Mechanics of Soft Active Materials (SAMs)

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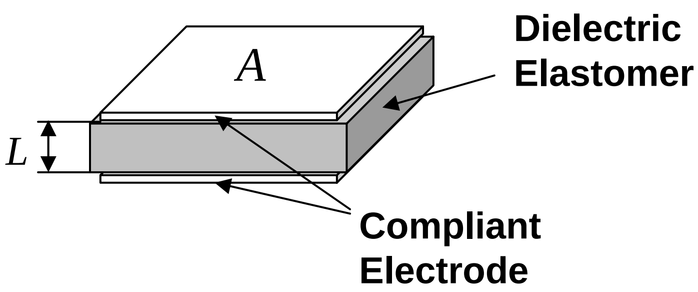
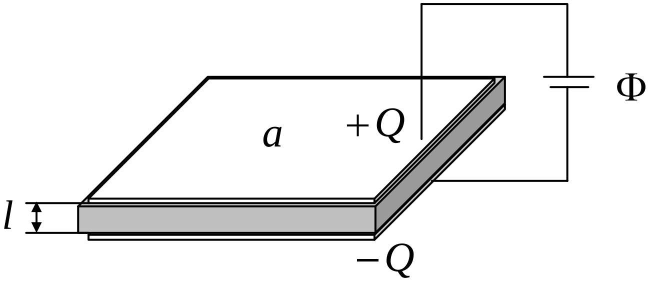
Work with
X. Zhao, W. Hong, J. Zhou, W. Greene

Dielectric elastomers



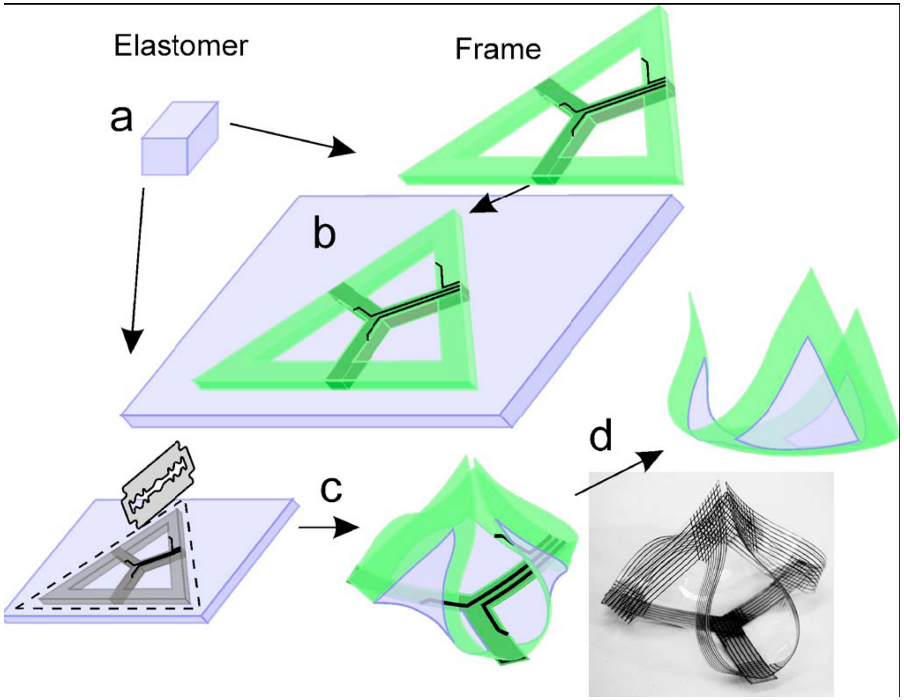


Reference State

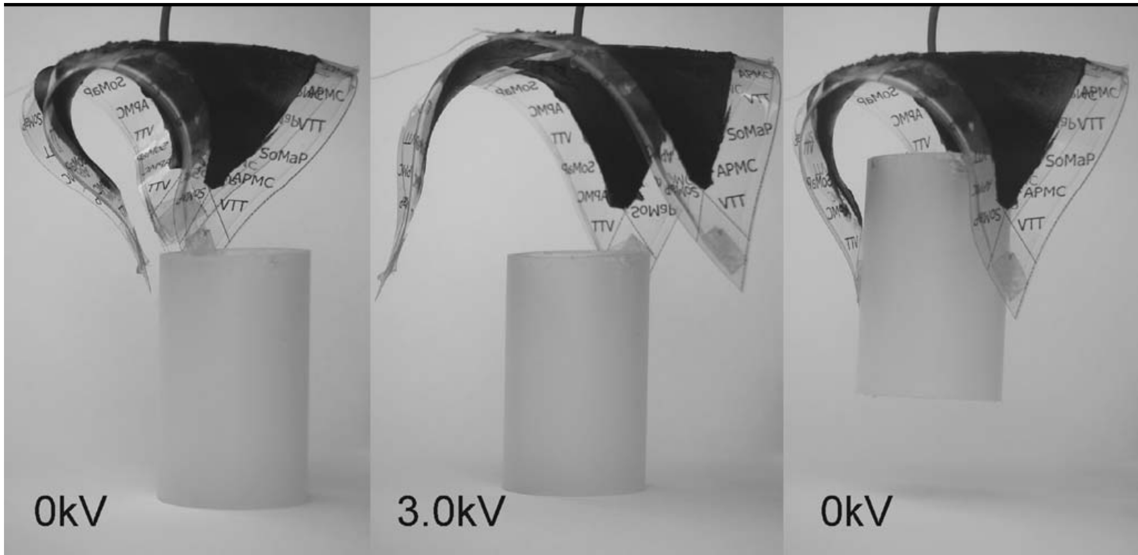
Current State

Pelrine, Kornbluh, Pei, Joseph
High-speed electrically actuated elastomers with strain greater than 100%.
Science 287, 836 (2000).

