Comparison between Selected Viscoplastic Models Used to Describe Solder Deformation

Original Model / Modified Model / Application to Solder	Phenomenological Basis and Key Assumptions	Forms of Important Constitutive Relations	State Variables (S.V.) Material Parameters (excl. E,ν,α)	Required Tests to Determine M.P.s Other Captured Phenomena	Comments
1. Hart (1976) / Jackson et. al. (1981) / Wilcox et. al. (1990)	- Dislocation pile-ups occur at two kinds of barriers: strong (macroplastic) and weak (microplastic) each described by only one state variable and corresponding anelastic modulii. Plastic behavior is represented by leakage of dislocations past these barriers. - Rheological model assumes elements for anelastic, microplastic, macroplastic and glide friction behavior.	$\ln\left(\frac{s_1}{\sigma_{a1}}\right) = \left(\frac{\dot{\varepsilon}_1^{*}}{\dot{\alpha}_1}\right)^{\lambda_1}$ $\dot{\varepsilon}_1^{*} = (s_1/G)^{m_1} f_1 \exp(-Q/R\theta)$ $d(\ln s_1)/dt = h(s_1,\sigma_{a1})\dot{\alpha} - r(s_1,\theta)$ $h = \left(\frac{\sigma_{a1}}{s_1}\right)^{\beta/s_1}\left(\frac{\beta}{s_1}\right)^{\delta}$ $s_2 = \sigma_{a2} e^{-C\sigma_{a2}}$	2 S.V.s: Hardness ‘s1’ and ‘s2’ (1 S.V. in original Hart Model) 18 M.P.s (10 in original model, 8 in Wilcox et al. model)	- tensile relaxation at room temperature - tension-compression cycling	⊕ Can predict important behavior including cyclic tests and transient effects ⊖ Does not include a definite form for grain boundary sliding, which comes into effect at temperatures higher than a third of the melting temperature. ⊖ The model in its useful form uses 2 state variables and 18 material parameters. ⊖ Limited range of validated test examples for solder deformation
2. Anand (1982) / Anand et. al. (1989) / Adams (1986), Wilde et al. (2000), Wang et al. (2001)	- Very similar to Hart model: describes a ‘resistance to deformation’ as a state variable that resists dislocation movement. - Model assumes all behavior as being captured by a single variable	$\dot{\varepsilon}^{p} = A\exp\left(\frac{-Q}{R\theta}\right)\left[\sinh\left(\xi\frac{\sigma}{s}\right)\right]^{1/m}$ $\dot{s} = h(\sigma,s,\theta)\dot{\varepsilon}^{p} - \dot{r}(s,\theta)$ $h = h_o\left\|\left(1-\frac{s}{s^*}\right)\right\|^{a} sign\left(1-\frac{s}{s^*}\right) \qquad a \ge 1$ $s^* = \tilde{s}\left[\frac{\dot{\varepsilon}^{p}}{A}\exp\left(\frac{Q}{R\theta}\right)\right]^{n}$	1 S.V.: Deformation resistance ‘s’ 9 M.P.s	- tensile testing at different strain rates and temperatures - strain hardening - steady state creep -thermal cycling hysteresis loops	⊕ Only one state variable and 9 Material parameters ⊕ ANSYS already offers it as a constitutive model ⊕ Extensive range of predicted behavior validated by several authors. ⊖ Does not account for Bauschinger effect

