Electromechanical instability of large deformation in dielectric elastomers

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Work with
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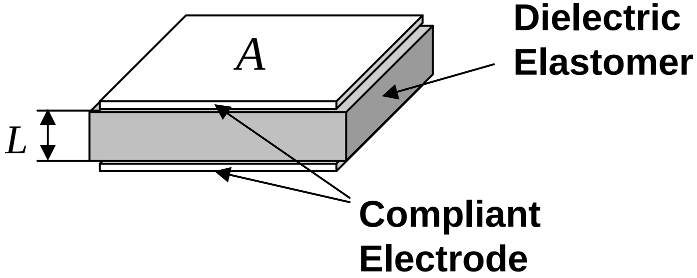
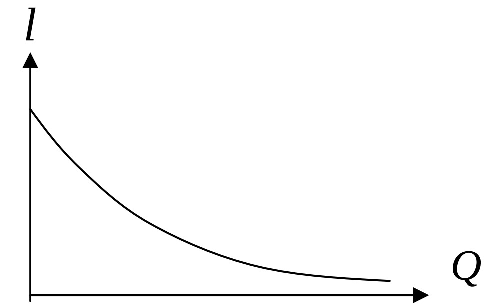
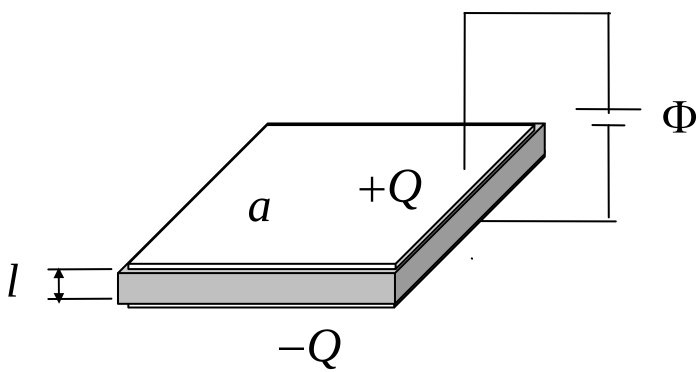
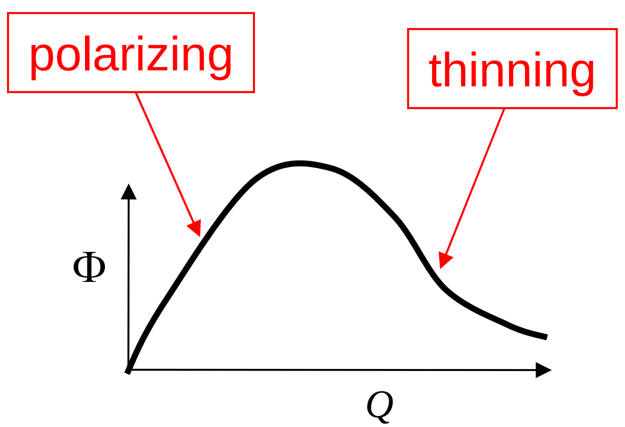
Pull-in instability









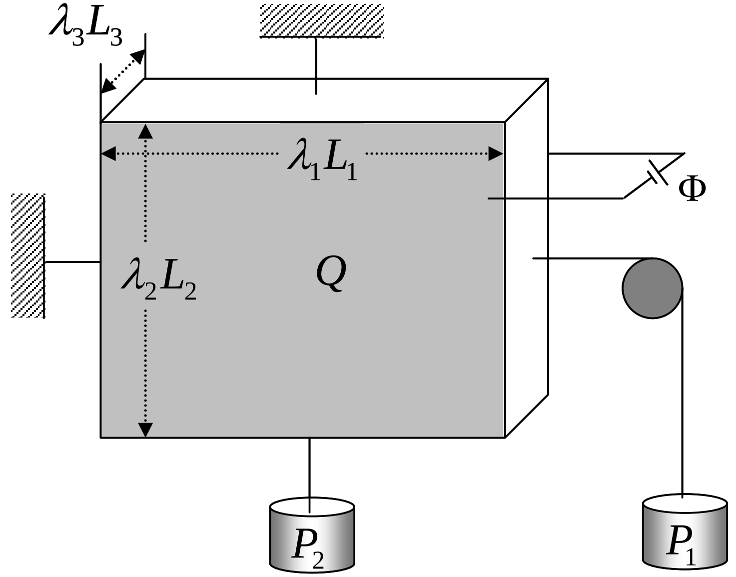
Stark & Garton
Nature 176, 1225 (1955)