Dielectric elastomer actuators

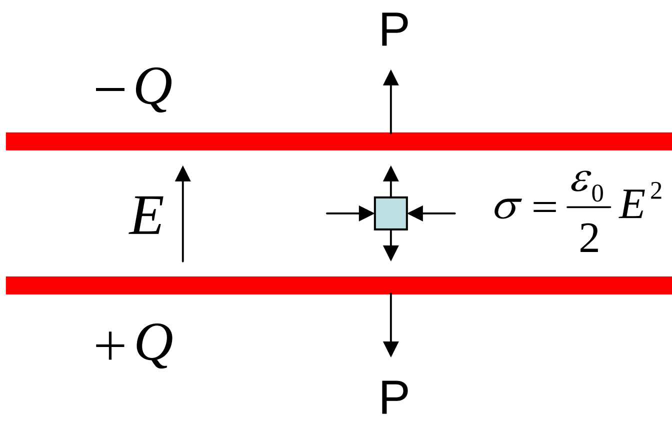


- Large deformation
- Compact
- Lightweight
- Low cost
- Low-temperature fabrication



Kofoda, Wirges, Paajanen, Bauer
APL **90**, 081916, 2007

Maxwell stress in vacuum (1873)



$$\sigma_{ij} = \varepsilon_0 E_j E_i - \frac{\varepsilon_0}{2} E_k E_k \delta_{ij}$$

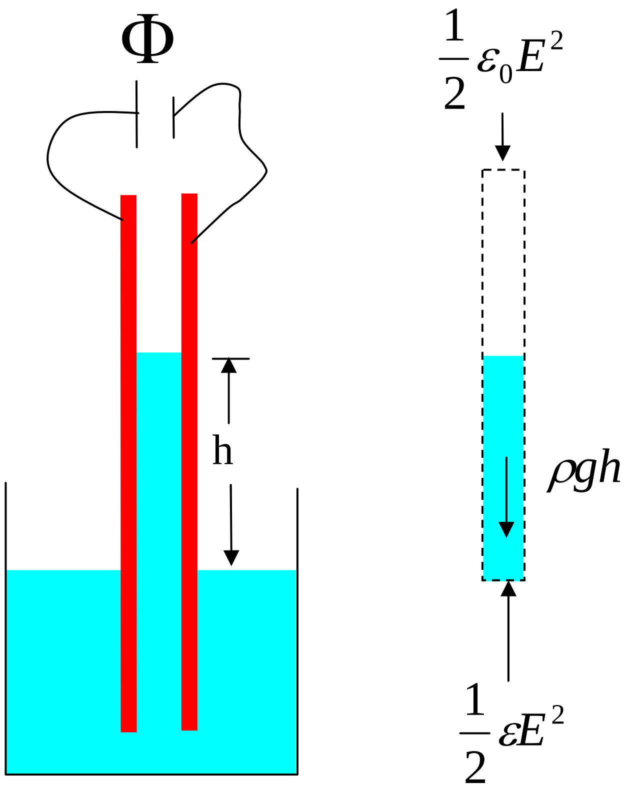
A field of forces needed to maintain equilibrium of a field of charges $F_i = qE_i$

Electrostatic field

$$E_i = -\frac{\partial \Phi}{\partial x_i} \qquad \frac{\partial E_i}{\partial x_i} = \frac{q}{\varepsilon_0}$$

$$F_i = \frac{\partial}{\partial x_j} \left(\varepsilon_0 E_j E_i - \frac{\varepsilon_0}{2} E_k E_k \delta_{ij} \right)$$

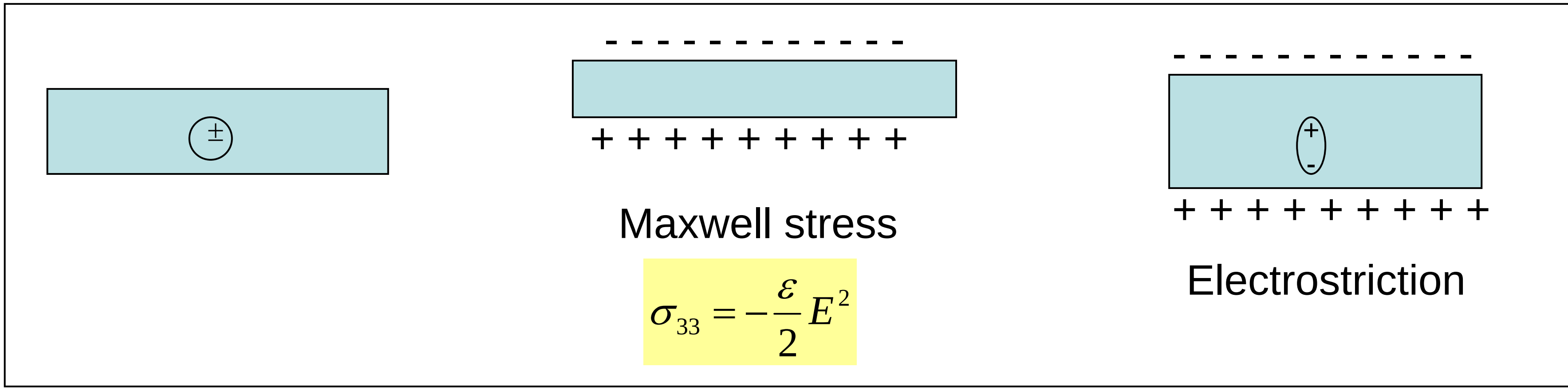
Include Maxwell stress in a free-body diagram



"Free-body" diagram

$$\rho g h = \frac{1}{2}\left(\varepsilon - \varepsilon_0\right)E^2$$

Trouble with Maxwell stress in dielectrics



$$\sigma_{33} = -\frac{\varepsilon}{2}E^2$$

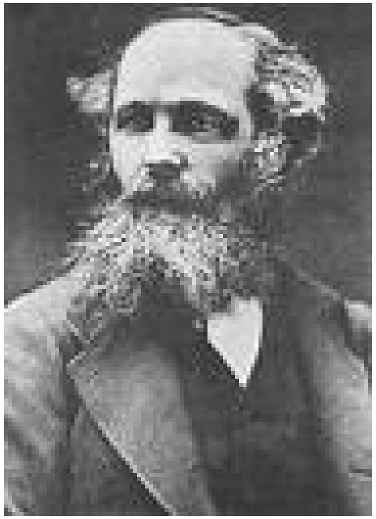
Our complaints:

- In general, ε varies with deformation.
- In general, E^2 dependence has no special significance.
- Wrong sign of the Maxwell stress?

In solid, Maxwell stress is not even wrong; it's a bad idea.

Suo, Zhao, Greene, JMPS (2007)

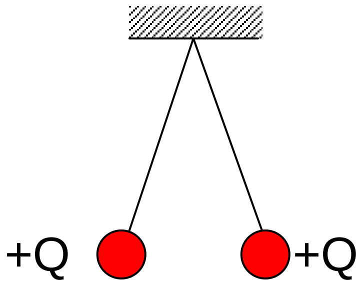
James Clerk Maxwell (1831-1879)



"I have not been able to make the next step, namely, to account by mechanical considerations for these stresses in the dielectric. I therefore leave the theory at this point…"

A Treatise on Electricity & Magnetism (1873), Article 111

Trouble with electric force in dielectrics



$$E_i = -\frac{\partial \Phi}{\partial x_i} \qquad \frac{\partial E_i}{\partial x_i} = \frac{q}{\varepsilon_0}$$

