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Rupture of Rubber. I. Characteristic Energy for Tearing

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1. INTRODUCTION

The resistance to tearing of a rubber vulcanizate is usually determined by loading in a specified manner a test-piece of the vulcanizate of standard shape, in which a notch has been produced, either in the molding process or by cutting the test-piece in a standard fashion. A wide variety of shapes of test-piece and notch and of methods of loading have been recommended by various authors (see, for example, Buist[1]).

All of these methods of determining tear resistance yield results which are characteristic of the method of test as well as of the particular type of test-piece used.

In the present paper an attempt is made to obtain a criterion for the tearing of a rubber vulcanizate which is independent of the form of the test-piece. This is desirable both with regard to the study of the relation between the resistance to tear of a vulcanizate and its structure and from the point of view of obtaining a better understanding of the mechanical factors involved in determining the tear of a test-piece.

2. THE CRITERION FOR TEARING

Two criteria for tearing of cut or notched elastic test-pieces have been used. For simplicity these will be discussed in relation to test-pieces which have the form of thin sheets of uniform thickness cut uniformly through their thickness.

According to one of these criteria—the critical stress criterion—the cut will spread when the stress at its tip reaches a critical value. For an ideally thin sheet the material at the tip of the cut is in simple extension, no matter how the test-piece as a whole is deformed, since the surface tractions on both the major and minor surfaces at the cut are zero.

According to the second criterion for tearing, originally suggested by Griffith,[3] a cut of length c will spread by an amount dc at a certain overall deformation of the test-piece, if, and only if, the energy W stored elastically in the test-piece is thereby decreased by an amount $-dW$ greater than the increase of surface free-energy due to the formation of new surface. If the area of new surface, formed by an increase in cut-length dc, is dA, then Griffith's criterion may be written

$$-(dW/dc) > T(dA/dc) \tag{2.1}$$

where T is the surface free-energy per unit area of the material.

Elliott[2] has attempted to reconcile the Griffith and critical stress criteria, for the spread of a crack in an elastic material showing brittle fracture, by assuming that the crack has a width of interatomic dimension.

In attempting to find a criterion for the spreading of a cut in a sheet of rubber vulcanizate the direct investigation of the critical stress criterion presents considerable difficulties, since the solution of the mathematical problem of determining the stress distribution in a cut sheet of highly-elastic material is intractable and the stress concentration in the neighborhood of the tip of the cut is limited to so small a region that measurements on it cannot be readily carried out. Consequently an attempt has been made to discover to what extent energetic considerations of the type involved in the Griffith criterion can be used as a criterion for tearing in vulcanized rubber.

The Griffith theory describes the slow propagation of a cut as the conversion of elastic energy stored in the bulk of the test-piece into surface free-energy, and it is implied that from the thermodynamic point of view the quasi-static propagation of a cut is a reversible process. It may, however, be that the reduction of elastically-stored energy due to increase in cut-length, at constant overall deformation of the test-piece, is balanced by changes in energy other than this increase in surface free-energy. It is to be expected that any such changes will be proportional to the increase in cut-length and, for a thin sheet of given material in which the cut-length is large compared with its width and with the thickness of the sheet, will be determined primarily by the state of deformation in the neighborhood of the tip of the cut at the instant of tearing. This state of deformation will in turn be determined by the shape of the cut in the neighborhood of its tip and the magnitude of the extension ratio at the tip at the instant of tearing, rather than by the shape of the test-piece as a whole and the detailed manner in which the forces are applied to it in regions sufficiently far from the tip of the cut.

It is therefore to be anticipated that the energy which must be expended, at the expense of the elastically stored energy, in causing a given small increase in the cut length at constant overall deformation of the test-piece (*i.e.*, under such conditions that the forces applied to the test-piece do no work) will be substantially independent of the shape of the test-piece and of the manner in which the deforming forces are applied to it. This energy will therefore be a characteristic energy for tearing of thin sheets of the material, although it may depend on the shape of the tip of the cut.

Let us consider a sheet of the vulcanizate of thickness t in its undeformed state, containing a cut of length c, to be deformed by applying displacements over part of its boundary. The region near the tip of the cut will, of course, be very highly deformed in comparison with the rest of the sheet. We shall assume that for the cut to spread by an amount dc, an amount of work $Tt \cdot dc$ must be done, where T is an energy characteristic of the material. Then during the quasi-static growth of the cut, during which no work is done by the external forces due to movement of the parts of the boundary over which they are applied, the change in the free energy of deformation of the sheet is given by

$$-\left(\frac{\partial W}{\partial c}\right)_l = Tt \qquad (2.2)$$

The suffix l indicates that the differentiation is carried out under conditions of constant displacement of the parts of the boundary which are not force-free. The criterion (2.2) is seen to be similar in form to Griffith's criterion (2.1), but T is no longer to be interpreted as a surface free-energy.

In the experiments described in this paper an attempt is made to assess the validity of the criterion (2.2) for the tearing of a natural rubber vulcanizate.

3. QUALITATIVE OBSERVATIONS ON THE PROCESS OF TEARING

All the experiments described in this paper were carried out using vulcanizates of natural rubber containing very little added ingredients apart from those necessary to effect the vulcanization. Their recipes are given in the Appendix. Such vulcanizates show substantially reversible elasticity except at very large deformations such as occur in the experiments only in the immediate neighborhood of the tip of a cut. Moreover, if tear tests are carried out on test-pieces of such vulcanizates, the results obtained for sufficiently slow rates of deformation of the test-piece are substantially independent of the speed of test.

If a cut is made with a razor blade in a test-piece cut from a thin sheet of one of these vulcanizates and the test-piece is slowly extended so that the cut opens, then it is seen under a low power microscope that even quite small forces cause a noticeable tearing to occur at the tip of the cut. In the early stages this slight tearing continues only so long as the overall deformation of the test-piece continues and ceases as soon as the deformation ceases. However, if the deformation of the test-piece is continued until the cut has grown by a few hundredths of a millimeter, catastrophic rupture occurs, the cut suddenly increasing in length by a few millimeters. Further deformation of the test-piece results in further catastrophic increases in the cut length of the same nature. In the cut growth preceding catastrophic rupture the initially smooth tip of the cut becomes increasingly irregular in appearance. Although these observations apply qualitatively to all unloaded natural rubber vulcanizates, the detailed quantitative behavior varies from one vulcanizate to another. They indicate that it is difficult to define a unique point of tear. In tear tests it is usual to take as the point of tear that point at which catastrophic rupture commences. For the most part it has been found convenient to follow this practice in the present experiments and the value of T which is then obtained refers to the process of catastrophic growth of a tear rather than to the process which precedes it.

In principle, however, if the same condition at the tip of the cut is always taken as indicating the point of tear, then comparison between different tear tests should be possible. The value of T obtained from these would then be a characteristic energy for tearing at the stage considered.

4. THE DIRECT VERIFICATION OF THE TEAR CRITERION FOR CATASTROPHIC TEARING

In this section experiments are described which have as their object the direct verification of the tear criterion (2.2) by measuring the quantity $(\partial W/\partial c)_l$ at the instant of tear for various types of test-piece cut from a sheet of the material. For a particular vulcanizate, if the criterion (2.2) applies, we should expect the values of $(\partial W/\partial c)_l/t$ obtained from these experiments to be constant. The value of the constant would then give the characteristic energy for tearing of the vulcanizate employed.

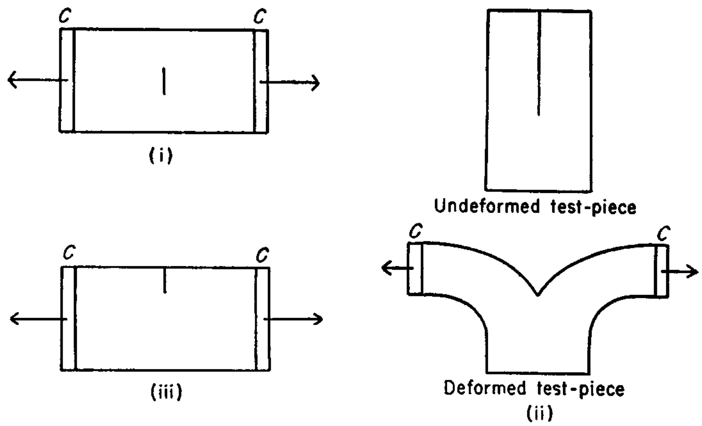

Fig. 1. Schematic diagrams of test-pieces.