3. Busso (1992) / - / Busso (1992, 1994)	- Similar to Anand model except that the activation energy is not treated as a material parameter, but is related to the stress state. - Also, state variable used is the back stress which is the internal stress developed due to dislocation pile-up at barriers. The back stress allows the model to describe hardening characteristics.	$\dot{\varepsilon}^p = \dot{\varepsilon}_0 \exp\left\{\frac{-F_o}{R\theta}\left[1-\left(\frac{\lvert\sigma-B\rvert}{\sigma_o G/G_o}\right)^p\right]^q\right\} sign(\sigma-B)$ $\sigma=\sigma_o\frac{G}{G_o}\left\{1-\left(\frac{\theta}{\theta_o}\right)^{1/q}\right\}^{1/p}$ $\theta_o=\frac{F_o}{R}\frac{1}{\ln(\dot{\varepsilon}_o/\dot{\varepsilon}^p)}$	One S.V.: Back/Internal stress 'B'	- temperature and rate dependent tensile testing - constant rate behavior at different temperatures	⊕ Only one state variable and 10 Material parameters ⊕ Accounts for Bauschinger effect ⊖ Limited follow up literature by authors employing model for solder deformation studies ⊖ Isothermal conditions assumed for all performed tests, material constants
			10 M.P.s: 5 to be determined from stress strain curves	- strain hardening - steady state creep - thermal cycling hysteresis loops - Bauschinger effect	
4. Yao and Krempl (1985) / Tachibana and Krempl (1995,1997, 1998) / Maciucescu at al. (1999)	- Viscoplasticity theory Based on Overstress (VBO) assumes a viscoplastic flow potential to relate plastic strain rate and stress. - State Variables defined to model specific behavior: kinematic stress for work hardening and isotropic stress for softening. In the absence of work hardening, the former can be set to zero.	$x=\sigma-B$ $\Phi=\left(\frac{g}{m+1}\right)\left(\frac{\bar{x}}{\bar{\sigma}}\right)^m \bar{x}$ $\dot{\varepsilon}^p=\frac{3}{2}g\left(\frac{\bar{x}}{\bar{\sigma}}\right)^m\left(\frac{x}{\bar{x}}\right)$ $\dot{B}=\bar{\Psi}'\dot{\theta}\sigma+\bar{\Psi}\dot{\sigma}+E\bar{\Psi}'\dot{p}\left(\frac{x}{\bar{x}}-\frac{B-f}{A}\right)$ $\dot{f}=\frac{2}{3}E_t\dot{\varepsilon}^p=E_t\dot{p}\frac{x}{\bar{x}}$ $\dot{A}=-\beta\left(\frac{A-A_2}{A_2}\right)p$	3 S. V.s: Back stress, kinematic stress and isotropic stress	- uniaxial testing at different strain rates and temperatures for 7 M.P.s - cyclic tests for 2 M.P.s	⊕ Only 9 Material Parameters ⊖ 3 State Variables ⊕ Accounts for Bauschinger effect ⊕ Temperature dependency of material constants assumed a priori ⊖ Limited follow up literature by authors employing model for solder deformation studies
			9 M.P.s in simplified VBO model	- strain hardening - steady state creep - Bauschinger effect	

Review of Selected Viscoplasticity Constitutive Models for the Description of Deformation Behavior of Solder Alloys

1. Introduction

1.1. Motivation

Several viscoplasticity models have been proposed to describe the high homologous temperature behavior of metals and the remarkable range of behaviors displayed by them in these conditions. In scope, this document is focusing on viscoplasticity models that have been applied specifically towards predicting solder behavior, even if that was not the original intention of the author of the model. The purpose of this work is to identify a few models that meet a given set of criteria and evaluate and critically assess them qualitatively. While it is impossible to identify one particular model as surpassing all others, it is hoped that a better understanding may be achieved of the trade-offs involved in choosing a particular model over the others and thereby assist a researcher in selecting a particular model based on his/her requirements and the model's limitations and strengths.

1.2. Criteria used for Selection of Models

The following section discusses qualitatively, four viscoplasticity models that have been used to describe solder behavior in microelectronic packaging. All four models are quoted by name of one of the authors (or the author first credited with the model). These four models constitute only a small section of the many constitutive models that have been presented to date. However, these 4 models were selected on the basis of certain criteria which are listed below.

a. The model has been applied successfully to describe deformation behavior of solder alloys
b. The model has a reasonably small number of material parameters (under 10, excluding 2 elastic constants and 1 thermal coefficient)
c. The model is able to predict a reasonable range of experimentally observed behavior including tensile testing at different temperatures and strain rates, creep behavior and cyclic testing related phenomena
d. There is evidence of the model having been tested for its accuracy in predicting experimentally observed behavior that was *not* used in estimating material parameters