In a vacuum,
force is needed to maintain equilibrium of charges
Define electric field by E = F/Q

$$F_i = qE_i$$

In a dielectric,
force between charges is
NOT an operational concept

$$F_i = qE_i$$

Historical work
- Toupin (1956)
- Eringen (1963)
- Tiersten (1971)
- ……

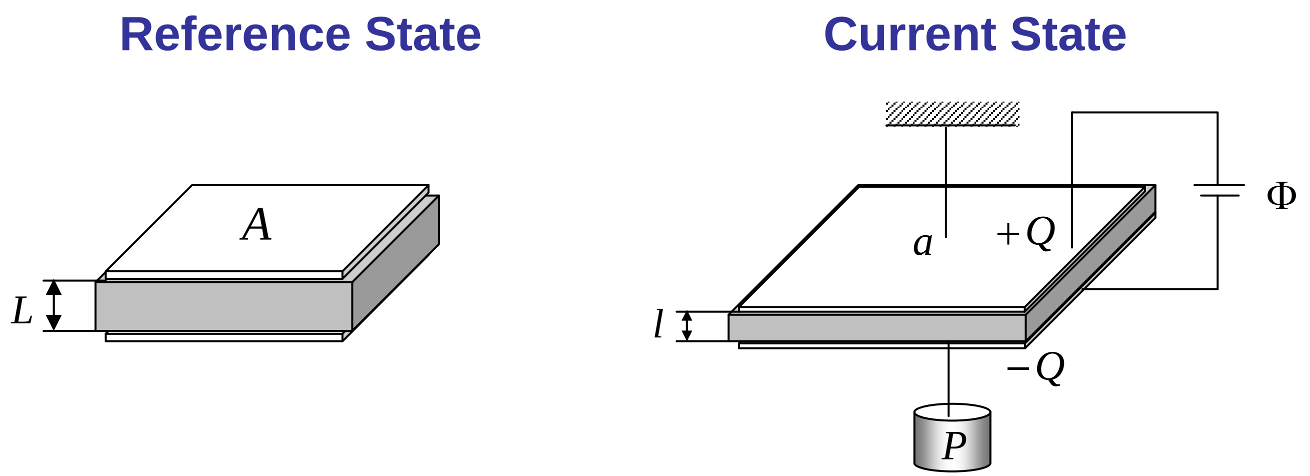
Recent work
- Dorfmann, Ogden (2005)
- Landis, McMeeking (2005)
- **Suo, Zhao, Greene (2007)**
- ……

The Feynman Lectures on Physics

Volume II, p.10-8 (1964)

“What does happen in a solid? This is a very difficult problem which has not been solved, because it is, in a sense, indeterminate. If you put charges inside a dielectric solid, there are many kinds of pressures and strains. You cannot deal with virtual work without including also the mechanical energy required to compress the solid, and it is a difficult matter, generally speaking, to make a unique distinction between the electrical forces and mechanical forces due to solid material itself. Fortunately, no one ever really needs to know the answer to the question proposed. He may sometimes want to know how much strain there is going to be in a solid, and that can be worked out. But it is much more complicated than the simple result we got for liquids.”

All troubles are gone if we use measurable quantities



$\lambda = l / L$

$s = P / A$

$\tilde{E} = \Phi / L$

$\tilde{D} = Q / A$

Weight does work $P\delta l$

Battery does work $\Phi \delta Q$

For elastic dielectric, work fully converts to free energy: $\delta U = P\delta l + \Phi \delta Q$

$$\frac{\delta U}{AL} = \frac{P\delta l}{AL} + \frac{\Phi \delta Q}{LA} \qquad \delta W = s\delta\lambda + \tilde{E}\delta\tilde{D}$$

Material laws

$$s = \frac{\partial W(\lambda, \tilde{D})}{\partial \lambda} \qquad \tilde{E} = \frac{\partial W(\lambda, \tilde{D})}{\partial \tilde{D}}$$

Suo, Zhao, Greene, JMPS (2007)

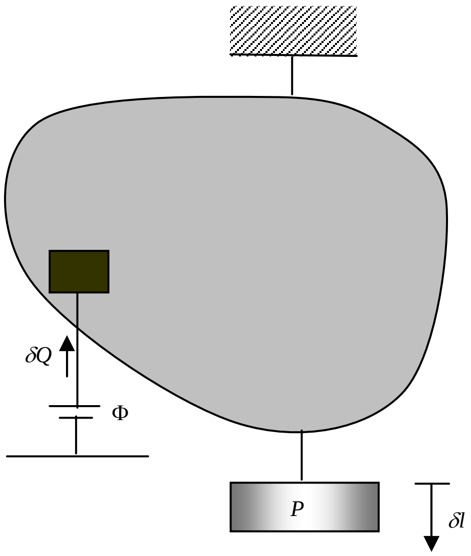
Game plan

- Extend the theory to 3D.
- Construct free-energy function *W.*
- Study interesting phenomena.
- Add other effects (stimuli-responsive gels).

3D inhomogeneous field

Linear PDEs

Suo, Zhao, Greene, JMPS (2007)



A field of weights $F_{iK}(\mathbf{X},t) = \frac{\partial x_i(\mathbf{X},t)}{\partial X_K}$,

$$\int s_{iK} \frac{\partial \xi_i}{\partial X_K} dV = \int \tilde{b}_i \xi_i dV + \int \tilde{t}_i \xi_i dA$$

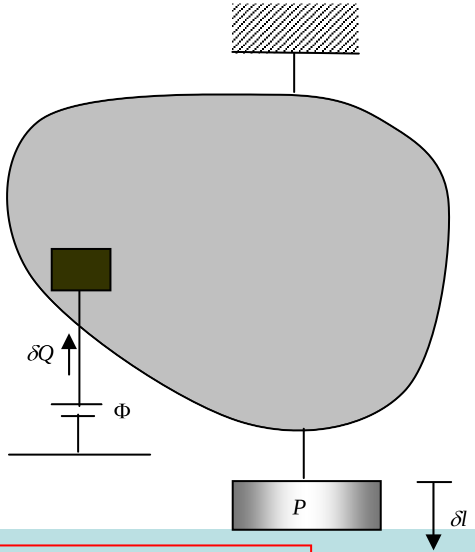
$$\frac{\partial s_{iK}(\mathbf{X},t)}{\partial X_K} + \tilde{b}_i(\mathbf{X},t) = 0 \qquad \left(s_{iK}^-(\mathbf{X},t) - s_{iK}^+(\mathbf{X},t)\right)N_K(\mathbf{X},t) = \tilde{t}_i(\mathbf{X},t)$$

A field of batteries $\tilde{E}_K(\mathbf{X},t) = -\frac{\partial \Phi(\mathbf{X},t)}{\partial X_K}$,

$$\int \left(-\frac{\partial \eta}{\partial X_K}\right)\tilde{D}_K dV = \int \eta \tilde{q} dV + \int \eta \tilde{\omega} dA$$

$$\frac{\partial \tilde{D}_K(\mathbf{X},t)}{\partial X_K} = \tilde{q}(\mathbf{X},t) \qquad \left(\tilde{D}_K^+(\mathbf{X},t) - \tilde{D}_K^-(\mathbf{X},t)\right)N_K(\mathbf{X},t) = \tilde{\omega}(\mathbf{X},t)$$