Three test-pieces were used. They are shown schematically in Figure 1. In Figures 1(i) and (iii) the test-pieces are shown only in their undeformed states, while in Figure 1(ii), they are shown in both their undeformed and deformed states. All of the test-pieces were prepared from rectangular specimens cut from sheets of vulcanized natural rubber of approximately the same thickness, prepared according to the recipe A given in the Appendix. The measured mean thicknesses of the test-pieces were 0.90, 0.87, and 0.85 mm. respectively. In each case a cut was made in the rectangular specimen and the tips of the cuts were formed with a razor blade. The ends of the test-pieces so obtained were held in clamps C and load-overall deformation relations were obtained. In each case the deformation was increased up to the point at which the specimen tore, this point being

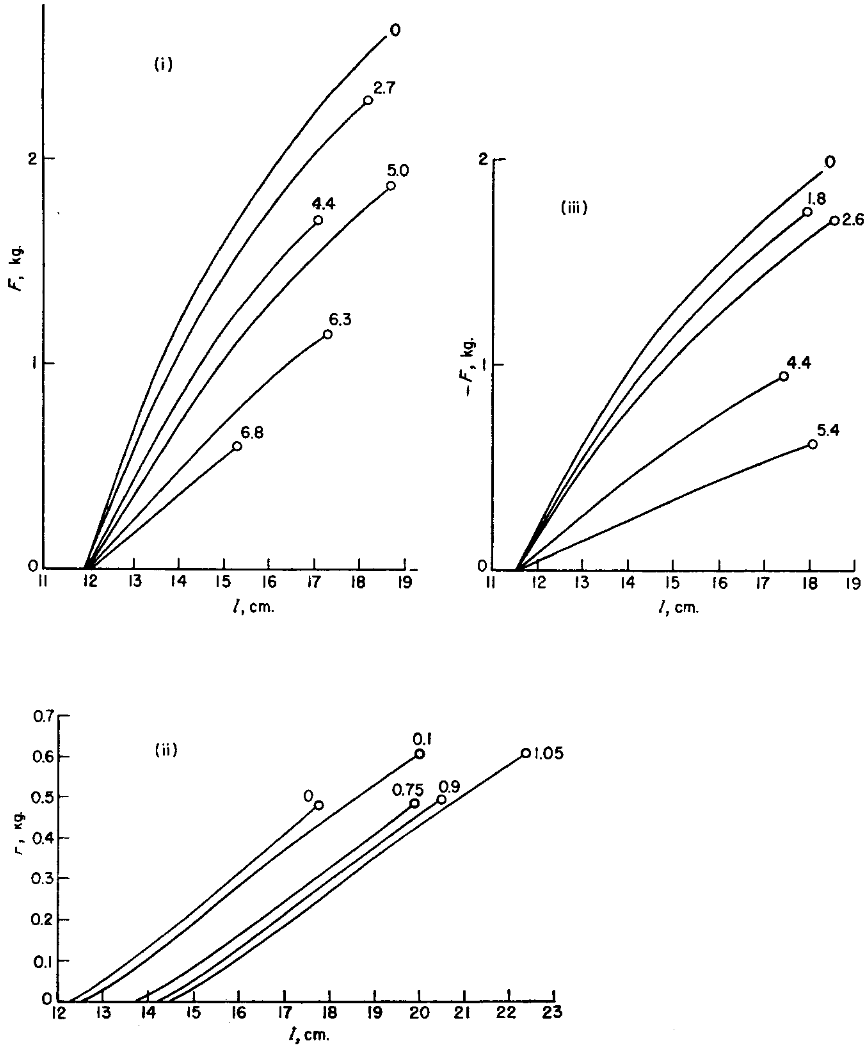

Fig. 2. Measured load-deformation relations for various cut-lengths *c*. Circles denote points at which catastrophic tearing occurred. Numbers against curves give values of *c* in cm.

defined by the occurrence of a catastrophic increase in cut-length at constant overall deformation of the test-piece. The test-pieces were deformed in a tensometer by separating the clamps in the directions shown by the arrows in Figure 1 and the force necessary to do this was measured. The overall deformation was measured by the change in the distance l between the clamps. The deformed test-pieces were, of course, not flat, but were subject to a greater or lesser degree of buckling.

The measured deforming forces F are plotted against the distances l between the clamps in Figures 2(i), (ii), and (iii) for test-pieces of the types shown in Figures 1(i), (ii), and (iii) respectively with various lengths of cut c. In each case the point at which the test-piece tore is marked by a small circle on the appropriate curve.

The work done in deforming a test-piece up to an overall length l is given by the area between its load-deformation curve and the l-axis from the point at which this curve intercepts the l-axis to the value of l considered. Since the vulcanizate used was substantially reversible elastically, this work is equal to the energy W stored elastically in the test-piece at the overall deformation l. Of course, when the test-piece is deformed, the material in the immediate neighborhood of the tip of the cut, being in a highly strained state, is partially crystalline. This results in some slight irreversibility, in addition to that resulting from the small amount of hysteresis to which the remainder of the test-piece is subject, but owing to the very lim-

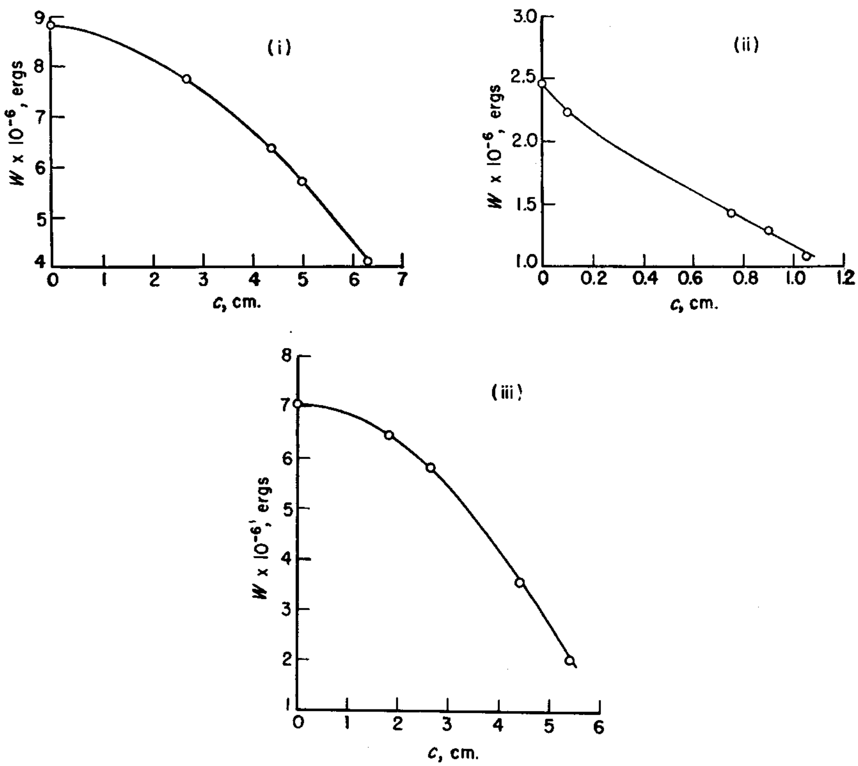

Fig. 3. Relations between W and c for a given value of the overall deformation. Calculated from the curves of Figure 2.

ited extent of the strain concentration the effect on the reversibility of the force-deformation curve is inappreciable.

From each of the sets of curves given in Figures 2(i), (ii), and (iii), the energy W stored elastically at a suitable value of l ($= l_0$) was found, for each cut-length c employed, in the manner described above. Values of l_0 of 18, 20, and 18 cm. were employed in the cases (i), (ii), and (iii) respectively. In the cases where the test-pieces tore before the overall deformation, at which W was evaluated, was reached, the appropriate force-deformation curve was extrapolated to this point. The values of W so obtained from Figures 2(i), (ii), and (iii) are plotted against c in Figures 3(i), (ii), and (iii) respectively. From Figures 2(i), (ii), and (iii), the values of l at which tearing occurred can be plotted against c for each of the three test-pieces. In each case there is a good deal of scatter in these points reflecting the inherent irreproducibility of the tearing process. However, straight lines may be drawn through them and, from these, the values of c for which tearing would have occurred at the overall deformations for which Figures 3(i), (ii), and (iii) are plotted, *i.e.*, for $l_0 = 18$, 20, and 18 cm. respectively, can be found. At these values of c the tangents to the corresponding W-c curves in Figures 3 are drawn and their slopes give the appropriate values of $(\partial W/\partial c)_l$ at the instant at which tearing occurs. From these and the measured thicknesses t of the sheets used, the characteristic energies T for tearing were found from equation (2.2). The values of c, at which the slopes of the curves in Figure 3 are taken, are only approximately determined in the manner outlined above, particularly in the case of the test-piece of the type shown in Figure 1 (iii). The resulting inaccuracies in the values of T are, however, lessened by the fact that the slopes of the W-c curves change rather slowly for values of c in the neighborhood of those concerned.

The values of T obtained for the test-pieces shown in Figures 1(i), (ii), and (iii) were 1.2×10^7, 1.2×10^7, and 1.4×10^7 ergs/cm.^{2} respectively. In view of the inaccuracies involved in the calculation of these figures from the experimental results and the inherent irreproducibility of the tearing process, these values of T, which are probably determined by the experiments to only $\pm 30\%$, may be considered to be in agreement.

