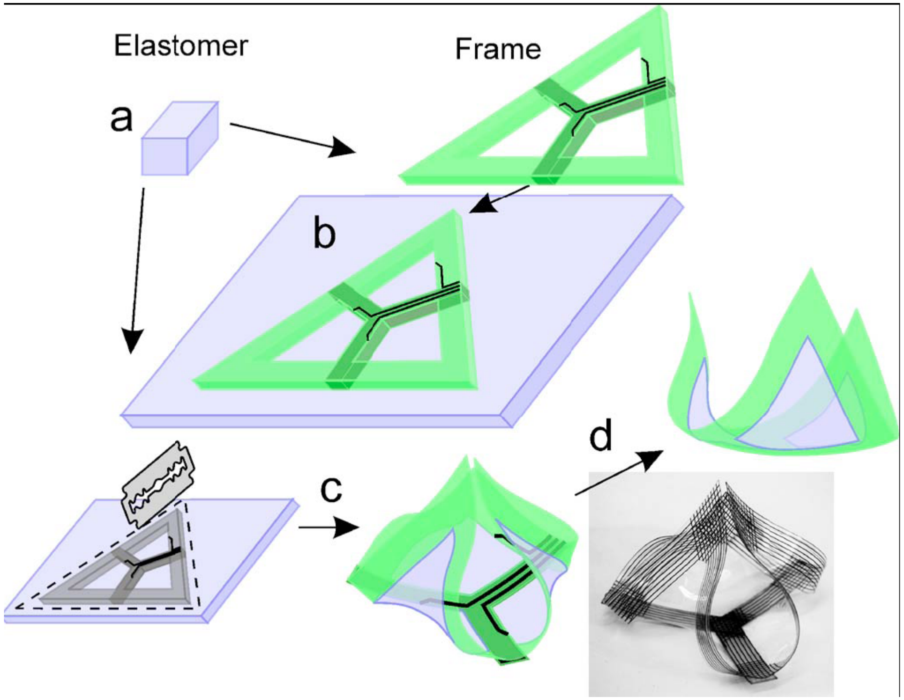
$$D_i = \frac{F_{iK}}{\det\left(\mathbf{F}\right)}\tilde{D}_K$$

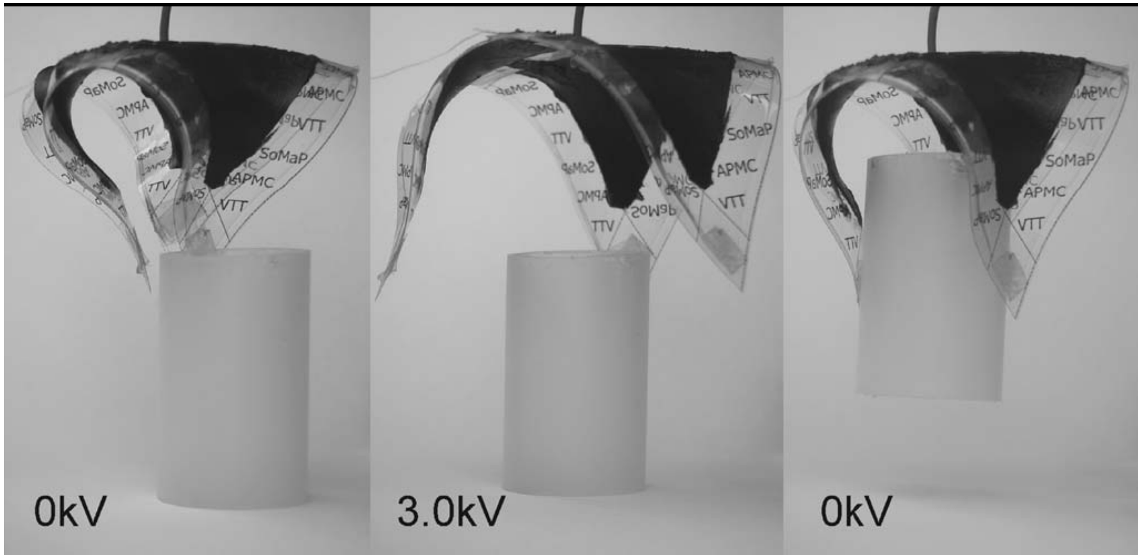
$$\tilde{E}_K\left(\mathbf{F},\tilde{\mathbf{D}}\right) = \frac{\partial W\left(\mathbf{F},\tilde{\mathbf{D}}\right)}{\partial \tilde{D}_K} \qquad s_{iK}\left(\mathbf{F},\tilde{\mathbf{D}}\right) = \frac{\partial W\left(\mathbf{F},\tilde{\mathbf{D}}\right)}{\partial F_{iK}}$$