Pelrine, Kornbluh, Pei, Joseph
Science 287, 836 (2000)

Plante, Dubowsky
Int. J. Solids and Structures 43, 7727 (2006)

Theory

Zhao, Suo, APL 91, 061921 (2007)



$$s = P / A$$

$$\tilde{E} = \Phi / L$$

$$\tilde{D} = Q / A$$

Free energy of the system

$$G\left(\lambda_1, \lambda_2, \tilde{D}\right) = L_1 L_2 L_3 W\left(\lambda_1, \lambda_2, \tilde{D}\right) - P_1 \lambda_1 L_1 - P_2 \lambda_2 L_2 - \Phi Q$$

System Dielectric Weights Battery

Equilibrium condition

$$0 = \frac{\delta G}{L_1 L_2 L_3} = \left(\frac{\partial W}{\partial \lambda_1} - s_1\right)\delta\lambda_1 + \left(\frac{\partial W}{\partial \lambda_2} - s_2\right)\delta\lambda_2 + \left(\frac{\partial W}{\partial \tilde{D}} - \tilde{E}\right)\delta\tilde{D}$$

Need free-energy function $W\left(\lambda_1, \lambda_2, \tilde{D}\right)$

Dielectric constant is insensitive to deformation

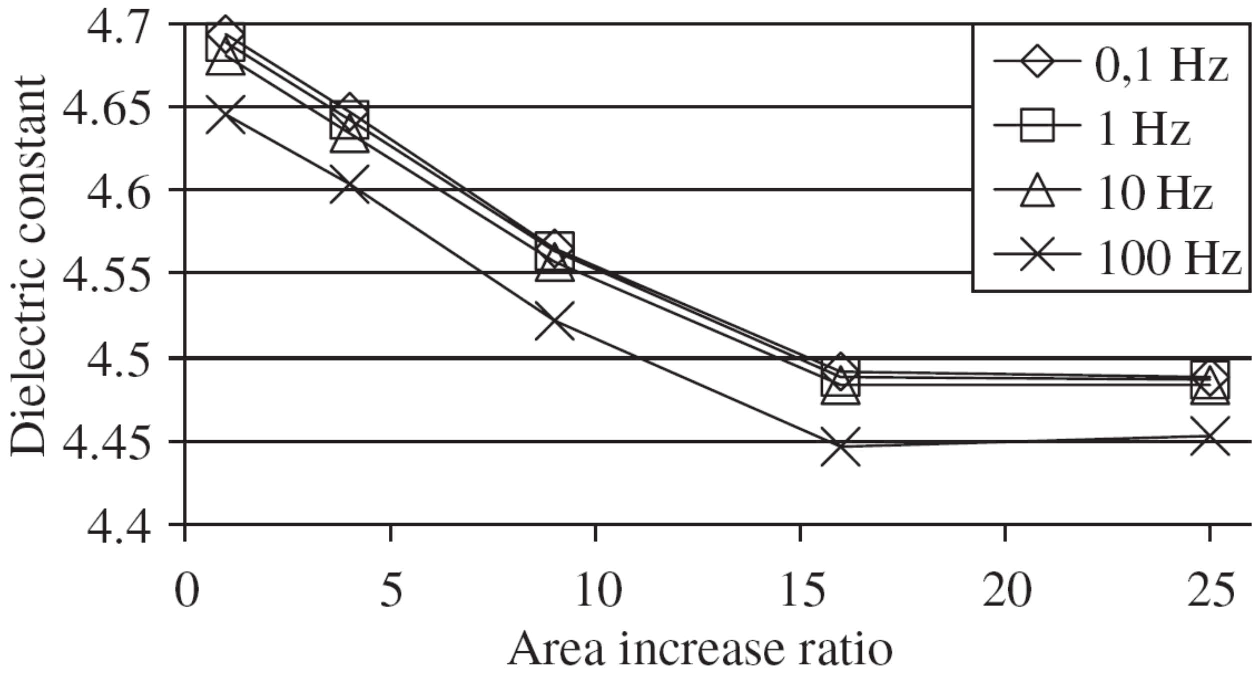