Thus, for the three test-pieces employed, the cut-lengths c at which tearing occurred at a specified overall deformation can be interpreted in terms of a substantially constant characteristic energy of magnitude 1.3×10^7 ergs/cm.^{2} approximately.

5. DIRECT VERIFICATION OF THE TEAR CRITERION FOR INCIPIENT TEARING

It has been remarked in Section 3 that when a test-piece of a natural rubber vulcanizate, having a cut in it, is deformed to the point at which it tears, catastrophic rupture, in which the cut-length increases suddenly by a finite amount at constant overall deformation of the test-piece, is preceded by a condition in which the cut-length increases continuously with the deformation.

In the experiments described in Section 4, the point at which tearing occurred was defined by the occurrence of catastrophic rupture. The value of T obtained from such experiments is, of course, a characteristic energy for catastrophic rupture of the material. In the present section the results will be given of similar experiments in which the instant of tearing is defined by the first visible occurrence of an increase in cut-length. This was determined by applying ink to the tip of the cut and observing it through a low-power microscope as the test-piece was deformed. The first appearance of freshly torn, uninked rubber in the inked area was taken as the point at which tearing occurred. Preliminary experiments indicated that this point was at least as reproducible as that at which catastrophic rupture occurred.

The test-pieces used in these experiments were similar to those used in the previous experiments both in shape and composition. The mean thicknesses of the test-pieces of the types shown in Figures 1(i), (ii), and (iii) were, however, 1.54, 1.55, and 1.48 mm. respectively. The use of the first visible occurrence of tearing, instead of the catastrophic increase in cut-length, as the criterion of tearing had the advantage that the load-overall deformation curves could be measured for each specimen well beyond this point, since the increase in cut-length between this point and that at which catastrophic rupture occurs is sufficiently small to have an inappreciable effect on the load-deformation relation. The work done up to the point at which tearing occurred could therefore be found by graphical integration, without having to extrapolate these curves.

The load-overall deformation curves measured for specimens of the type shown in Figures 1(i), (ii), and (iii) are shown in Figures 4(i), (ii), and (iii) respectively. The bars against each curve denote the overall lengths of the test-pieces at which the first visible signs of tearing occurred in each case. Their extensions are a measure of the uncertainty in the determination of these points. In each case W, the work done to extend the test-piece to given overall lengths l, was found by graphical integration for the values of cut-length c used in the F-l curves of Figures 4(i), (ii), and (iii). The results obtained are plotted in Figures 5(i), (ii), and (iii) respectively.

We could proceed to find the values of T, the characteristic energy corresponding to the first visible occurrence of tearing, in each of the experiments and for each value of l used in drawing the curves of Figure 5, as was done in Section 3. However, an alternative course has been adopted here.

A value for T, the characteristic energy for tearing of the vulcanizate, is assumed and the corresponding critical value of $(\partial W/\partial c)_l$ is obtained from equation (2.2). For each of the W-c curves of Figure 5, the value of the cut-length c is found at which the slope of the curve has this critical value of $(\partial W/\partial c)_l$. The corresponding values of F are then found by interpolation from the curves of Figure 4. We thus obtain, for the assumed value of T, corresponding values of the deforming force F and cut-length c at which the tear criterion (2.2) predicts that tearing should occur for each of the test-pieces. This procedure is repeated for a number of different values of T. In Figures 6(i), (ii), and (iii), the F-c relations so obtained

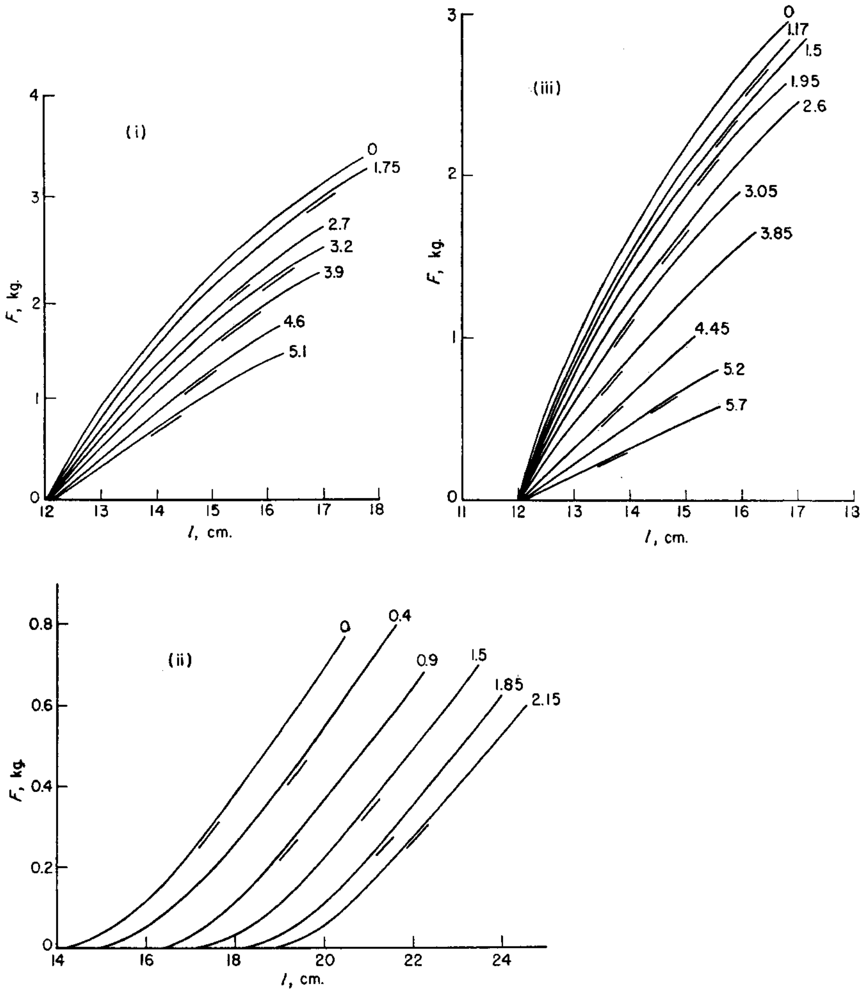

Fig. 4. Measured load-deformation relations for various cut-lengths c. Bars denote points at which incipient tearing occurred. Numbers against curves give values of c in cm.

with an assumed value of T of 3.7×10^6 ergs/cm.^{2}, which was found to give the best agreement with the experimental points, are shown by the full lines for the test-pieces shown in Figures 1(i), (ii), and (iii) respectively. The bars denote observed values of F, obtained from Figure 4, at which tearing occurred, at various values of c, for the three test-pieces. It is