Material law



Elastic dielectric, defined by a free energy function $W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)$

$$\delta W = \frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial F_{iK}} \delta F_{iK} + \frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial \tilde{D}_K} \delta \tilde{D}_K$$

Free energy of the system

Free energy of dielectric

Potential energy of weights

Potential energy of batteries

$$\delta G = \int \delta W dV - \int \tilde{b}_i \delta x_i dV - \int \tilde{t}_i \delta x_i dA - \int \Phi \delta \tilde{q} dV - \int \Phi \delta \tilde{\omega} dA$$

A little algebra

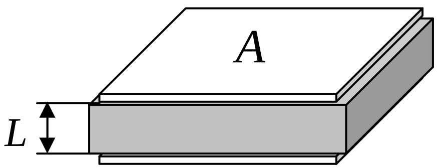
$$\delta G = \int \left[\frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial F_{iK}} - s_{iK} \right] \delta F_{iK} dV + \int \left[\frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial \tilde{D}_K} - \tilde{E}_K \right] \delta \tilde{D}_K dV$$

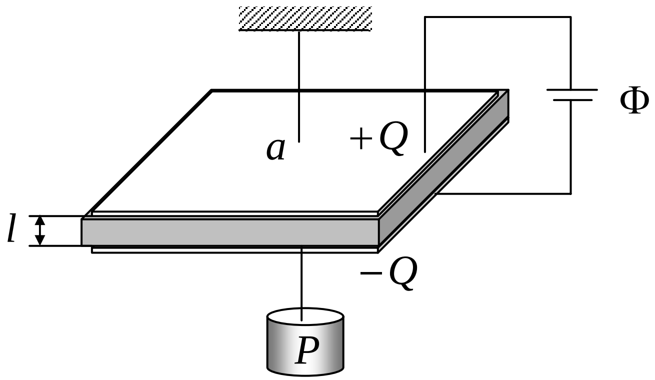
Thermodynamic equilibrium: $\delta G = 0$ for arbitrary changes δF_{iK} and $\delta \tilde{D}_K$

Material laws

$$s_{iK}\left(\mathbf{F}, \tilde{\mathbf{D}}\right) = \frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial F_{iK}}, \qquad \tilde{E}_K\left(\mathbf{F}, \tilde{\mathbf{D}}\right) = \frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial \tilde{D}_K}$$

Work-conjugate, or not





$\tilde{E} = \Phi / L \qquad (E = \Phi / l)$

$\tilde{D} = Q / A \qquad (D = Q / a)$

Nominal electric field and nominal electric displacement are work-conjugate

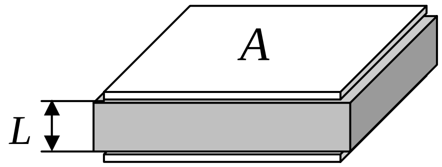
Battery does work $\Phi \delta Q = (\tilde{E}L)\delta(\tilde{D}A) = (AL)\tilde{E}\delta\tilde{D}$

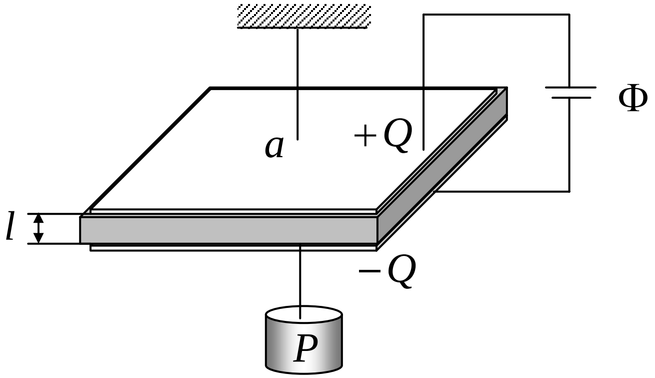
True electric field and true electric displacement are NOT work-conjugate

Battery does work $\Phi \delta Q = (El)\delta(Da) = (la)E\delta D + EDl\delta a$

True vs nominal

Reference State | **Current State**





$s = P / A$	$\sigma = P / a$	$\sigma_{ij} = \dfrac{F_{jK}}{\det(\mathbf{F})} s_{iK}$
$\tilde{E} = \Phi / L$	$E = \Phi / l$	$E_i = H_{iK} \tilde{E}_K \qquad (\mathbf{H} = \mathbf{F}^{-1})$
$\tilde{D} = Q / A$	$D = Q / a$	$D_i = \dfrac{F_{iK}}{\det(\mathbf{F})} \tilde{D}_K$

Dielectric constant is insensitive to stretch

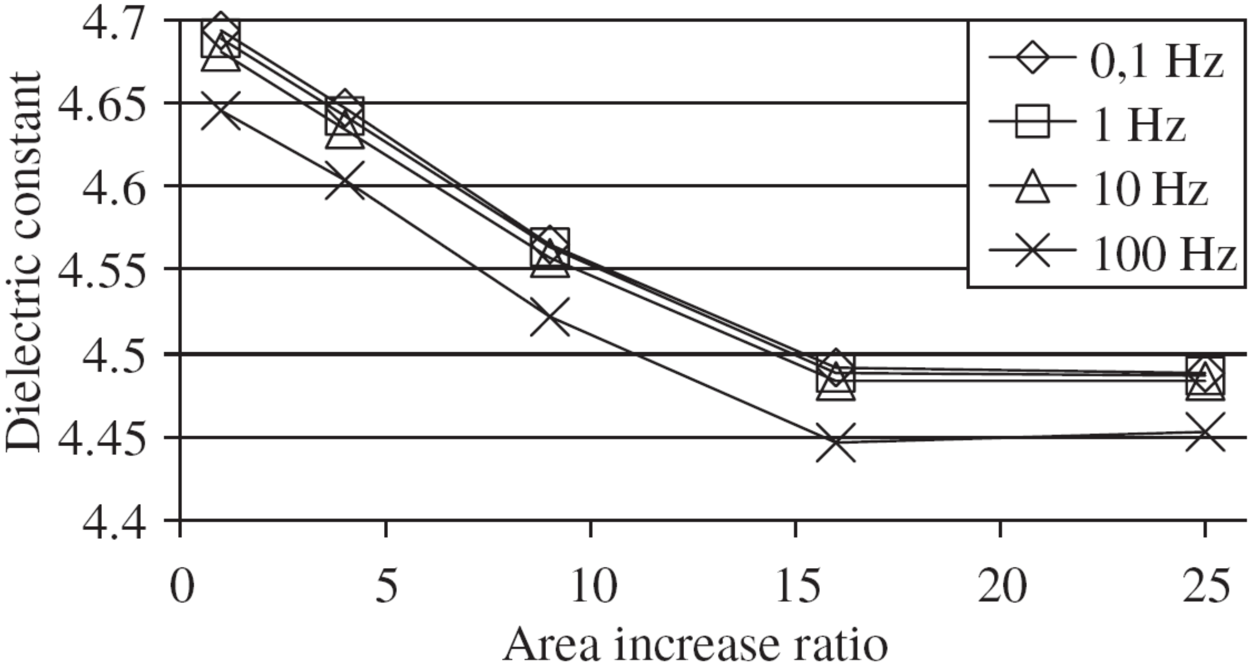


Figure 5. *The relative dielectric constant of VHB^{TM} 4910 drops, when it is stretched.*