The four models that were found to satisfy these criteria were the Hart, Anand, Busso and Krempl models and form the core of the following discussion. Other models that satisfied some (but not all) of the criteria are those proposed by Pao et al. (1992), McDowell et al. (1994) and Ishikawa et al. (2001). While all 3 models have been applied to studying solder deformation, they were not selected for different reasons: Pao et al. describe only steady-state creep phenomena since their model is based on a creep law. McDowell et al. and Ishikawa et al. use the yield surface approach for modeling solder behavior as opposed to the selected four models that use constitutive theory without the formal assumption of a yield surface. Most yield surface based plasticity theory has been applied to studying rate-independent plasticity. Moreover, the existence of a well defined yield point (and an elastic limit for that matter) is under debate and yield can be better thought of and described as a mechanism that can be explained by constitutive laws, see Lubliner (1990). Also, the Ishikawa model, while fairly impressive in it's range of material behavior prediction, is not 'unified' in the strictest sense (since it decomposes inelastic strain rates into plastic and creep components) and perhaps more importantly, needs estimation of as many as 14 material parameters. Very little supporting literature was found for the model proposed by McDowell et al.

1.3. Phenomenological Basis

The phenomenological basis for most of the proposed models arises from Kocks' discussion of the thermodynamics of slip. Hart was the first to propose a unified viscoplasticity model based on these principles. Broadly speaking, Hart bases his derivations on the existence of barriers to dislocations and suggests the existence of two stresses: one due to the pile-up of these dislocations at the barriers and the other due to the glide resistance/friction to dislocation movement within barriers. The Hart model was modified by Jacskon et al. (1981) to consider transient effects by introduction of two kinds of dislocation barriers: strong (such as sub-grain walls) and weak (such as dislocation tangles). Leakage of dislocations across these barriers yields macro- and microplastic behavior respectively. Hart's original model did not capture the latter and worked with a single state variable. However, as Jackson et. al. showed, this could be rectified by consideration of a two state variable model.

Anand's model is based on the same physics as the Hart model: the internal variable 's' represents an averaged isotropic resistance to macroscopic plastic flow offered by the underlying isotropic strengthening mechanisms such as dislocation density, solid solution strengthening, subgrain, and grain size effects, etc. The deformation resistance s is consequently proportional to the equivalent stress and is called the hardness. Busso too, uses a single state variable that derives from the same physical basis of dislocation pile-ups at barriers creating a back stress.

Krempl's model differs from the other three models in the fact that it works with a viscoplastic flow potential and there is no thermodynamic 'activation energy' associated with the plastic strain rate, as is seen in all other models. Krempl's model accounts for hardening and softening phenomena by introduction of one state variable each. This is in addition to the basic state variable which is the so-called overstress which is discussed later in detail.

While the authors' initial equations and statements may be rooted in physically valid theory, as the model is constructed, there are several 'curve fit' parameters introduced to suit the nature of the curves that arise after experimental results are obtained. While this may not seem a very 'scientific' way of approaching the problem, it is nonetheless the only real method that can be applied to a problem of this nature.

1.4. Overview

The discussion of each model in this review begins by listing the primary assumptions made by the author(s) and discusses possible outcomes and limitations arising from the same. It also presents the important constitutive equations with the specific internal/state variables and material parameters. It lists, for each model, the experiments used to determine the material parameters and the validation used for the resulting model (typically by testing it with experiments *not* used in determination of the material parameters). The discussion of each model closes with a perceived list of advantages and limitations of the model with an additional objective of modeling solder deformation behavior in a finite element setting which would require an extension of most models to three dimensional space.

An attempt has been made to use the same notation throughout this discussion: the notation varies from author to author and should be related to the original publications with caution. For the sake of brevity, terms are defined explicitly only when not clear from context. References are not always explicitly cited for the same reason.