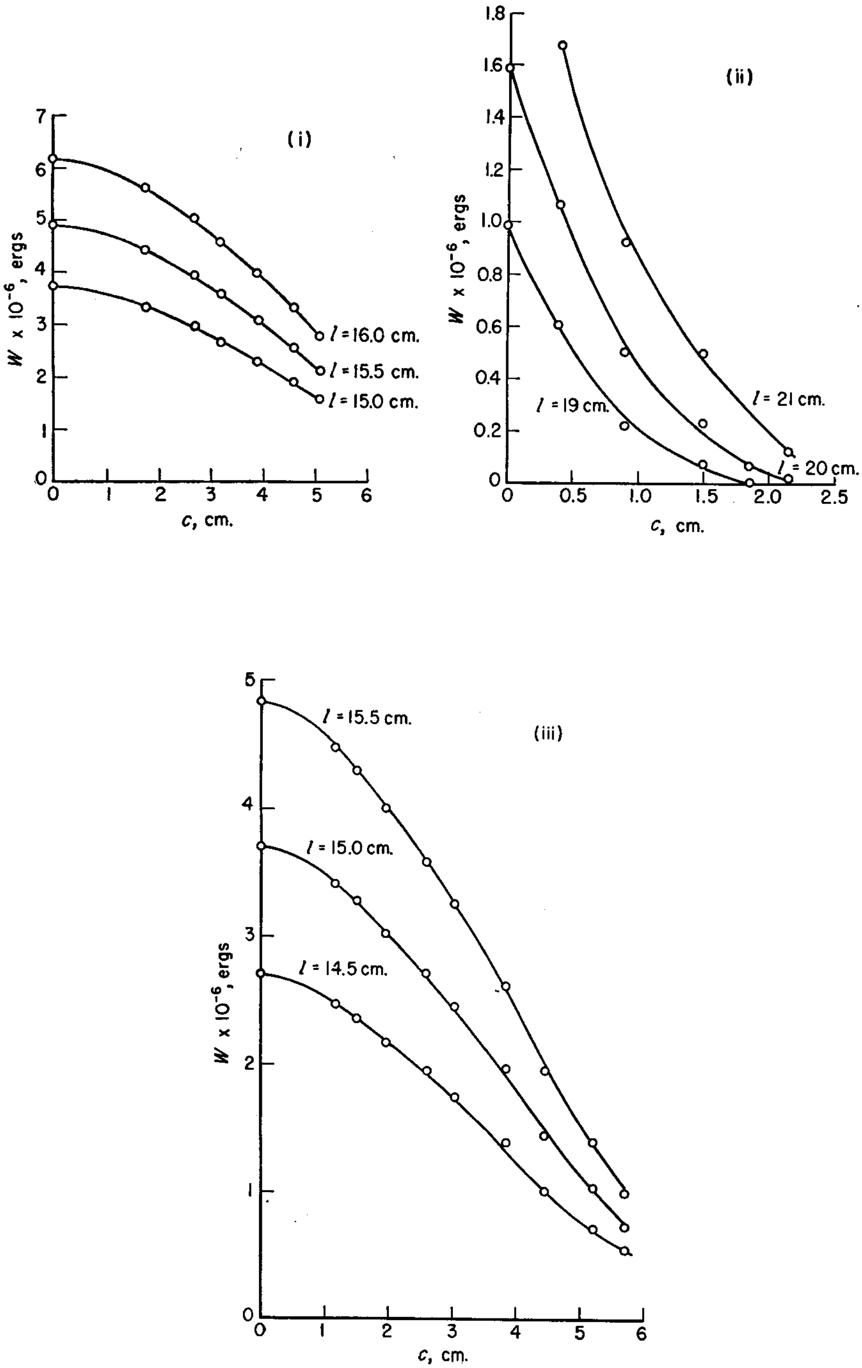

Fig. 5. Relations between W and c for given values of the overall deformation. Calculated from the curves of Figure 4.

seen that the F-c curves calculated on the basis of an assumed value of T of 3.7×10^6 ergs/cm.^{2} give an adequate representation of the experimental

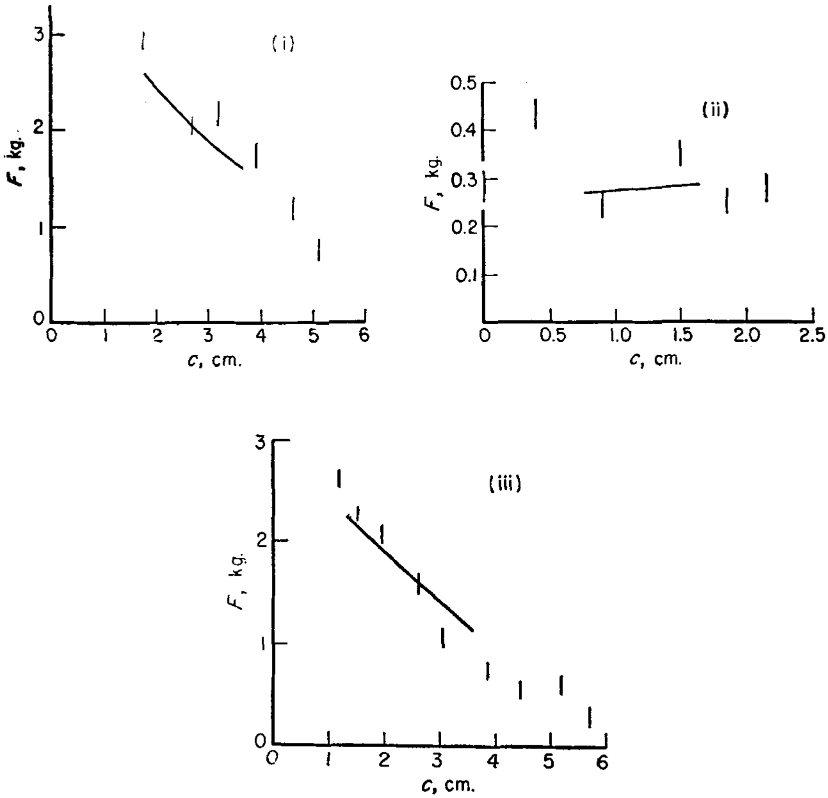

Fig. 6. Values of F at which tearing occurred for various values of c. The bars denote experimental values. The curves represent the calculated relations on the assumption that $T = 3.7 \times 10^6$ ergs/cm.^{2}.

points in view of the inherent inaccuracies involved in determining the latter. It is noteworthy that the qualitative dependence of F on c at the instant of tearing is reasonably well represented in all three cases, as are the absolute magnitudes of the tearing forces F.

6. THE CALCULATION OF $(\partial W/\partial c)_l$

In the experiments described in Sections 4 and 5 the relations between the elastically stored energy W and cut length c for various overall lengths l of the test-pieces were obtained experimentally. From these a characteristic energy for tearing T at a given overall length can be found from the slope of the corresponding W-c curve at the value of c at which tearing occurs. Alternatively, from a knowledge of T and the measured W-c

curves, the points at which tearing would occur for a given value of c can be calculated. Either process is somewhat laborious and subject to considerable inaccuracy as it involves the measurement of force-overall length relations for a test-piece with a range of values of cut-length and graphical integration of the curves so obtained.

In this section it will be shown that, by suitable choice of the form of the test-piece, $(\partial W/\partial c)_l$ can be calculated as a function of c and l from the known elastic behavior of the vulcanizate.

We shall first consider a test-piece of the type shown in Figure 1(ii). If the length of the cut is sufficiently large compared with the half-width of the specimen, then when it is deformed a considerable region (denoted A

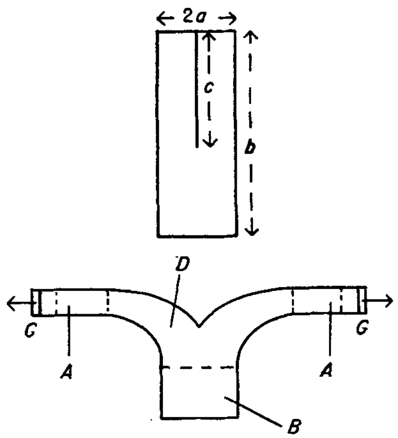

Fig. 7. Schematic diagram of "simple extension" tear test-piece.

in Figure 7) of each of the arms by which it is held is substantially in simple extension. Let λ be the extension ratio in this region. The strain distributions in the neighborhood of the clamps C and in the neighborhood D of the tip of the cut are complicated in character. However, provided the uncut portion of the test-piece is also sufficiently long, there is a region B which is substantially undeformed. A small increase dc in the cut-length c at constant applied force F increases the size of the region A in simple extension at the expense of the undeformed region B while leaving the strain distributions in the neighborhood of the tip of the cut and of the clamps unaltered. A test-piece of the type considered will therefore be referred to as a "simple extension" test-piece. The increase in volume of the regions A at the expense of the region B is $A_0 \cdot dc$, where A_0 is the cross-sectional area of the test-piece in its undeformed state. (The cross-sectional area of each arm by which it is held is $\frac{1}{2}A_0$.) The extension ratio of the material in the region A is unaltered since it depends only on the applied force F. It is readily seen that the overall length l between the clamps will increase by an amount dl, given by $2\lambda \cdot dc$, so that we have

$$\left(\frac{\partial l}{\partial c}\right)_F = 2\lambda \tag{6.1}$$

The energy W stored elastically in the test-piece is a function of c and l and therefore the change dW in this energy, due to changes dc and dl in cut-length c and overall length l respectively, is given by

$$dW = \left(\frac{\partial W}{\partial c}\right)_l dc + \left(\frac{\partial W}{\partial l}\right)_c dl$$

which yields, with $F = (\partial W/\partial l)_c$,

$$\left(\frac{\partial W}{\partial c}\right)_F = \left(\frac{\partial W}{\partial c}\right)_l + F\left(\frac{\partial l}{\partial c}\right)_F \tag{6.2}$$

Equation (6.2) yields, with (6.1),

$$\left(\frac{\partial W}{\partial c}\right)_l = \left(\frac{\partial W}{\partial c}\right)_F - 2\lambda F \tag{6.3}$$

From this equation $(\partial W/\partial c)_l$ can be determined as a function of λ if F and $(\partial W/\partial c)_F$ can be determined in terms of λ. It can then be determined as a function of F from the relation between λ and F.

Rivlin[4] has shown that for a virtually incompressible elastic material, such as a rubber vulcanizate, which is isotropic in its undeformed state, the energy W_0 stored elastically per unit volume of the material, when it is subjected to a pure homogeneous deformation for which the extension ratios are λ_1, λ_2, and λ_3 respectively, is a function of I_1 and I_2, which are defined by

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad \text{and} \quad I_2 = \lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2} \tag{6.4}$$

For a simple extension of extension ratio λ, we have

$$I_1 = \lambda^2 + (2/\lambda) \quad \text{and} \quad I_2 = (1/\lambda^2) + 2\lambda \tag{6.5}$$

The force F necessary to maintain this simple extension in a test-piece of cross-sectional area $^1/_2\, A_0$ in its undeformed state is given by

$$F = A_0\left(\lambda - \frac{1}{\lambda^2}\right)\left(\frac{\partial W_0}{\partial I_1} + \frac{1}{\lambda}\frac{\partial W_0}{\partial I_2}\right) \tag{6.6}$$

and this expression therefore gives the relation between the applied force and extension ratio λ in the regions A for the test-piece shown in Figure 7.

