

1 Summary: Rankine-Hugoniot Equations

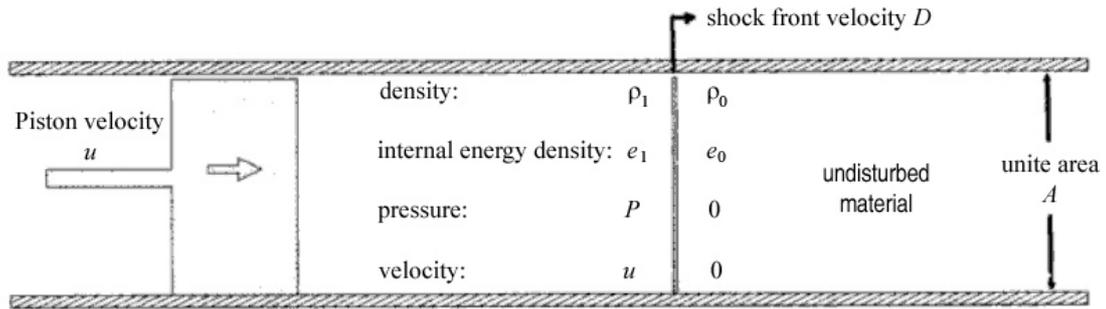


Figure 1. Constant velocity piston-generated shock transition from state 0 to state 1.

(1) Density ratio across the shock wave

$$\frac{\rho_1}{\rho_0} = \frac{D}{D-u} . \tag{1}$$

(1) Pressure applied to the shock

$$P = \rho_0 D u . \tag{2}$$

(3) Change of the specific energy

$$e_1 = e_0 + \frac{1}{2} u^2 . \tag{3}$$

2 Hugoniot Equations

By an ideal shock, one means that a planar pusher is driven at constant velocity into the sample. The resulting shock wave is characterized by a planar shock front that travels at constant velocity during the measurement. Furthermore, one assumes that unshocked material remains at known initial conditions. Under these assumptions, one can determine the equation of state (EOS) from the measured velocities D and u using Eqs. (1), (2) and (3).

The term Hugoniot used here means the sequence of thermodynamic equilibrium states reached behind each shock for a sequence of different-strength shocks from a given initial state.

The Hugoniot equation could be determined from experimental data, and viewed from different planes (such as D - u plane, pressure-volume plane).

2.1 Shock – particle velocity plane

It is found that the shock velocity D is linearly related to the particle velocity u for lots of materials, and can be expressed as

$$D = c_0 + su . \quad (4)$$

If we determined D and u experimentally for a particular material, at a number of shock states, we could plot the data at the D - u plane, strike a straight line through the data points and easily find the D - u Hugoniot equation.

Problem: Study the Water shock Hugoniot in the shock-particle velocity plane.

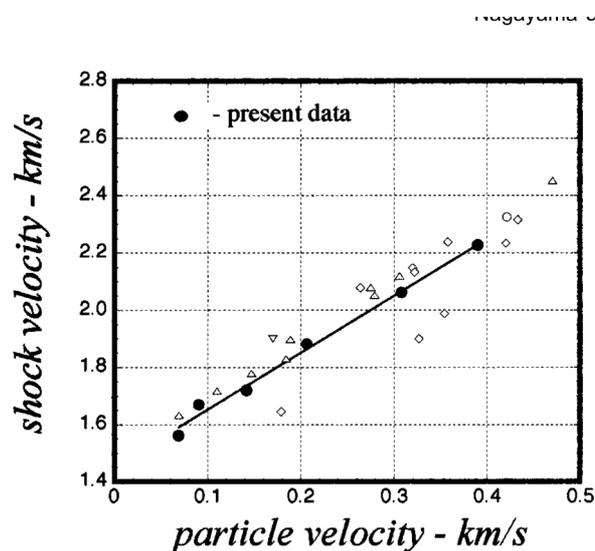


Figure 2. Water shock Hugoniot in the shock-particle velocity plane. Closed circles are the data from Nagayama et al. (2002), while open symbols are data from other sources.

Solution: From the curve, $c_0 = 1.45 \text{ km/s}$, $s = 1.99$.

2.2 Pressure – volume plane

The specific volume, volume per unit mass, is defined as

$$v = \frac{1}{\rho} \quad (5)$$

From the D - u Hugoniot equation, the specific volume v_1 and the pressure P after the shock can be derived from Eqs. (1) and (2), respectively.

Problem: From the shock-particle velocity plane in Figure 2, calculate the specific volume and the pressure when the particle velocity is $u = 0.3\text{km/s}$, respectively. The water density at the initial state is $\rho_0 = 1000\text{kg/m}^3$.

Solution:

From $D = c_0 + su$, $c_0 = 1.45\text{km/s}$ and $s = 1.99$. One has the shock velocity

$$D = 2.05\text{km/s} \text{ (which is 4600 mph).}$$

From Eq. (2), the pressure is

$$P = 0.615\text{GPa}.$$

From Eq. (1), the specific volume is:

$$\begin{aligned} v_1 &= \frac{1}{\rho_0} \frac{D-u}{D} \\ &= \frac{1}{1000} \frac{2.05-0.30}{2.05} \text{ m}^3/\text{kg}, \\ &= 0.85 \times 10^{-3} \text{ m}^3/\text{kg} \end{aligned}$$

Comparing with the specific volume for water in the initial state (zero pressure and zero velocity)

$$v_0 = 1 \times 10^{-3} \text{ m}^3/\text{kg},$$

the change of specific volume is not that dramatic.

Problem: From the shock-particle velocity plane in Figure 2, draw the curve in the Hugoniot pressure-volume plane.

Solution:

Let the particle velocity u change from 100m/s to 500m/s. As in the last problem, the pressure can be calculated from Eq. (2), and the ratio of the specific volume across the shock v_1/v_0 can be calculated from Eq. (1) with $v_1 = 1/\rho_1$ and $v_0 = 1/\rho_0$. The Hugoniot curve in the pressure-volume plane can therefore be calculated and is shown in Figure 3.

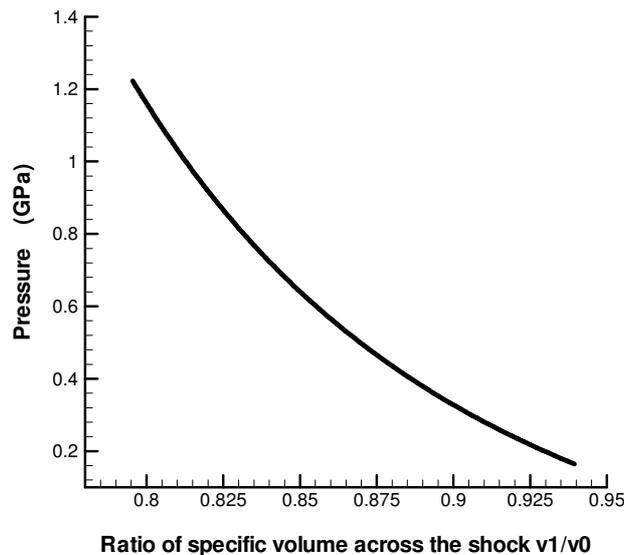


Figure 3. Hugoniot pressure-volume data for water.

References

Nagayama, K., Mori, Y., Shimada, K., and Nakahara, M. (2002) Shock Hugoniot compression curve for water up to 1 GPa by using a compressed gas gun, J. Appl. Phys. 91, 476-482.