2 Selected Viscoplasticity Models

2.1. The Hart Model

Proposed by Hart (1976)
Extended by Jackson et al. (1981) and Korhonen et al. (1987)
Applied to solder deformation by Wilcox et al. (1990)

Background

According to Korhonen et al. (1987), Hart was the first who suggested the existence of a plastic equation of state in terms of stress, non-elastic strain rate and a hardness parameter. Hart's model has been quite successful in describing several deformation behavior such as creep, load relaxation and constant extension tensile rates in the macroplastic region (Jackson et al., 1981) but does not contain the ability to predict microplasticity effects and transient behavior like the Bauschinger effect. The model has since been modified by at least two authors to describe these behavior and this modified model was employed in a study of solder deformation by Wilcox et al. (1990). The rest of this discussion pertains to the use of this modified model and not the original Hart model: while the phenomenological basis and treatment remain the same, the form of the constitutive model is different.

- The model assumes all barriers to dislocations to be of two kinds (strong and weak) with uniform strengths and spacing. However, many of the essential features of the model are still derivable from this stand point.
- Hart introduces the *hardness parameter* as a state variable that measures the characteristics of the barrier structure to plastic flow. If the hardness parameter is uniquely specified by operating stress and non-elastic strain rate, the relationship obtained is a plastic equation of state.
- Wilcox et al. (1990) apply the model to solder deformation after ignoring viscous effects for the purpose of simplification (reduction of material parameters). Such an assumption holds good for small stresses and low temperatures, since for larger applied stresses and higher temperatures, glide friction controls almost all observed behavior (Korhonen et al., 1987)
- Hart discusses deformation purely due to grain matrix deformation (as against due to grain boundary sliding). Accounting for grain boundary sliding requires, according to Hart, minor yet poorly understood modifications. While Korhonen et al. address this fact, they conclude that there exists no definite form for the grain boundary flow law. Since the grain matrix deformation model is essentially valid only for temperatures under a third of the melting temperature of the material under consideration, this raises doubts regarding the comprehensiveness of the model for describing solder deformation accurately. All experimental work done seems to be below a third of this temperature. In fact, Jackson et al. only discuss results at 25 ^{o}C. Korhonen et al. conduct high temperature tests to a maximum of 400 ^{o}C for stainless steel, which meet's the criterion of Hart's grain matrix deformation model.

Important to this work, Wilcox et al. make several additional simplifying assumptions in addition to ignoring grain boundary sliding effects:
- Work hardening is ignored. This assumption can be made only after experimentally identifying if strain hardening effects on the stress strain curve are very small (Adams, 1986 makes the same assumption when applying the Anand model). Wilcox et al.'s work with eutectic Sn-Pb solder showed this was a reasonable assumption.
- Wilcox et al. acknowledge that the extension of data obtained experimentally from bulk samples may not always be applicable to all solder joints as dimensions decrease. Such a conclusion can only be arrived at after experimental work is performed.