Since an increase in cut-length dc under conditions of constant applied force F transfers a volume $A_0 \cdot dc$ of rubber from the undeformed state to one of simple extension, the change dW in the energy stored elastically in the test-piece is $W_0 A_0 \cdot dc$. Thus,

$$\left(\frac{\partial W}{\partial c}\right)_F = W_0 A_0 \tag{6.7}$$

Introducing the relation (6.7) into (6.3), we obtain

$$\left(\frac{\partial W}{\partial c}\right)_l = W_0 A_0 - 2\lambda F \tag{6.8}$$

We shall now consider a test-piece of the form shown in Figure 1(iii). If the width of the test-piece is sufficiently great compared with its length l_0 (*i.e.*, the separation between the clamps) and the length of the cut is also sufficiently great compared with the latter, as shown in Figure 8, then when

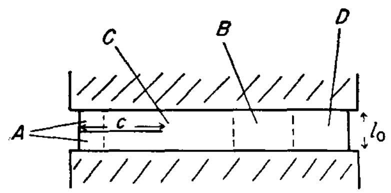

Fig. 8. Schematic diagram of "pure shear" tear test-piece.

the test-piece is deformed, by separating the clamps, in a direction parallel to the dimension l_0, the regions A of the test-piece are substantially undeformed, the region B is in a state of pure shear, and the region C lying between A and B is in a complicated state of strain. Further, a slight departure from pure shear takes place close to the force-free edge of the region D. Provided that the overall separation between the clamps is unchanged, so that the extension ratio λ defining the amount of pure shear in the region B is unchanged, an increase in the cut length of amount dc (measured in the undeformed state of the test-piece) does not alter the state of strain in the region C but merely shifts this region parallel to the direction of the cut, causing the regions A to grow at the expense of the region B. Thus, an increase in cut-length dc transfers a volume $l_0 t \cdot dc$ of the rubber from a state of pure shear to the undeformed state, l_0 being the length of the test-piece between the clamps and t its thickness, both quantities being measured in the undeformed state. A tearing test-piece of the type shown in Figure 8 will therefore be referred to as a "pure shear" test-piece.

For a pure shear, the extension ratios in the three principal directions are λ, 1, and $1/\lambda$, λ (> 1) being that in the direction of the applied force F. For such a deformation we readily see that I_1 and I_2 in the expression for the stored-energy function are given by

$$I_1 = I_2 = \lambda^2 + 1 + (1/\lambda^2) \tag{6.9}$$

The change dW in the energy W, stored elastically in the test-piece due to a change in cut-length dc, is thus given by $-W_0 l_0 t \cdot dc$, where W_0 is now the energy stored elastically per unit volume of the material in a state of pure shear of amount defined by an extension ratio λ. W_0 is found from the expression for the stored-energy function of the material in terms of I_1 and I_2 by employing the expressions (6.9) for I_1 and I_2. We then have

$$\left(\frac{\partial W}{\partial c}\right)_l = -W_0 l_0 t \qquad (6.10)$$

In general, W_0 can be obtained by graphical integration under the load-deformation curve for pure shear of a test-piece of the material considered.

7. EXPERIMENTAL RESULTS FOR THE "SIMPLE EXTENSION" TEST-PIECE

For the "simple extension" test-piece, shown schematically in Figure 7, it has been shown that $(\partial W/\partial c)_l$ is given in terms of the extension ratio λ in the regions A of the arms by which the test-piece is held, or the applied force F, by the expressions (6.8) and (6.6). The critical value of λ, or of F, at which tearing occurs is then that for which $(\partial W/\partial c)_l$ satisfies the relation (2.2), in which the characteristic energy for tearing T has the value appropriate to the particular vulcanizate employed. We can then find the value of T for a particular vulcanizate by measuring the value of F at which a test-piece of the type shown in Figure 7 tears. The value of $(\partial W/\partial c)_l$ corresponding to this value of F may then be found from equations (6.8) and (6.6) and, from (2.2), the value of T for the vulcanizate may be found. Experiments of this type were carried out for a number of test-pieces of the type shown in Figure 7 having different widths, so that tearing occurred at different values of F and λ.

In order to find from equations (6.8) and (6.6) the values of $(\partial W/\partial c)_l$, corresponding to various values of F and λ for the test-pieces used, it is necessary to find experimentally, for the particular vulcanizate, the relations between load L and extension ratio λ, and between the energy W_0 stored elastically per unit volume and λ, when a strip of the vulcanizate is subjected to simple extension. It will be seen that the relation between W_0 and λ can be found with sufficient accuracy from the relation between L and λ by calculation.

In order to determine the values of F for which tearing occurs, a number of test-pieces of the type shown in Figure 7 of different widths were prepared from a single sheet of the vulcanizate of thickness about 2 mm. This was prepared by a molding process according to recipe A given in the Appendix. The mean thickness t was measured for each of the test-pieces. The lengths of the test-pieces in their undeformed state (b in Figure 7) were about 15 cm. and their widths ($2a$ in Figure 7) varied from 0.75 to 3.2 cm. The cut-length c varied from 8 to 12 cm. These dimensions are such as to satisfy the condition that the regions A are substantially in simple extension and the region B is substantially undeformed when the test-piece is subjected to a deformation. Each of the test-pieces was extended slowly in the tensometer until the point was reached at which catastrophic tearing first occurred and the corresponding value of the force F was measured. In each case the tip of the cut was formed with a razor blade. For each test-piece six such measurements were made and their mean taken. In order to do this it was, of course, necessary to use increasing values of c. It will, however, be noted from equations (6.8) and (6.6) that $(\partial W/\partial c)_l$ is not dependent on the value of c and, within the range of cut-lengths employed, no such dependence of the force F necessary to produce tearing was observed. The means of the measured values of F at which tearing occurred for the various test-pieces are given in Table I together with their mean thicknesses t and widths $2a$.

TABLE I

Reference number of test-piece:	1	2	3	4	5
Thickness t, mm.	1.93	1.93	1.93	1.85	1.93
Width $2a$, cm.	3.2	2.4	1.6	1.2	0.75
Mean applied force for tearing F, kg.	1.01	0.92	0.78	0.71	0.62

In order to obtain the required experimental relation between load L and extension ratio λ for simple extension, a test-piece of the vulcanizate was used having the form of a thin strip cut from the same sheet as the test-pieces used in the tearing experiments. The technique of measurement was similar to that employed by Rivlin and Saunders.[5] From equation (6.6) the relation between L and λ is given by

$$L = 2A_0\left(\lambda - \frac{1}{\lambda^2}\right)\left(\frac{\partial W_0}{\partial I_1} + \frac{1}{\lambda}\frac{\partial W_0}{\partial I_2}\right)$$

where A_0 is now the cross-sectional area of the strip in its undeformed state. In this expression I_1 and I_2 are given in terms of λ by (6.5). The results obtained are shown in Figure 9 in which

$$[\partial W_0/\partial I_1 + (1/\lambda)(\partial W_0/\partial I_2)] = [L/2A_0(\lambda - 1/\lambda^2)]$$

is plotted against $1/\lambda$. It is seen that the relation is linear. It has been pointed out by Rivlin and Saunders[5] that this does not necessarily imply that $\partial W_0/\partial I_1$ and $\partial W_0/\partial I_2$ are constant. Nevertheless, for the purposes of the interpretation of the present experiments, we can write as an empirical fact

$$\frac{L}{2A_0(\lambda - 1/\lambda^2)} = \frac{\partial W_0}{\partial I_1} + \frac{1}{\lambda}\frac{\partial W_0}{\partial I_2} = C_1 + \frac{1}{\lambda}C_2 \qquad (7.1)$$

where C_1 is the intercept of the straight line in Figure 9 on the straight line $1/\lambda = 0$ and $(C_1 + C_2)$ is its intercept on the line $1/\lambda = 1$. From Figure 9, we obtain $C_1 = 1.3$ kg./cm.^{2} and $C_2 = 0.96$ kg./cm.^{2}. From (7.1) we can obtain the work required to extend the test-piece to extension ratio λ and hence the energy W_0 stored elastically per unit volume at this extension ratio. Thus,

$$W_0 = \frac{1}{A_0}\int_1^\lambda L\cdot d\lambda = C_1\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + C_2\left(\frac{1}{\lambda^2} + 2\lambda - 3\right) \quad (7.2)$$

The relation (7.1) is empirically valid only over the range of values of λ covered by the experiment of which the result is given in Figure 9, so that in deriving (7.2) an extrapolation of the relation (7.1) is involved which is not strictly valid. However, the work of Rivlin and Saunders[5] indicates that only a very slight error will be involved in making this extrapolation. It may be noted, too, that if λ is nearly unity then in equation (6.8) F and W_0 are approximately proportional to $(\lambda - 1)$ and $(\lambda - 1)^2$ respectively so that, for sufficiently small values of $(\lambda - 1)$, the term $2\lambda F$ is predominant in determining $(\partial W/\partial c)_l$.