Figure 5. *The relative dielectric constant of VHB^{TM} 4910 drops, when it is stretched.*

Kofod, Sommer-Larsen, Kornbluh, Pelrine
Journal of Intelligent Material Systems and Structures **14**, 787-793 (2003).

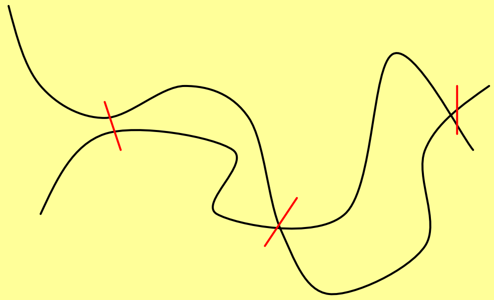
Material law: Ideal dielectric elastomers

Zhao, Hong, Suo, Physical Review B 76, 134113 (2007)

Dielectric behavior is liquid-like, unaffected by deformation.

$$W\left(\lambda_1, \lambda_2, \tilde{D}\right) = W_s\left(\lambda_1, \lambda_2\right) + \frac{D^2}{2\varepsilon}$$

Stretch Polarization



Ideal electromechanical coupling is purely a geometric effect:

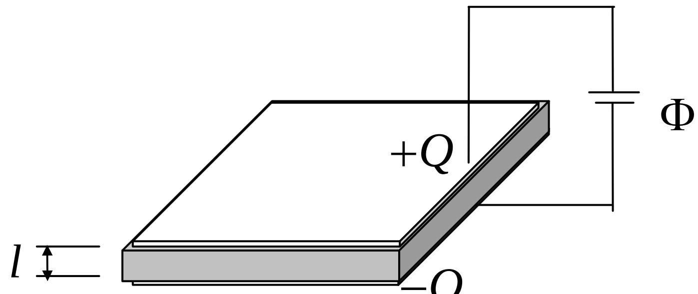
$$D = \frac{Q}{a} \qquad \tilde{D} = \frac{Q}{A} \qquad D = \frac{\tilde{D}}{\lambda_1 \lambda_2}$$

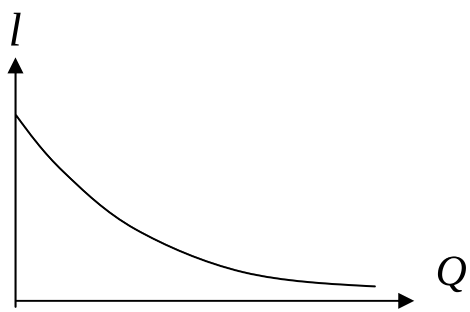
Choose a free energy of stretching. For example, neo-Hookean law:

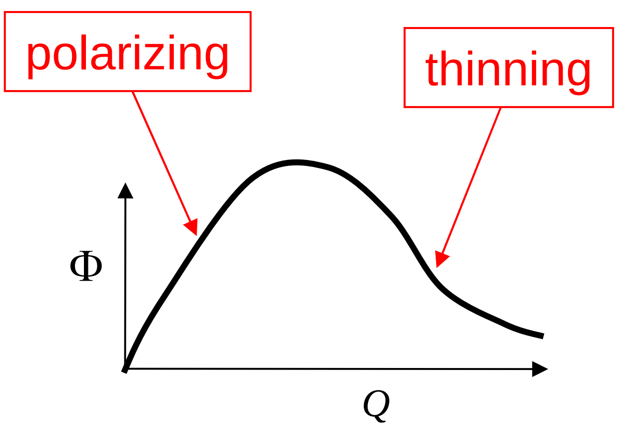
$$W_s\left(\lambda_1, \lambda_2\right) = \frac{\mu}{2}\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right)$$

incompressibility $\lambda_3 = 1/\left(\lambda_1 \lambda_2\right)$

Theory of pull-in instability







$$W\left(\lambda,\tilde{D}\right)=\frac{\mu}{2}\left(\lambda^2+2\lambda^{-1}-3\right)+\frac{\lambda^2\tilde{D}^2}{\varepsilon}$$

$$s=\frac{\partial W\left(\lambda,\tilde{D}\right)}{\partial\lambda}=0 \qquad \tilde{E}=\frac{\partial W\left(\lambda,\tilde{D}\right)}{\partial\tilde{D}}$$

$$\lambda=\left(1+\frac{\tilde{D}^2}{\varepsilon\mu}\right)^{-1/3}$$

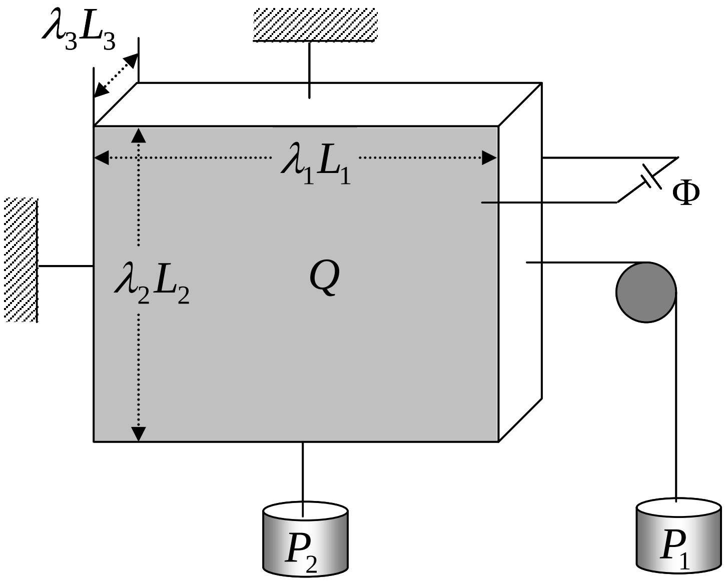
$$\frac{\tilde{E}}{\sqrt{\mu/\varepsilon}}=\frac{\tilde{D}}{\sqrt{\varepsilon\mu}}\left(1+\frac{\tilde{D}^2}{\varepsilon\mu}\right)^{-2/3}$$