$$D_i = \varepsilon E_i \qquad \sigma_{ij} = \frac{F_{iK}}{\det\left(\mathbf{F}\right)}\frac{\partial W_s\left(\mathbf{F}\right)}{\partial F_{jK}} + \varepsilon\left(E_i E_j - \frac{1}{2}E_k E_k \delta_{ij}\right)$$

Zhao, Hong, Suo, Physical Review B 76, 134113 (2007)

Programmable deformation





Design:
Kofod, Wirges, Paajanen, Bauer
APL **90**, 081916 (2007)

Simulation:
Zhao, Suo
APL **93**, 251902 (2008)

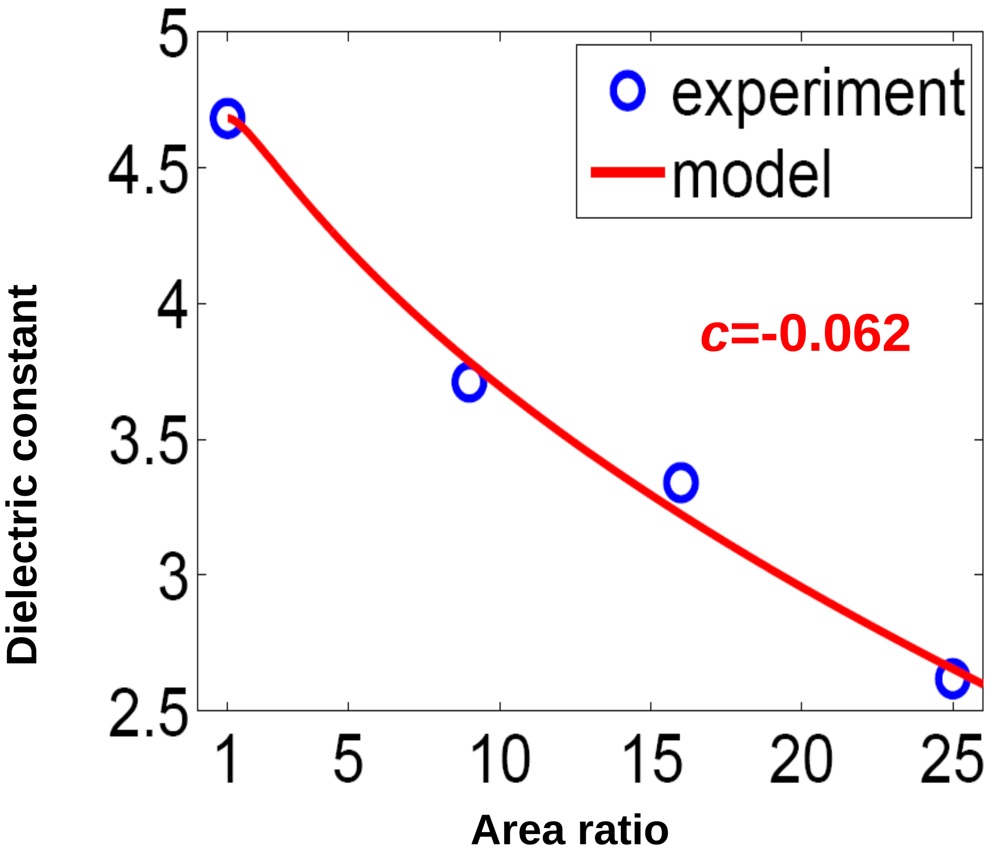
Electrostriction

Maxwell stress

$$\sigma_{33} = -\frac{\varepsilon}{2}E^2$$

Electrostriction

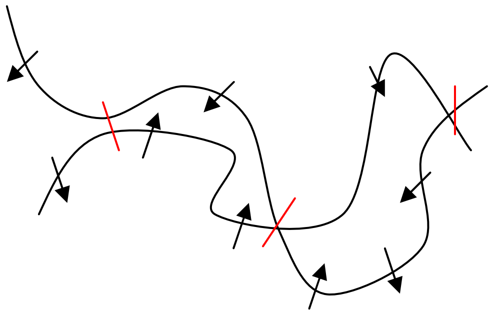