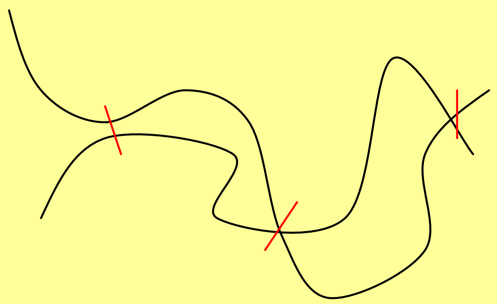
Kofod, Sommer-Larsen, Kornbluh, Pelrine
Journal of Intelligent Material Systems and Structures **14**, 787-793 (2003).

Ideal dielectric elastomers

Zhao, Hong, Suo, Physical Review B 76, 134113 (2007).

$$W\left(\mathbf{F}, \tilde{\mathbf{D}}\right) = W_s\left(\mathbf{F}\right) + \frac{D^2}{2\varepsilon}$$

Stretch Polarization



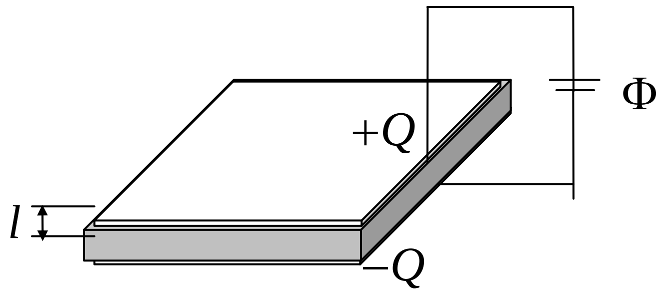
$$W_s\left(\mathbf{F}\right) = \frac{\mu}{2}\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right)$$

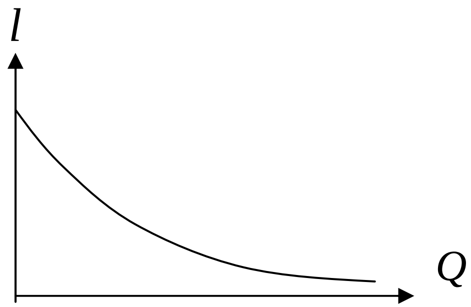
$$D_i = \frac{F_{iK}}{\det\left(\mathbf{F}\right)} \tilde{D}_K$$

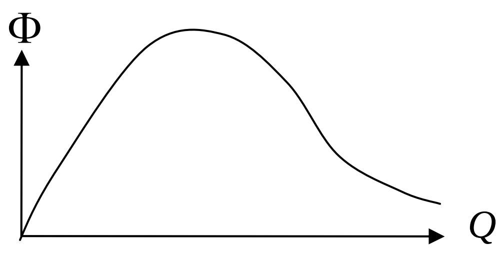
$$\tilde{E}_K\left(\mathbf{F}, \tilde{\mathbf{D}}\right) = \frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial \tilde{D}_K} \qquad s_{iK}\left(\mathbf{F}, \tilde{\mathbf{D}}\right) = \frac{\partial W\left(\mathbf{F}, \tilde{\mathbf{D}}\right)}{\partial F_{iK}}$$

$$D_i = \varepsilon E_i \qquad \sigma_{ij} = \frac{F_{iK}}{\det\left(\mathbf{F}\right)} \frac{\partial W_s\left(\mathbf{F}\right)}{\partial F_{jK}} + \varepsilon\left(E_i E_j - \frac{1}{2} E_k E_k \delta_{ij}\right)$$

Electromechanical instability







$$\tilde{E}_c \sim \sqrt{\frac{\mu}{\varepsilon}} \sim \sqrt{\frac{10^6\, N/m}{10^{-10}\, F/m}} = 10^8 V/m$$

Stark & Garton, Nature 176, 1225 (1955).

$$W\left(\lambda, \tilde{D}\right) = \frac{\mu}{2}\left(\lambda + 2\lambda^{-1} - 3\right) + \frac{\lambda^2 \tilde{D}^2}{\varepsilon}$$

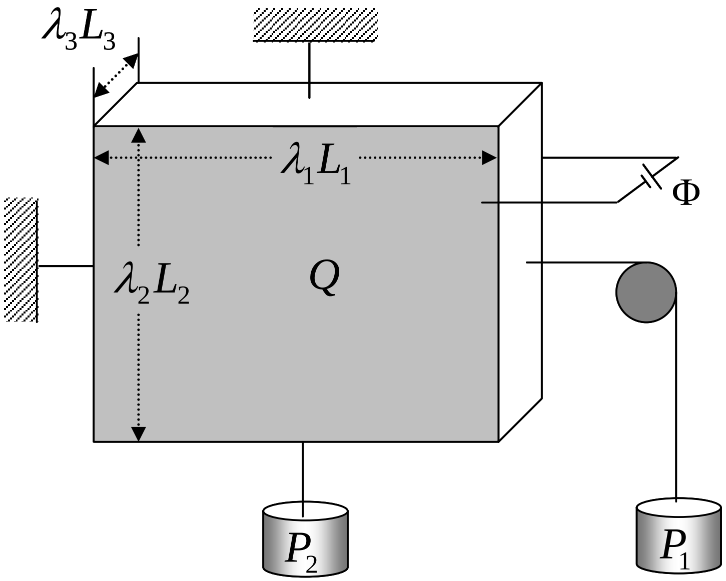
$$s = \frac{\partial W\left(\lambda, \tilde{D}\right)}{\partial \lambda} = 0 \qquad \lambda = \left(1 + \frac{\tilde{D}^2}{\varepsilon\mu}\right)^{-1/3}$$