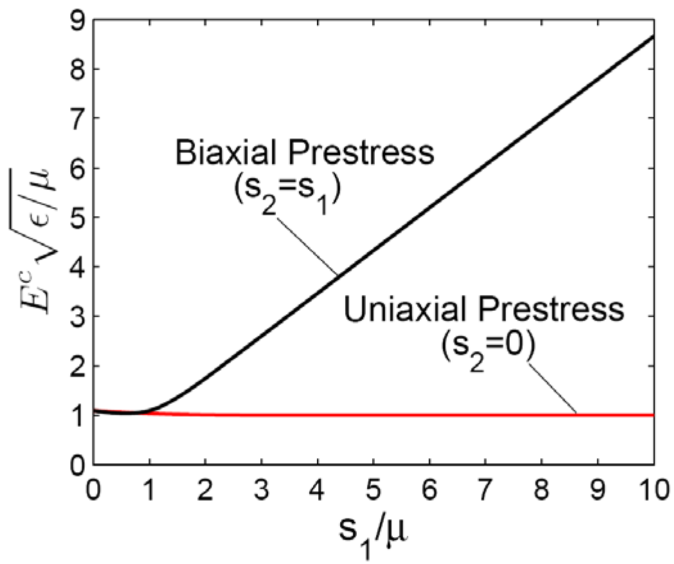
$$\lambda_c\approx 0.63 \qquad \tilde{E}_c\sim\sqrt{\frac{\mu}{\varepsilon}}\sim\sqrt{\frac{10^6\,N/m}{10^{-10}\,F/m}}=10^8\,V/m$$

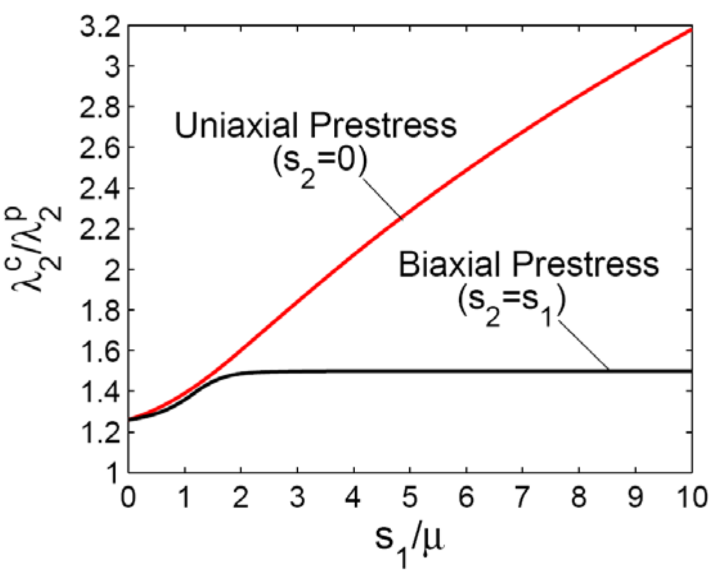
Stark & Garton, Nature 176, 1225 (1955)

Zhao, Suo, APL 91, 061921 (2007)