Non-ideal dielectric elastomer



deformation **affects** dielectric constant

Wissler, Mazza, Sens. Actuators, A **138**, 384 (2007).

Quasi-linear dielectric elastomer



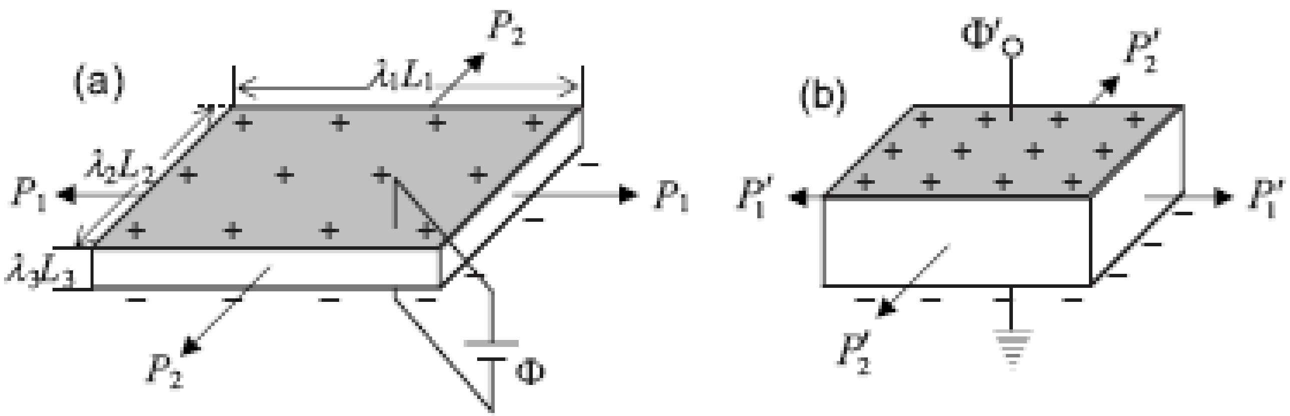
$$\varepsilon = \bar{\varepsilon}\left(1 + c\left(\lambda_1 + \lambda_2 + \lambda_3 - 3\right)\right)$$

$$W\left(\lambda_1, \lambda_2, \tilde{D}\right) = \frac{\mu}{2}\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right) + \frac{D^2}{2\varepsilon}$$

$$s_1 = \frac{\partial W\left(\lambda_1, \lambda_2, \tilde{D}\right)}{\partial \lambda_1}$$

Zhao, Suo, JAP **104**, 123530 (2008).