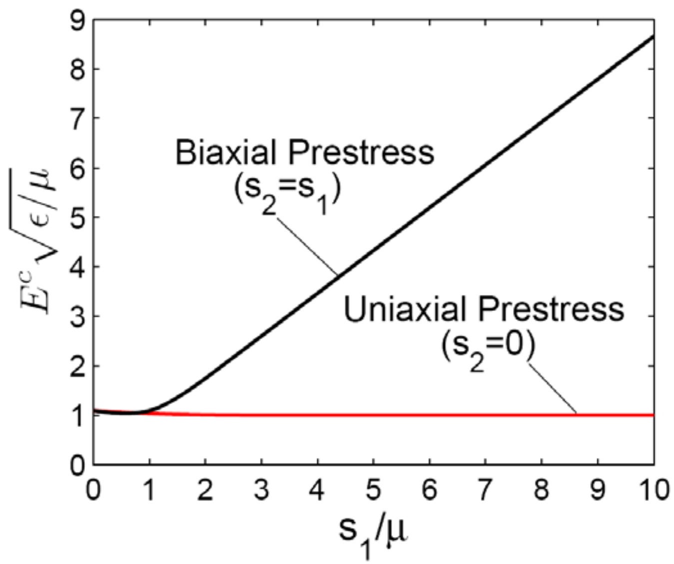
$$\tilde{E} = \frac{\partial W\left(\lambda, \tilde{D}\right)}{\partial \tilde{D}} \qquad \tilde{E} = \frac{\tilde{D}\lambda^2}{\varepsilon}$$

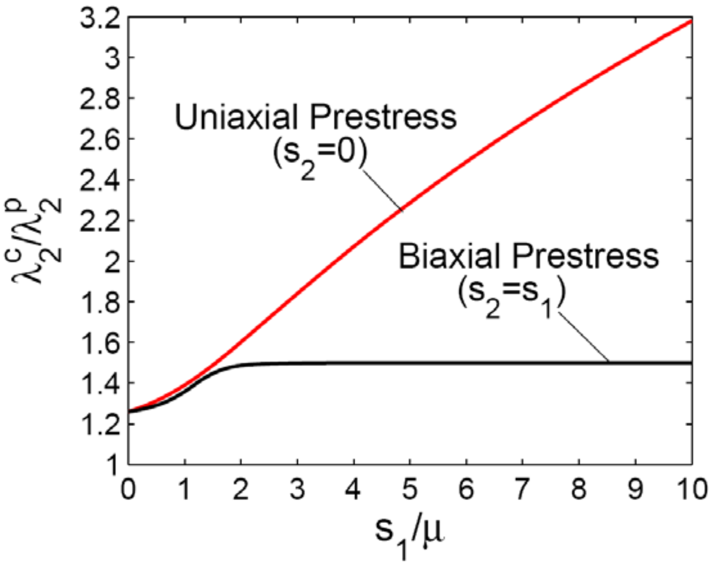
$$\frac{\tilde{E}}{\sqrt{\mu/\varepsilon}} = \frac{\tilde{D}}{\sqrt{\varepsilon\mu}}\left(1 + \frac{\tilde{D}^2}{\varepsilon\mu}\right)^{-2/3}$$

Zhao, Suo, APL 91, 061921 (2007)

Pre-stresses enhance actuation



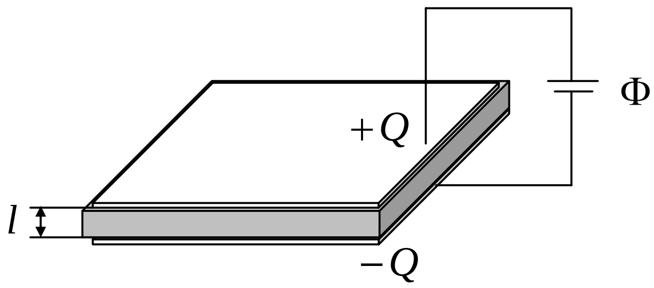


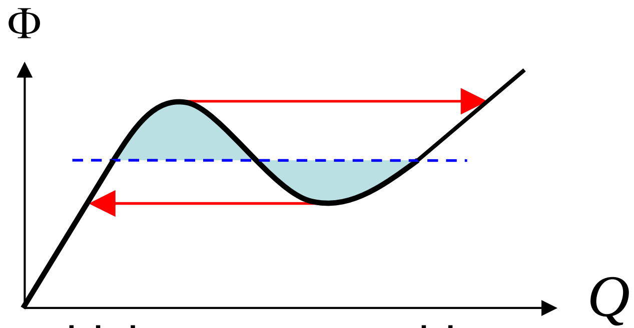


Experiment: Pelrine, Kornbluh, Pei, Joseph
Science 287, 836 (2000).

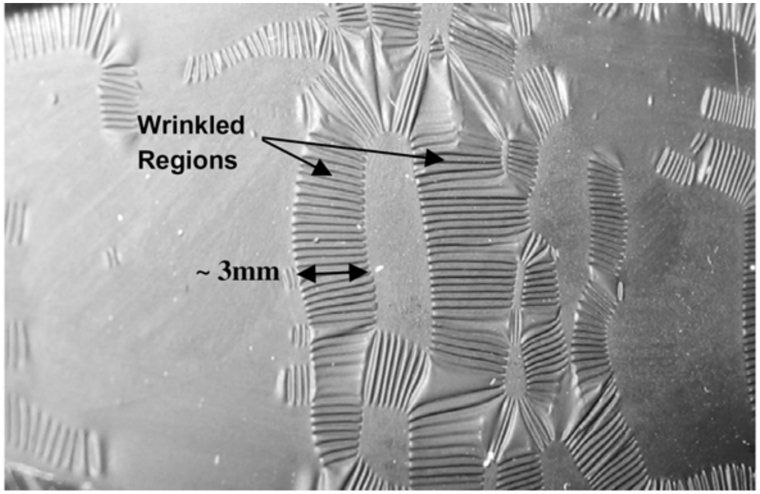
Theory: Zhao, Suo
APL 91, 061921 (2007)

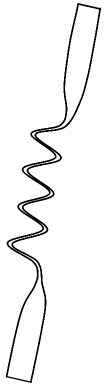
Coexistent states





Top view





Cross section

Coexistent states: flat and wrinkled

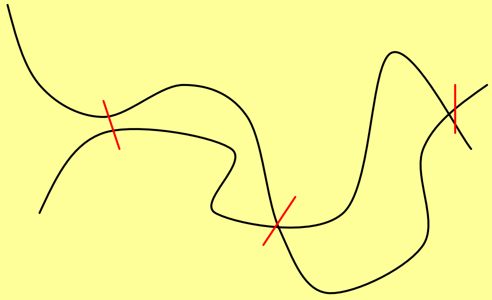
Experiment: Plante, Dubowsky,
Int. J. Solids and Structures **43**, 7727 (2006).

Theory: Zhao, Hong, Suo
Physical Review B 76, 134113 (2007)..