Pre-stretch increases actuation strain



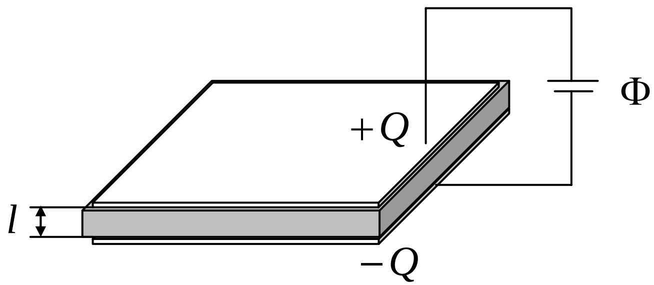


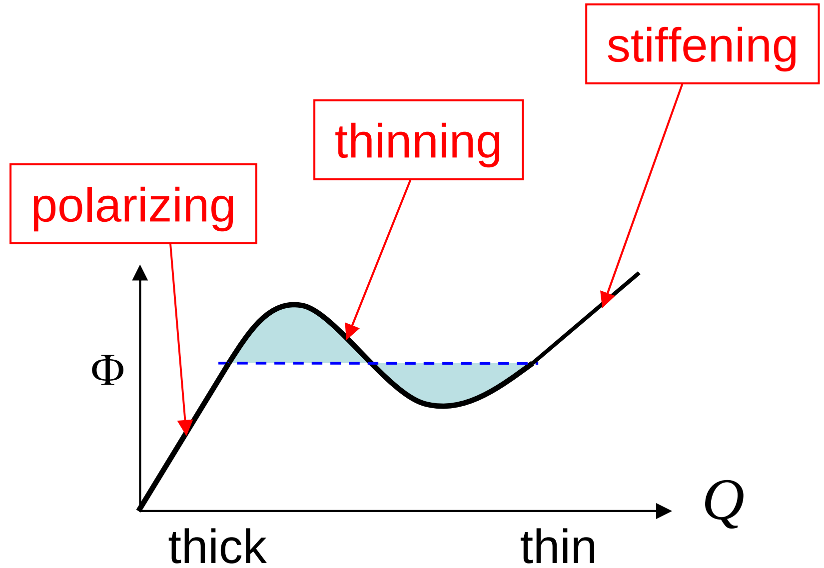


Experiment: Pelrine, Kornbluh, Pei, Joseph
Science 287, 836 (2000).

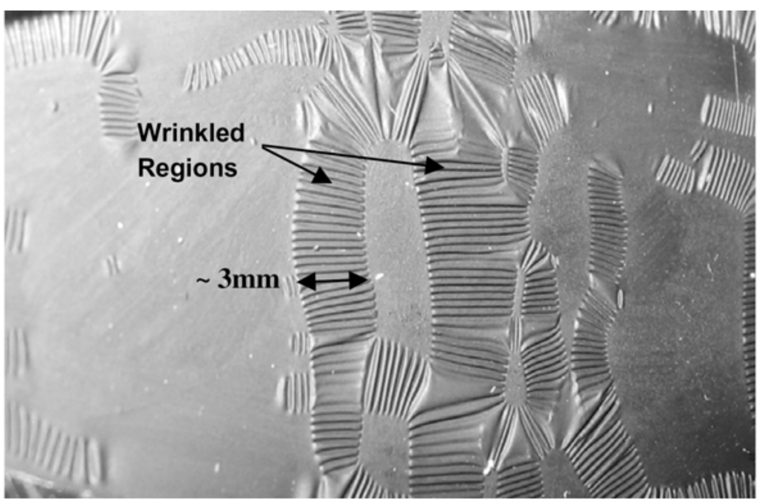
Theory: Zhao, Suo
APL 91, 061921 (2007)

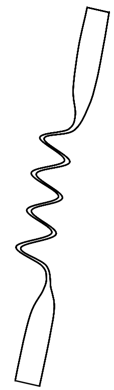
Coexistent states





Top view





Cross section

Coexistent states: flat and wrinkled

Experiment: Plante, Dubowsky,
Int. J. Solids and Structures **43**, 7727 (2006)

Theory: Zhao, Hong, Suo
Physical Review B 76, 134113 (2007)