Constitutive Equations, State Variables and Material Parameters

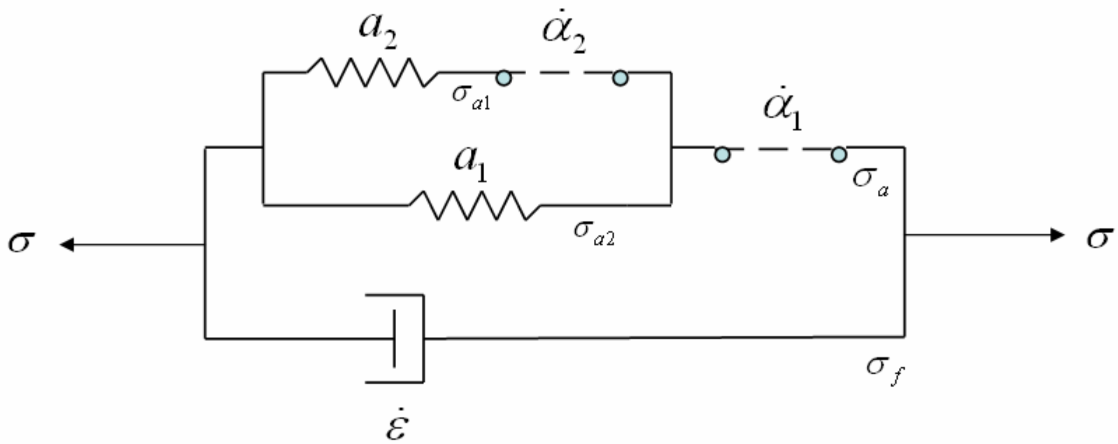

Fig 1. Rheological model depicting the extended Hart's model

The rheological model originally conceived by Hart had an anelastic element a, a plastic element $\dot{\alpha}$ and an element representing glide friction $\dot{\varepsilon}$. The extension of this model is depicted in Figure 1 above and contains two anelastic elements a_1 and a_2, two plastic elements $\dot{\alpha}_1$ (macroplastic behavior) and $\dot{\alpha}_2$ (microplastic behavior). The glide friction element was retained from the original Hart model. This rheological model represents the grain matrix deformation aspect of the constitutive behavior. Constraint equations that are implied from the diagram are:

$$\sigma = \sigma_f + \sigma_a = \sigma_f + \sigma_{a1} + \sigma_{a2}$$

and

$$\dot{\varepsilon} = \dot{\alpha}_1 + \dot{a}_1 = \dot{\alpha}_1 + \dot{\alpha}_2 + \dot{a}_2$$

Development of Constitutive Equations

The anelastic elements are described as linear elements with a modulii such that:

$$\sigma_{a_1} = \mu_1 a_1$$

and

$$\sigma_{a_2} = \mu_2 a_2$$

The plastic elements are represented by a flow rule relating the hardness parameters s_1 and s_2 to the strain rate and stress state.

$$\ln\left(\frac{s_1}{\sigma_{a1}}\right) = \left(\frac{\dot{\varepsilon}_1^*}{\dot{\alpha}_1}\right)^{\lambda_1}$$

where $\dot{\varepsilon}_1^*$ is a rate parameter and λ_1 is a constant and a similar equation can be written for the second element.

The mechanical equation of state relating strain rate and stress in terms of activation energy Q, Boltzmann's constant R, temperature θ is commonly given by an Arrhenius form as (written for one element, similar for the other)

$$\dot{\varepsilon}_1^* = (s_1 / G)^{m_1} f_1 \exp(-Q / R\theta)$$

where f is a material parameter, G is shear modulus.

Finally, the macroplastic hardness parameter s_1 is written as being governed by work-hardening and static recovery terms:

$$d(\ln s_1)/dt = h(s_1, \sigma_{a1})\dot{\alpha} - r(s_1, \theta)$$

The recovery term may be ignored at moderate homologous temperatures and work hardening function can be expressed as

$$h = \left(\frac{\sigma_{a1}}{s_1}\right)^{\beta/s_1} \left(\frac{\beta}{s_1}\right)^{\delta}$$

where β and δ are additional material constants.

An expression for the microplastic hardness parameter is not obtainable as in the above method since it's evolution is a sampling effect rather than the result of work-hardening. Jackson et al. derived an expression for the evolution of s_2 based on a curve fit to an exponential equation:

$$s_2 = \sigma_{a2} e^{-C\sigma_{a2}}$$

where C is yet another material constant. They used this result to express the parameter for 3 different bounds.

State Variables and Material Parameters

While Hart's original model specifies only 1 state variable and 10 material parameters, the extended version has two state variables to account for microplasticity effects. This increases the number of material parameters as well and for the model described by Jackson et al. there are as many as 18 material parameters. Wilcox et al. make several simplifying assumptions as discussed previously, which have the effect of reducing these parameters to just 8. However, this does not include the anelastic modulii, which they choose outside of the experimental data.

Validation

Several tests by Korhonen et al. (constant extension rate tension, constant load creep, load relaxation, creep recovery) and Jackson et al. (uniaxial tension, load relaxation, load-hold-unload profiles) point towards the validity of the Hart model at low temperatures relative to melting point. It is Wilcox's results that assume greater importance for this work. Despite several simplifying assumptions, Wilcox et al. are able to obtain good agreement between model and experimental data for tensile relaxation and tension-compression tests. As will be seen in future models, this range of testing is by no means extensive since it excludes discussions of strain rate jump and creep tests, to name a few.