Elastomer: extension limit

$$W\left(\mathbf{F},\tilde{\mathbf{D}}\right)=W_s\left(\mathbf{F}\right)+\frac{D^2}{2\varepsilon}$$

Stretch Polarization



Stiffening as each polymer chain approaches its fully stretched length (e.g., Arruda-Boyce model)

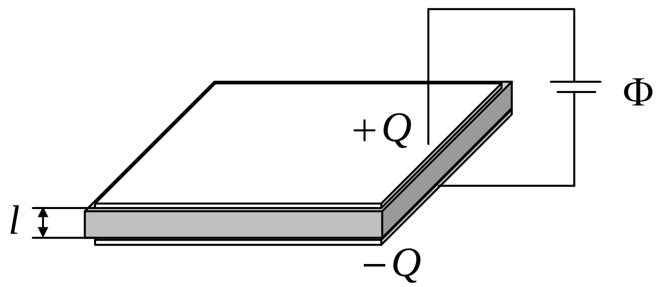
$$W_s=\mu\left[\frac{1}{2}(I-3)+\frac{1}{20n}\left(I^2-9\right)+...\right]$$

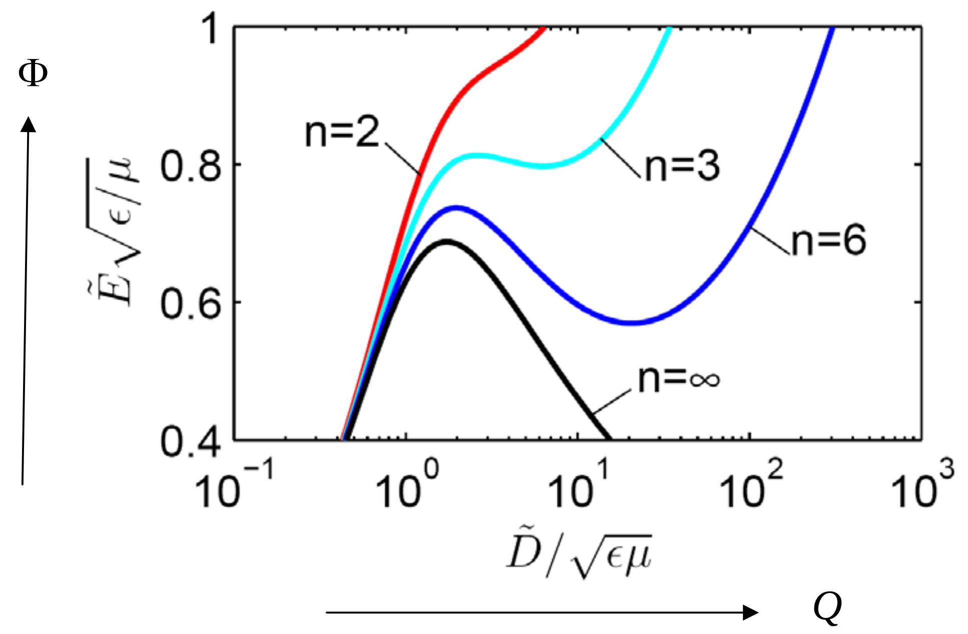
$$I=\lambda_1^2+\lambda_2^2+\lambda_3^2$$