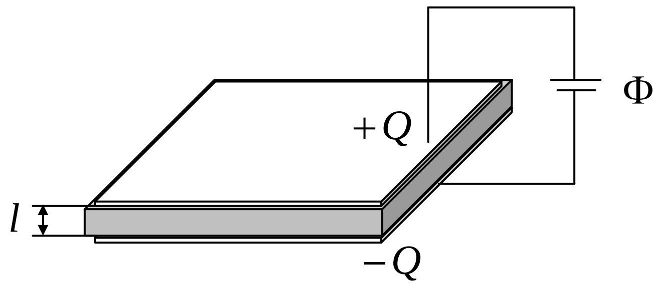
Stiffening due to extension limit

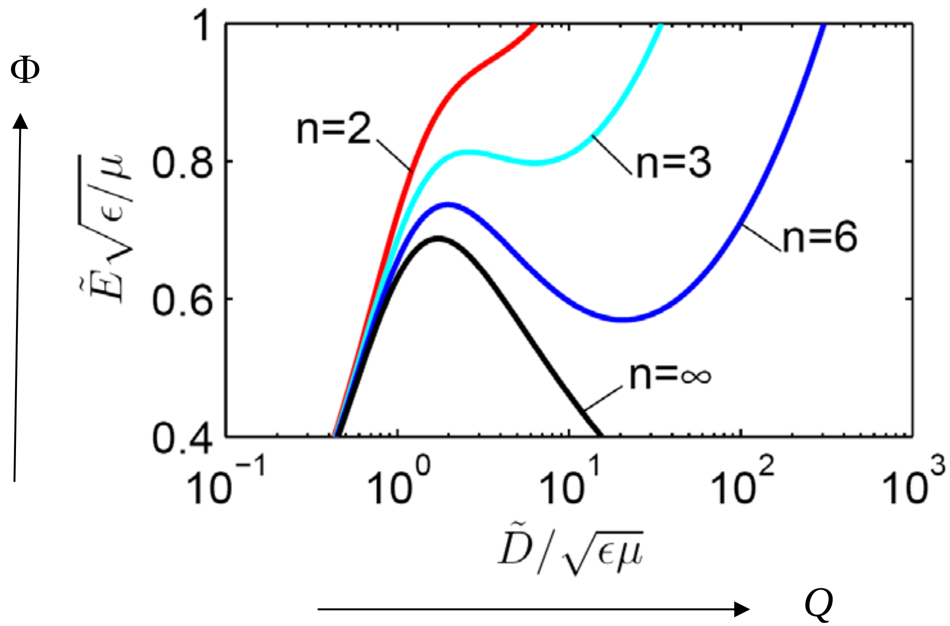
Stiffening as each polymer chain approaches its fully stretched length (e.g., Arruda-Boyce model)

$$W_s = \mu\left[\frac{1}{2}(I-3)+\frac{1}{20n}\left(I^2-9\right)+\ldots\right]$$

$$I = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

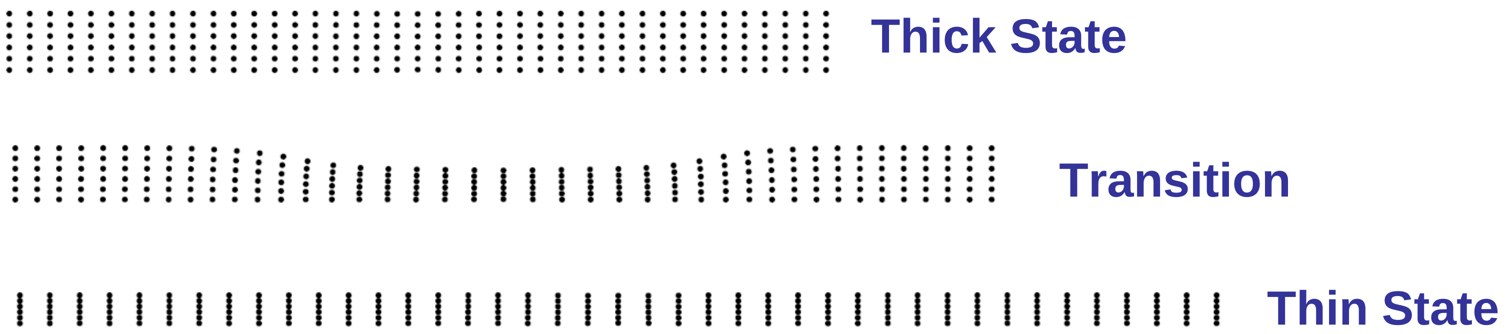
μ: small-strain shear modulus
n: number of monomers per chain