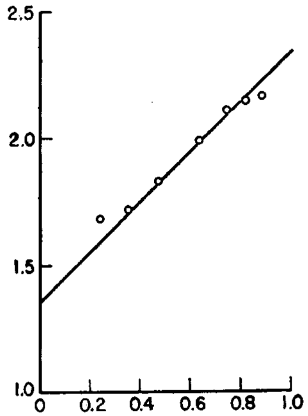

Fig. 9. Experimental relation between $\partial W_0/\partial I_1 + (1/\lambda)(\partial W_0/\partial I_2)$ (in kg./cm.^{2}, ordinate) and $1/\lambda$ (abscissa) for simple extension of vulcanizate A.

Introducing the relations (7.1) and (7.2) into equations (6.6) and (6.8), and writing $\alpha = C_2/C_1$ and $A_0 = 2\,at$, we obtain

$$\frac{F}{2\,atC_1} = \left(\lambda - \frac{1}{\lambda^2}\right)\left(1 + \frac{\alpha}{\lambda}\right)$$

and

$$\frac{(\partial W/\partial c)_l}{2\,atC_1} = -\frac{\lambda F}{atC_1} + \left[\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + \alpha\left(\frac{1}{\lambda^2} + 2\lambda - 3\right)\right] \quad (7.3)$$

By assuming various values of λ these relations can be used to plot $(\partial W/\partial c)_l/2\,atC_1$ against $F/2\,atC_1$ for the value of $\alpha(= C_2/C_1) = 0.74$ which was found experimentally for the vulcanizate. The curve so obtained is shown in Figure 10. For each of the test-pieces on which the force F necessary to produce tearing was measured, we can find the value of $(\partial W/\partial c)_l$ at

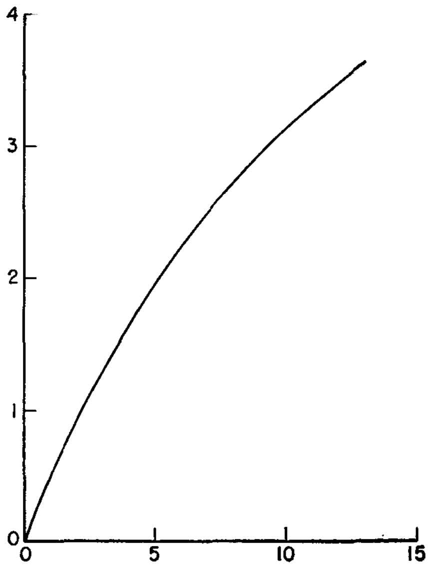

Fig. 10. Theoretical relation between $(\partial W/\partial c)_l/2atC_1$ (abscissa) and $F/2atC_1$ (ordinate) for "simple extension" test-piece.

which tearing occurred from the curve of Figure 10, employing the experimentally determined value of $C_1(= 1.3$ kg./cm.$^2)$. Hence, by means of equation (2.2), the characteristic energy for tearing T can be found. The values obtained for T are given in Table II.

TABLE II

Reference number of test-piece:	1	2	3	4	5
Characteristic energy for tearing T in 10^7 ergs/cm.^{2}	1.2	1.2	1.0	1.1	1.1

Introducing the relation (2.2) into the second of equations (7.3), we obtain

$$\frac{\lambda F}{atC_1} = \frac{T}{2\,aC_1} + \left[\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + \alpha\left(\frac{1}{\lambda^2} + 2\lambda - 3\right)\right] \tag{7.4}$$

in which T is substantially constant for test-pieces of a given vulcanizate. Substituting from the first of equations (7.3) in (7.4), we obtain the following equation for the value of λ at which the test-piece tears:

$$\left(\lambda^2 - \frac{4}{\lambda} + 3\right) + 3\alpha\left(1 - \frac{1}{\lambda^2}\right) = \frac{T}{2\,aC_1} \tag{7.5}$$

For sufficiently large values of a, at which $T/2aC_1 \ll 1$, λ becomes very nearly unity and is given approximately by

$$\lambda - 1 = \frac{T}{12aC_1(1 + \alpha)} \tag{7.6}$$

Then, since the term $\left[\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + \alpha\left(\frac{1}{\lambda^2} + 2\lambda - 3\right)\right]$ is of the order $(\lambda - 1)^2$, while $T/2aC_1$ is of the order $\lambda - 1$, we can neglect the former in comparison with the latter. We then see from (7.4) that, since $\lambda \approx 1$, F is very nearly independent of a and of C_1 for sufficiently large values of a, having the value $^1/_2Tt$.

8. EXPERIMENTAL RESULTS FOR THE "PURE SHEAR" TEST-PIECE

The tear criterion could be verified and a value for the characteristic energy for tearing T could be obtained for the "pure shear" test-piece, shown schematically in Figure 8, in a manner somewhat similar to that described for the "simple extension" test-piece in Section 7. Here, however, we shall predict the results of tear experiments carried out on "pure shear" test-pieces from the values of T found from measurements on "simple extension" test-pieces of the same vulcanizates by the method described in Section 7. The predicted values of the extension ratio λ, defining the amount of pure shear at which tearing occurs, will be compared with those measured. By this means we can not only verify the internal consistency of the tear criterion when applied to tear specimens of a particular type, as was done in the experiments described in Section 7, but can show that a single tear criterion can be applied to predict the results of experiments on test-pieces of different forms.

In these experiments two vulcanizates were used. They were prepared by a molding process as sheets about 1 mm. thick, recipes B and C given in the Appendix being used. In each case, "simple extension" test-pieces and "pure shear" test-pieces were cut from a single sheet.

It can be seen from equations (6.10) and (2.2) that for a "pure shear" test-piece tearing occurs when

$$W_0 l_0 = T \tag{8.1}$$

where W_0 is the elastically stored energy, per unit volume of the material, at the extension ratio λ defining the pure shear at which tearing occurs in the test-piece. The manner in which W_0 depends on λ can be found from the load-deformation relation for pure shear determined experimentally for the vulcanizate considered.

The force F necessary to maintain a pure shear defined by the extension ratio λ is given by

$$F = 2A_0\left(\lambda - \frac{1}{\lambda^3}\right)\left(\frac{\partial W_0}{\partial I_1} + \frac{\partial W_0}{\partial I_2}\right) \tag{8.2}$$

in which I_1 and I_2 are given by equations (6.9) and A_0 is the cross-sectional area of the test-piece in its undeformed state. The relation between F and λ was determined experimentally for each of the vulcanizates, in a manner similar to that adopted by Rivlin and Saunders,[5] using test-pieces cut from the same sheets as those from which the "pure shear" and "simple extension" test-pieces used for the tearing experiments were cut. From these measurements F/A_0 was plotted against $(\lambda - 1/\lambda^3)$ for each of the vulcanizates, as shown in Figure 11. It is seen that in each case a substantially linear relation is obtained. From an empirical point of view we may

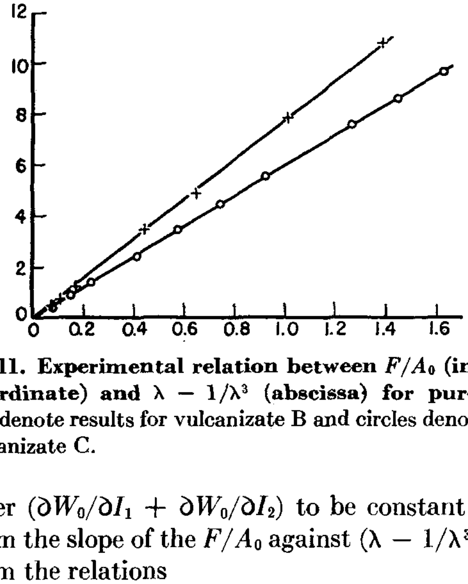

Fig. 11. Experimental relation between F/A_0 (in kg. per cm.^{2}, ordinate) and $\lambda - 1/\lambda^3$ (abscissa) for pure shear. Crosses denote results for vulcanizate B and circles denote results for vulcanizate C.

therefore consider $(\partial W_0/\partial I_1 + \partial W_0/\partial I_2)$ to be constant in each case and find its value from the slope of the F/A_0 against $(\lambda - 1/\lambda^3)$ curve. We can then find W_0 from the relations

$$W_0 = \int_1^\lambda \frac{F}{A_0}\,d\lambda = \left(\frac{\partial W_0}{\partial I_1} + \frac{\partial W_0}{\partial I_2}\right)\left(\lambda^2 + \frac{1}{\lambda^2} - 2\right) \tag{8.3}$$

It might at first sight appear that the experimental results shown in Figure 11 disagree with the load-deformation characteristics in pure shear obtained for a rubber vulcanizate by Rivlin and Saunders.[5] These indicated a decrease in $(\partial W_0/\partial I_1 + \partial W_0/\partial I_2)$ with increasing λ. However, it must be borne in mind that the vulcanizates employed in the present experiments are considerably harder than that to which the results of Rivlin and Saunders[5] apply and it has been shown[6] that the fractional deviation from constancy in $(\partial W_0/\partial I_1 + \partial W_0/\partial I_2)$ decreases with increase in the hardness of the vulcanizate. In the light of these experiments it is not to be expected that the deviation from linearity of the F/A_0 against $(\lambda - 1/\lambda^3)$ relationship would be detectable in experiments of the present degree of accuracy.