2.2. The Anand Model

Proposed by Anand (1982)
Revised by Anand et al. (1989)
Applied to solder deformation by Adams (1986), Wilde et al. (2000), Wang et al. (2001) etc.

Background

The Anand model is by far the most popular viscoplastic model to describe solder deformation. While Anand's original work was in the area of hot working of metals, Adams (1986) used his model with simplifications to describe tests on Pb-Sn solder and it has since then been applied by more than one author for the same purpose. The reasons for the popularity of the Anand model are several:
- At the outset, it involves only one state variable, which is the scalar non-zero variable called deformation resistance (similar to the hardness term discussed by Hart).
- The Anand model needs no explicit yield condition and no unloading/loading criterion.
- The Anand model in its most often used form for solder deformation has 9 material parameters (that require determination by curve fit methods), which is a reasonable number of parameters to work with for a model of this nature.
- Perhaps the most important reason for the popularity of the model compared to similar models by Busso can be linked to the fact that the ANSYS code offers it as a constitutive model, which has been a very useful tool for those interested in FE modeling of solder joints in electronic packages as it avoids detailed coding of the model into the FE software.
- Finally, several authors have successfully applied the Anand model to describing solder deformation such as Wilde et al. (2000) and Wang et al. (2001). The range of behavior described by the model is enviable, including strain hardening, constant strain rate behavior at different temperatures, steady state creep and thermal cycling hysteresis loops.

- Anand assumes one state variable can completely describe constitutive behavior. While this is a gross oversimplification, it has proven to be adequate to describe most experimentally observed phenomena reasonably well.

Development of Constitutive Equations

Anand's first assumption introduces the state variable 's' at all times in a ratio to the equivalent tensile stress as expressed below. This is primarily due to its attractiveness in this form from dimensional considerations.

$$\dot{\varepsilon}^p = f(\frac{\sigma}{s}, \theta)$$

Like Hart, Anand employs an evolution equation that is in terms of work hardening and static recovery terms.

$$\dot{s} = h(\sigma, s, \theta)\dot{\varepsilon}^p - \dot{r}(s, \theta)$$

Adams (1986) does not include the evolution expression for s on the grounds that the experimental results showed little or no strain hardening after strains of approximately 5%. Anand (1989) includes the effects of both terms in his discussion of deformation of iron and aluminum alloys at high temperatures. For his test results however, he concludes that the static recovery contribution to the evolution expression can be neglected. Indeed, all the literature reviewed used models that excluded any static recovery terms. Performing load-unload-hold-reload tests is the only way to really determine the influence of static recovery and justify its exclusion from the model (if stress curves do not change much after reloading).

Anand also uses a similar form for the plastic strain rate expression (from Sellars and Tegart) which accommodates both the power law and the exponential dependence of strain rate on stress for constant state (structure). It is given by:

$$\dot{\varepsilon}^p = A\exp(\frac{-Q}{R\theta})[\sinh(\xi\frac{\sigma}{s})]^{1/m}$$

where m and ξ are expectedly, material parameters.

This gives the following:

$$\sigma = cs$$

where

$$c = \frac{1}{\xi}\sinh^{-1}\left[\left\{\frac{\dot{\varepsilon}^p}{A}\exp(\frac{Q}{R\theta})\right\}^m\right]$$

And from the evolution equation for 's':

$$\dot{\sigma} = ch(\sigma, s, \theta)\dot{\varepsilon}^p - c\dot{r}(s, \theta)$$

which on integrating gives,

$$\frac{d\sigma}{d\varepsilon^p} = ch - \frac{c}{\dot{\varepsilon}^p}\dot{r}$$

Ignoring the static recovery term makes the second term in the above expression vanish.

Anand put forth (without any apparent physical reasoning) the following expression for the work hardening term:

$$h = h_o\left|(1-\frac{s}{s^*})\right|^a sign(1-\frac{s}{s^*}) \qquad a \geq 1$$

$$s^* = \tilde{s}\left[\frac{\dot{\varepsilon}^p}{A}\exp(\frac{Q}{R\theta})\right]^n$$

State Variables and Material Parameters

As mentioned earlier, only one state variable is needed here: s. There are seemingly 9 material parameters (after excluding the recovery term) that need to be determined from experimental data being applied to the equations discussed above. The material parameters can be sufficiently determined from isothermal constant strain tension tests spanning the range of temperatures and strain rates of interest and from strain rate jump tests.