μ: small-strain shear modulus

n: number of monomers per chain

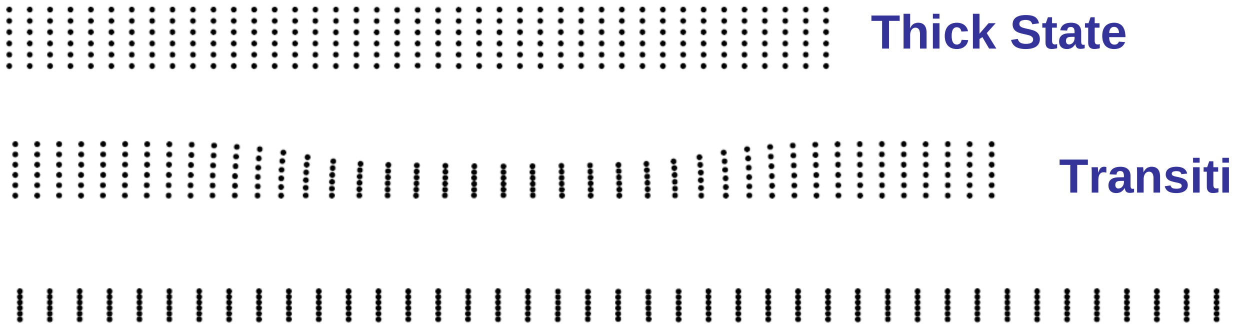
Coexistent states

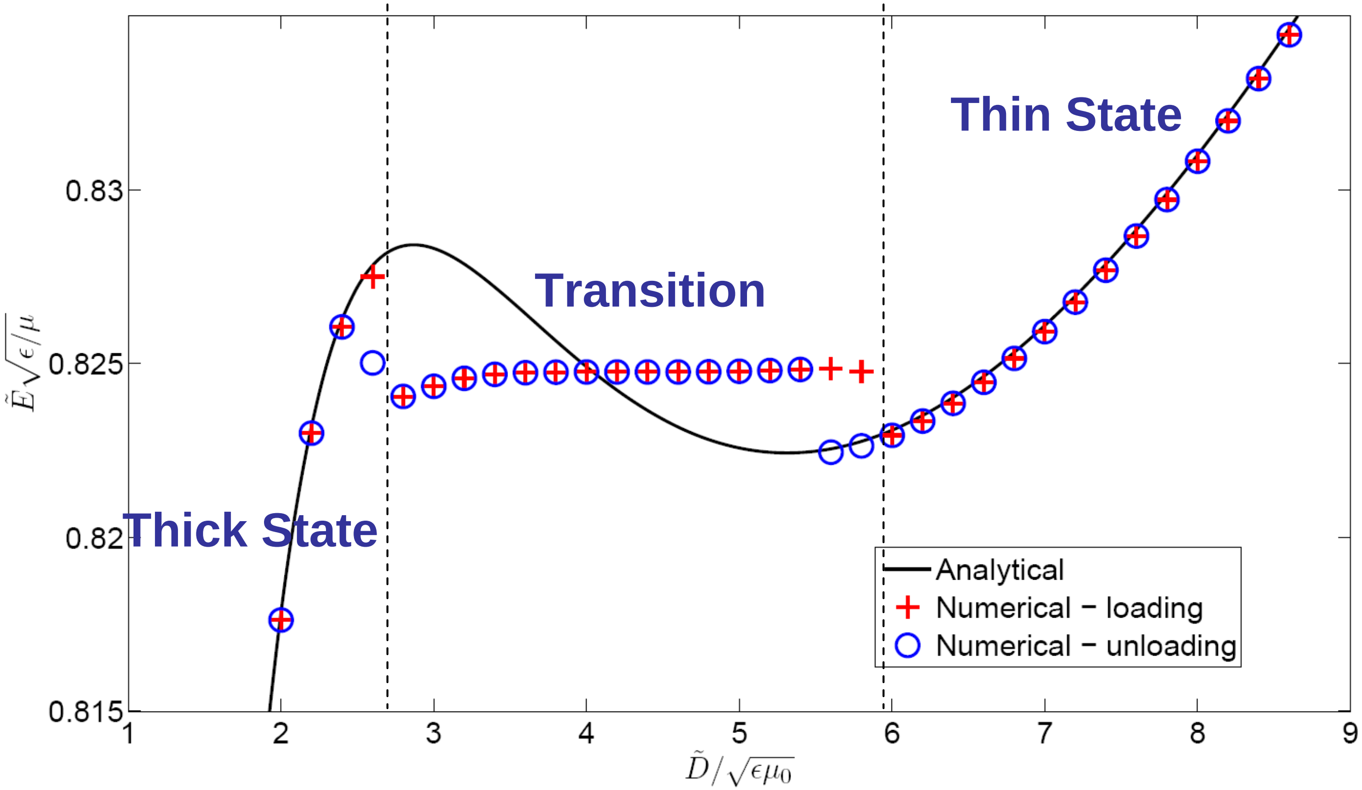




Zhao, Hong, Suo, Physical Review B 76, 134113 (2007).

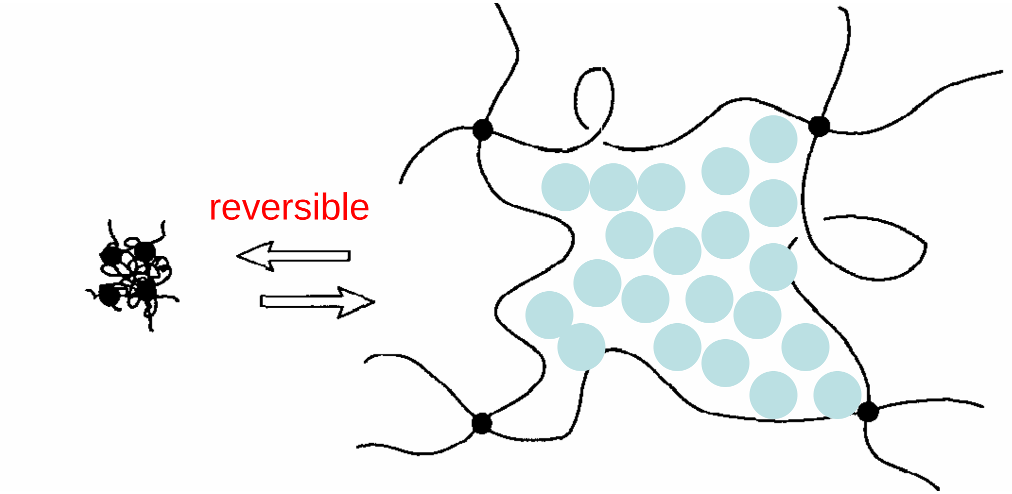
Finite element method





Zhou, Hong, Zhao, Zhang, Suo, IJSS, 2007

Stimuli-responsive gels

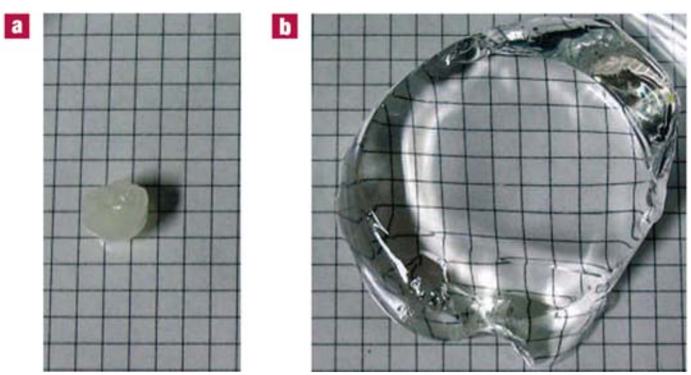


collapsed swollen

Gel

- long polymers (cross-linked but flexible)
- small molecules (mobile)

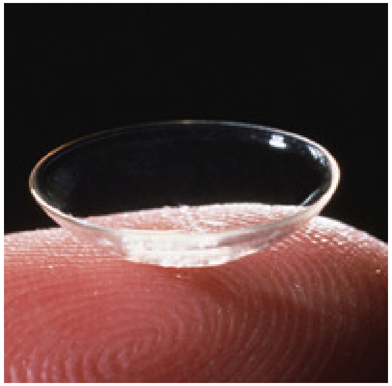
Stimuli



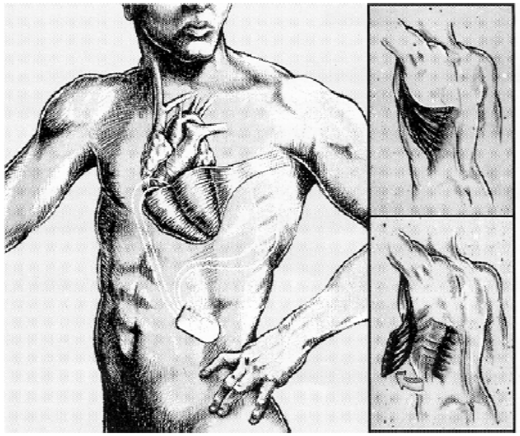
Ono et al, Nature Materials, 2007

- temperature
- electric field
- light
- ions
- enzymes

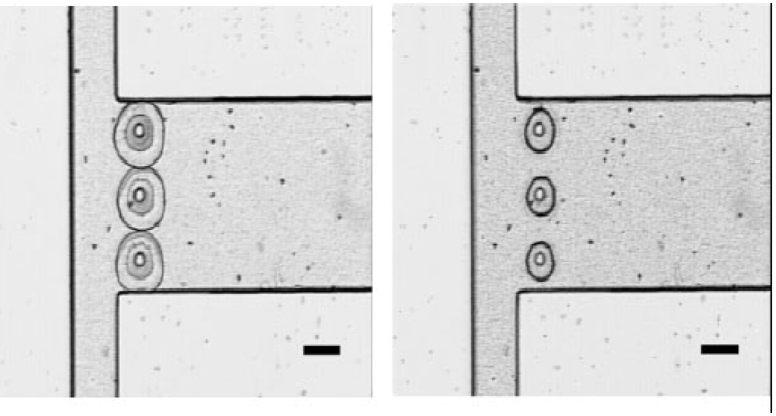
Applications of gels



Contact lenses



Artificial tissues



Gates in microfluidics

Drug delivery

Summary

- A nonlinear field theory. No Maxwell stress. No electric body force.
- Effect of electric field on deformation is a part of material law.
- Ideal dielectric elastomers: Maxwell stress emerges.
- Electromechanical instability: large deformation and electric field.
- Add other effects (solvent, ions, enzymes…)