In carrying out the experiments for the determination of the deformation necessary to produce tearing in "pure shear" test-pieces, test-pieces about 10 cm. wide and of lengths (measured between the lines along which they were clamped) varying from 2.2 to 0.65 cm. were used. Two reference lines were drawn on each test-piece parallel to its width and the test-piece was held in wide clamps along these lines. A cut about 4 cm. long was made parallel to the width and its tip was formed with a razor blade. The test-piece was supported vertically by one clamp and stretched slowly by applying increasing loads to the other clamp until catastrophic rupture occurred. During this stretching process care was taken to maintain parallelism of the clamps. The value of λ at catastrophic rupture was found as the ratio between the separation of the reference lines, in the region B of the test-piece (see Figure 8), at the instant of rupture and in the undeformed state. For each test-piece a number of such measurements were made and the mean value of λ was taken as characterizing the deformation at which tearing occurred. In successive measurements on the same test-piece it was perforce necessary to employ increasing cut-lengths, a procedure which is admissible since the value of λ at tear, for this type of test-piece, is not dependent on the cut-length. In each case, however, the tip of the cut was formed afresh with a razor blade.

In Table III the mean values of λ at the instant of tearing, found for various values of the length l_0 between the clamps in the undeformed state, are given. The values of $\partial W_0/\partial I_1 + \partial W_0/\partial I_2$ found from the curves of Figure 11 for the vulcanizates prepared according to recipes B and C were 3.9 and 3.0 kg./cm.^{2} respectively. The values of T obtained from measurements on the "simple extension" tear test-pieces prepared from these vulcanizates were 1.24×10^6 and 5.7×10^6 ergs/cm.^{2} respectively. It was found possible to make at least six measurements on each of the test-pieces prepared according to recipe B, while for those prepared according to recipe C fewer measurements were possible, in general, on account of the greater increase in cut length at each catastrophic tearing.

TABLE III

Recipe B	l_0, cm.	0.65	1.1	1.55	2.2	
	λ at instant of tearing	1.42	1.285	1.26	1.215	
Recipe C	l_0, cm.	1.0	1.0	1.2	1.4	1.8
	λ at instant of tearing	1.74	1.87	1.75	1.65	1.76

From equations (8.1) and (8.3) the criterion for tearing can be written in the form

$$\left(\frac{\partial W_0}{\partial I_1} + \frac{\partial W_0}{\partial I_2}\right)\frac{l_0}{T} = \frac{1}{\lambda^2 + \lambda^{-2} - 2} \tag{8.4}$$

The full line in Figure 12 represents a plot of $1/(\lambda^2 + \lambda^{-2} - 2)$ against λ and hence of the values predicted for $(\partial W_0/\partial I_1 + \partial W_0/\partial I_2)(l_0/T)$ against λ. The points denote the values of $(\partial W_0/\partial I_1 + \partial W_0/\partial I_2)(l_0/T)$ calculated from the experimental data given in Table III. It is seen that there is good agreement between the theoretically predicted and experimentally determined values.

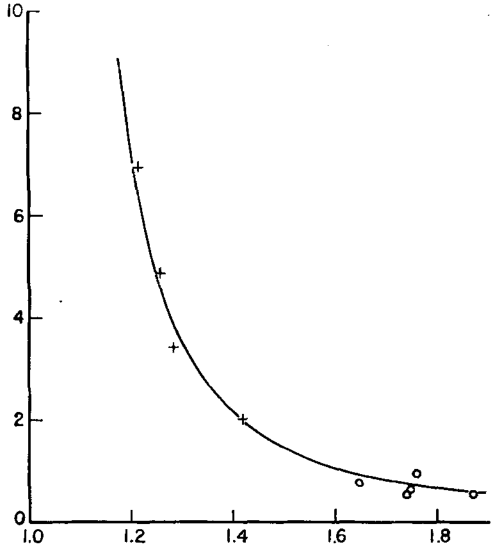

Fig. 12. Experimental and theoretical relations between $(\partial W_0/\partial I_1 + \partial W_0/\partial I_2)(l_0/T)$ **(ordinate) and** λ **(abscissa).** Crosses denote experimental results for vulcanizate B and circles denote experimental results for vulcanizate C. Curve represents theoretical relation.

9. THEORETICAL CONSIDERATIONS FOR TEST-PIECE WITH A SMALL CUT

We shall now consider a test-piece of the type shown in Figure 1(iii) in which the cut-length c is small compared with the width of the test-piece, which is in turn small compared with its length. If the cut-length were zero, the central region of the test-piece would be substantially in simple extension. The presence of a small cut of length c on one edge produces a region in its immediate neighborhood where this is no longer the case. However, the central region of the sheet away from the cut is still in simple extension. We shall assume that λ is the extension ratio in this region.

In the neighborhood of the cut the strain distribution is complicated in character, but it is worth noting that at the intersection of the cut and the free edge of the test-piece the material is unstrained. If the cut is ideally sharp in the undeformed state of the test-piece, then it can be seen from dimensional considerations that the change in the elastically stored energy due to the presence of the cut will be proportional to c^2, since the pattern of strain distribution is uninfluenced by the cut-length but the linear scale on which this distribution must be mapped is proportional to c. Although this conclusion is only strictly correct if the cut is ideally sharp and is made in the edge of a semi-infinite sheet, it will be substantially valid for the dimensional assumptions made above, provided that the radius of curvature at the tip of the cut is small compared with its length c.

Let us denote the energy stored elastically in the test-piece in the absence of the cut by W', and with a cut of length c by W. Then

$$W' - W = Kc^2t \tag{9.1}$$

where K is a constant of proportionality and t is the thickness of the test-piece in its undeformed state. The proportionality of $W' - W$ with t is valid only if $t \ll c$, so that plane stress conditions obtain over substantially the whole of the sheet. The value of K will, in general, depend on λ.

If the material of the test-piece obeyed classical elasticity theory, then the displacements at all points due to the deformation would be proportional to $(\lambda - 1)$. Consequently, the energy $W' - W$ would be proportional to $(\lambda - 1)^2$, or to the energy W_0 stored elastically per unit volume of material in simple extension.

We could then write (9.1) in the form

$$W' - W = K'c^2tW_0, \tag{9.2}$$

where K' is a constant independent of λ. Of course, since the deformations to which the test-pieces are subjected are large, classical elasticity theory cannot be applied to them. However, it will still be convenient to write equation (9.1) in the form (9.2), where K' is now a function of λ.

As has already been seen in Section 7, for the pure gum vulcanizates employed, W_0 may be expressed in terms of λ by the formula

$$W_0 = C_1\left[\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + \alpha\left(\frac{1}{\lambda^2} + 2\lambda - 3\right)\right] \tag{9.3}$$

where C_1 and α are constants which can be determined experimentally from the load-deformation relation for the simple extension of a strip of the vulcanizate.

From equations (9.2) and (9.3), we see that

$$-\left(\frac{\partial W}{\partial c}\right)_l = 2K'ctC_1\left[\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + \alpha\left(\frac{1}{\lambda^2} + 2\lambda - 3\right)\right] \tag{9.4}$$

which yields with (2.2) the following criterion for tearing of the test-piece considered:

$$T = 2K'cC_1\left[\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + \alpha\left(\frac{1}{\lambda^2} + 2\lambda - 3\right)\right] \tag{9.5}$$

In order to make this criterion for tearing explicit, it is strictly necessary to know the value of K' and the manner in which this depends on λ. This problem can, in principle, be solved mathematically but in practice it proves intractable. Again, it is in principle possible to find the values of K' at various values of λ experimentally, by determining the dependence of W on c and λ from load-deformation relations for test-pieces with cuts of various lengths, as was done for the test-pieces employed in the experiments described in Section 4. However, with the present type of test-piece the cut-length c must always be small compared with the width of the test-piece. Consequently, the changes in the elastically stored energy due to the presence of the cut are only a few per cent and the slight irreversibility in the elastic properties of the rubber makes it impossible to obtain measurements of sufficient accuracy to yield an accurate value for K'. Such measurements as have been made indicate that K' has a value lying between 2 and 3.

When there is no cut in the test-piece its central region across its entire width is in simple extension, and the applied force F is given by

$$F = 2\,al\left(\lambda - \frac{1}{\lambda^2}\right)\left(\frac{\partial W_0}{\partial I_1} + \frac{1}{\lambda}\frac{\partial W_0}{\partial I_2}\right) \tag{9.6}$$

in which a is the width of the test-piece in its undeformed state and I_1 and I_2 are given by equations (6.5). As has already been seen in Section 7, we may express this relation by the formula

$$F = 2\,alC_1\left(\lambda - \frac{1}{\lambda^2}\right)\left(1 + \frac{\alpha}{\lambda}\right) \tag{9.7}$$

for the vulcanizates used in the experiments on simple extension test-pieces. If the cut-length c is small compared with the width a, then the magnitude of the applied force is changed to only a very small extent and equation (9.7) may still be taken as giving the relation between the applied force F and extension ratio λ in the region A.