Energy harvesting

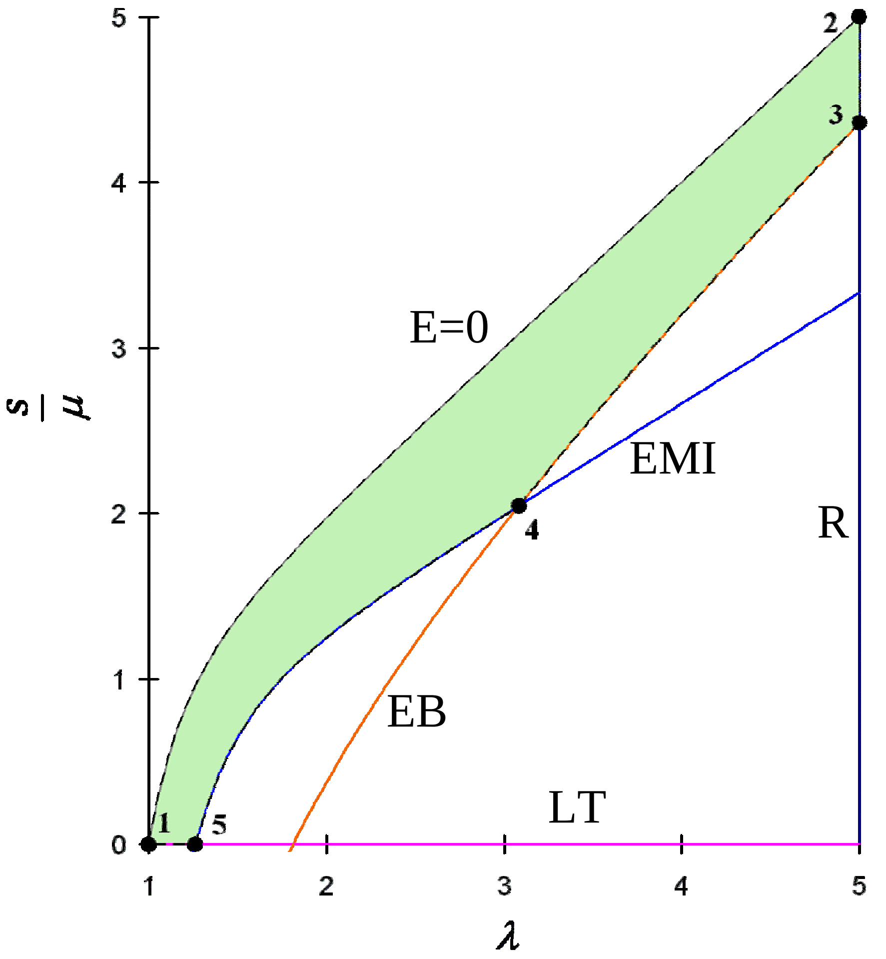


Generate electricity from walking

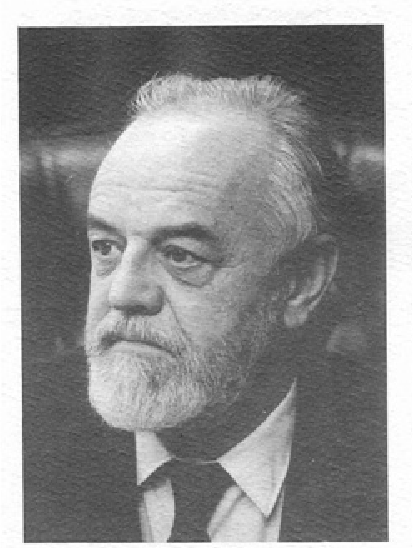
Generate electricity from waves

Pelrine, Kornbluh…

Maximal energy that can be converted by a dielectric elastomer



6 J/g, Elastomer (theory)
0.4 J/g, Elastomer (experiment)
0.01 J/g ceramics



Really that large? Someone should do more experiments.

Koh, Zhao, Suo, Appl. Phys. Lett. 94, 262902 (2009)

Summary

- Convergence of the two worlds (soft biology, hard engineering).
- Soft active materials (SAMs) have many functions (soft robots, adaptive optics, self-regulating fluidics, programmable haptics, oil recovery, energy harvesting, drug delivery, tissue regeneration, low-cost diagnosis...).
- Mechanics of SAMs is interesting and challenging (mechanics, electricity, chemistry; large deformation, mass transport, instability).
- The field is wide open (fabrication, computation, application).

3 lectures on SAMs: http://imechanica.org/node/3215
•Dielectric elastomers
•Gels
•Polyelectrolytes