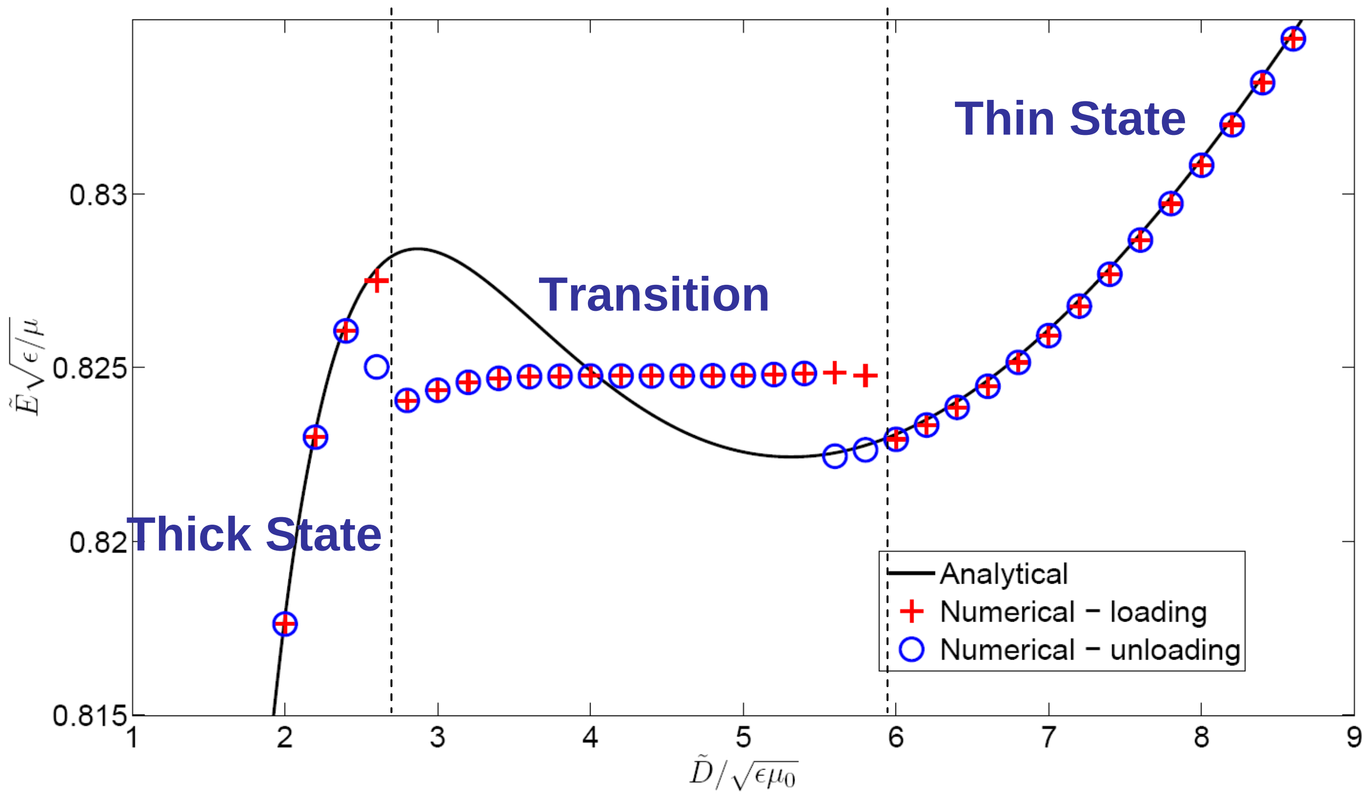




Zhao, Hong, Suo
Physical Review B 76, 134113 (2007)

FEM: inhomogeneous deformation





Zhou, Hong, Zhao, Zhang, Suo, IJSS, 2007

Summary

- Ideal dielectric elastomers: dielectric behavior is liquid like, unaffected by deformation.
- Pull-in instability: coupling large deformation and electric charge.
- Pre-stretch increases actuation strain.
- Co-existent states: stiffening due to extension limit.

Game plan

- Inhomogeneous deformation.
- Quantitative comparison with experiments.
- Effect of viscosity on instability.
- Nonideal electromechanical coupling.
- Adding other effects (solvents, ions, enzymes…): Soft Active Materials (SAMs)

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