If the value of α is known for the particular vulcanizate with which we are concerned, then a relation between $F/2atC_1$ and $2K'cC_1/T$ can be found from equations (9.5) and (9.7).

10. EXPERIMENTAL RESULTS FOR A TEST-PIECE WITH A SMALL CUT

It has been found that with test-pieces of the type considered in Section 9 the central region is substantially in simple extension except in the neighborhood of the cuts, when the clamps are separated, provided that in the unstrained state the length of the specimen is more than twice its width. Accordingly length/width ratios satisfying this condition were chosen for the test-pieces. It is more difficult to estimate the greatest permissible ratio of cut-length to width which will leave λ, the extension ratio in this region of the test-piece, and hence W_0 in equation (9.2), independent of the cut-length. However, it was found experimentally that if an uncut specimen is stretched, so that the value of λ is about 2 (say), and then cut about a fifth of the way across from one edge, the value of λ at the center of the opposite edge changed by less than $^1/_2$ per cent, while the force F necessary to maintain the overall deformation changed by less than 5%. It was considered that these changes in λ and F were sufficiently small to allow the relations (9.5) and (9.7) to be applied to the test-piece provided that c/a was less than about $^1/_5$ and accordingly such values of the cut length were not exceeded in the experiments.

In order to verify the relations derived in Section 9, three different vulcanizates were used. A thin sheet of each of these was prepared by a molding process according to the recipes A, D, and E given in the Appendix.

The elastic constants C_1 and α were found for each sheet, using a small test-piece cut from it, in the manner described in Section 7. A "simple extension" type of tear test-piece, similar to those employed in the experiments in Section 7, was then cut from each sheet and the characteristic energy T for each of the vulcanizates was found from measurements similar to those described in Section 7.

Test-pieces of the type considered in Section 9 were then prepared from each of the three sheets. A cut was then made in each of them with a razor blade. Their widths a, mean thicknesses t, and cut-lengths c were then measured, the measurement of c being made with a vernier microscope.

Each test-piece so prepared was then extended in a tensometer until catastrophic tearing occurred and the corresponding force F was measured. The measurement was repeated with various values of the cut-length, the tip of the cut being formed in each case with a razor blade.

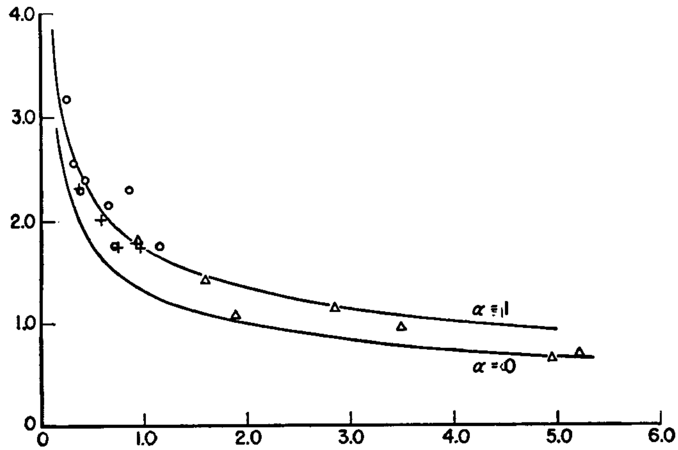

Fig. 13. Experimental and theoretical relations between $F/2aC_1t$ (ordinate) and $2K'cC_1/T$ (abscissa) for test-piece with small cut. Circles denote results for vulcanizate A, crosses for vulcanizate D, triangles for vulcanizate E.

The experimental results are given in Tables IV and V.

In Figure 13 the full lines represent the relation between $F/2aC_1t$ and $2K'cC_1/T$ predicted theoretically in Section 9 for values of α of 0 and 1. Corresponding values of these quantities, calculated from the results given in Tables IV and V, are also given in Figure 13, a value of $K' = 2$ which gave the closest agreement with the theoretical predictions being used.

TABLE IV

Vulcanizate	C_1, kg./cm.^{2}	α	T, ergs/cm.^{2}
A	1.00	0.9	5.2×10^6
D	1.42	0.74	6.9×10^6
E	1.85	0.54	2.3×10^6

TABLE V

Vulcanizate	a, cm.	t, mm.	c, cm.	F, kg.
A.............	6.1	0.80	0.55	2.38
	6.1	0.80	1.50	1.72
	6.5	0.90	0.93	2.05
	6.5	0.90	1.10	2.66
	2.9	0.90	0.33	1.66
	2.9	0.82	0.49	1.20
	5.0	0.82	0.53	2.10
	5.0	0.82	0.84	1.77
D.............	6.0	1.60	1.103	4.9
	5.5	1.60	1.148	4.4
	4.65	1.60	0.691	4.3
	2.7	1.61	0.462	2.88
	4.3	1.57	0.888	3.38
E.............	9.0	0.81	1.54	1.77
	9.0	0.81	1.61	1.88
	8.13	0.81	0.88	2.82
	8.13	0.81	1.08	2.33
	7.35	0.81	0.49	3.16
	7.35	0.81	0.58	2.38
	3.95	0.80	0.29	2.10

CONCLUSIONS

It has been seen from the foregoing experiments that for thin sheets of a natural rubber vulcanizate the growth of a cut is determined by an equation similar to that put forward by Griffith as the criterion for the growth of a crack in a hard solid. This criterion involves a characteristic energy for tearing of the material which, for the vulcanizates used, has a magnitude between 10^6 and 2×10^7 ergs/cm.^{2}.

This cannot, of course, be interpreted as twice the surface free-energy of the vulcanizate as was the corresponding characteristic energy involved in Griffith's equation. It must rather be considered more generally as representing the work expended irreversibly per centimeter increase in cut length and per centimeter thickness of the test-piece. We should therefore expect it to depend on the shape of the tip of the cut and on the particular vulcanizate employed. However, if the shape of the tip of the cut is standardized by forming it in some specified manner, then it appears that if the characteristic energy for tearing of the vulcanizate is known it can be employed to predict the force at which tearing will take place in a test-piece of the vulcanizate. Where the shape and disposition of the cut are such that the change in stored elastic energy resulting from increase in cut-length can be calculated from the known elastic characteristics of the vulcanizate, the tearing force can be calculated from the characteristic energy for tearing. Otherwise it is necessary to determine this change of stored energy experimentally.

APPENDIX

	A	B	C	D	E
Natural rubber (smoked sheet)	100	100	100	100	100
Sulfur	2	3	3	3	3
Zinc oxide	2	5	5	2	2
M.B.T.S.	1	—	—	1	4
M.B.T.	—	1	0.5	—	—
Stearic acid	0.5	1	1	1	0.5
D.P.G.	—	1	1	—	—
Nonox	0.5	1	1	0.5	—
Time of cure, mins.	45	40	6	45	40
Temperature of cure, °C.	140	140	140	140	140

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Synopsis

A criterion for tearing of test-pieces cut from thin sheets of a natural rubber vulcanizate, similar in form to the Griffith criterion for spreading of a crack, is formulated. This criterion involves a characteristic energy for tearing which is independent of the shape of the test-piece and of the disposition of the cut. It is shown how this characteristic energy can be found experimentally for a particular vulcanizate and used to predict the force required to tear test-pieces of the vulcanizate.

Résumé

On a mis au point une méthode de test pour le déchirement de pièces de vulcanisats de caoutchouc naturel, provenant de feuilles minces; cette méthode est similaire à celle de Griffith pour l'élargissement d'une déchirure préalable. La méthode décrite suppose l'existence d'une énergie caractéristique de déchirement, qui est indépendante de la forme de la pièces soumise à l'essai et de sa position. On a montré en outre comment cette énergie caractéristique peut être déterminée expérimentalement pour des vulcanisats particuliers et comment elle permet de prédire la force nécessaire pour déchirer des pièces-échantillons de vulcanisats à examiner.

Zusammenfassung

Es wird eine Reissprüfung für Teststücke, die aus dünnen Blättern von natürlichen Kautschukvulkanisaten herausgeschnitten werden, beschrieben, die in der Art der Griffith-Prüfung für Ausbreiten eines Risses ähnlich ist. Diese Prüfung benutzt eine charakteristische Reissenergie, die von der Form des Teststückes und von der Lage des Schnittes unabhängig ist. Es wird gezeigt, wie diese charakteristische Energie experimentell für ein bestimmtes Vulkanisat gefunden werden kann, und wie sie benutzt werden kann, um die Kraft vorauszusagen, die nötig ist, um ein Teststück vom Vulkanisat abzureissen.

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