Validation

Anand et al. (1989) validated the model for the original experiments (constant strain at different temperatures and strain rates) and demonstrated its usefulness on strain rate jump tests, strain rate decrement tests and a load controlled experiment. The model was also validated by other authors for similar tests (with the constant strain tests being the most popular) and also for steady state creep behavior (Wang et al.).

2.3. The Busso Model

Proposed by Busso (1992)

Applied to solder deformation by Busso (1992, 1994)

Background

Busso's model primarily differs from Anand's model in two respects:

- Busso does not use a deformation resistance (or hardness) term as a state variable but instead defines a back stress B to explain observed macroscopic kinematic hardening and explain the Bauschinger effect. The back stress can be thought of as an internal stress arising from dislocation pile-ups at weak and strong barriers as discussed by Hart.
- Busso also does not treat the activation energy term as a material parameter, but expresses it in terms of the applied stress.

Development of Constitutive Equations

Busso represents the constitutive law as a function g as:

$$g\{\sigma, \dot{\sigma}, \dot{\varepsilon}^p, \theta, \dot{\theta}, B, \dot{B}\} = 0$$

As with the Anand model, Busso writes the evolution of B in terms of hardening and static recovery terms, except he treats *h* and *r* as material parameters, unlike Anand who treats these terms as functions of applied stress and temperature.

$$\dot{B} = h\,\dot{\varepsilon}^p - rB\left|\dot{\varepsilon}^p\right|$$

The plastic strain rate is related by an exponential form to the activation energy Q

$$\dot{\varepsilon}^p = \dot{\varepsilon}_0 \exp(\frac{-Q}{R\theta})$$

$\dot{\varepsilon}_0$ is some pre-exponential factor that is treated as a material parameter.

As mentioned previously, Q is expressed in terms of applied stress as:

$$Q = F_0\left\{1 - \left(\frac{\sigma}{\sigma_o G / G_o}\right)^p\right\}^q$$

Where $\sigma_0 G / G_0$ is the flow stress scaled by ratio of shear modulii to reduce all elastic interactions to 0 K and F_0 is the total free energy of activation under vanishingly small stress. *P* and *q* are selected to best fit the stress dependence to the activation energy.

In terms of stress we can obtain then:

$$\sigma = \sigma_o \frac{G}{G_o}\left\{1 - \left(\frac{\theta}{\theta_o}\right)^{1/q}\right\}^{1/p}$$

where

$$\theta_o = \frac{F_o}{R}\frac{1}{\ln(\dot{\varepsilon}_o / \dot{\varepsilon}^p)}$$

Finally, the expression for plastic strain rate can be expressed as:

$$\dot{\varepsilon}^{p} = \dot{\varepsilon}_0 \exp\left\{\frac{-F_o}{R\theta}\left[1-\left(\frac{|\sigma - B|}{\sigma_o G / G_o}\right)^p\right]^q\right\} sign(\sigma - B)$$

where the applied stress term σ is replaced by $\sigma - B$ to account for kinematic deformation and cyclic deformation.

State Variables and Material Parameters

The Busso model too has just one state variable, back stress B. There are 10 material parameters, 5 associated with the intrinsic properties of the material and 5 that can be determined by fitting to stress strain data.

Validation

Busso simulates stress strain behavior and reproduces Adams' (1986) data very well. Additionally, the predictive capability of the model is tested against strain rate dependence tests which were not included in estimating material parameters. Finally, the Busso model is able to predict Bauschinger effect but the hysteresis loop fails to show a smooth elastic-plastic transition, though the onset of yielding is reasonably well predicted. Busso attributes this to using a scalar hardening term as opposed to a more complex function: use of more complex functions for *h* would however require more material parameters. Finally, the Busso model is tested for steady state creep behavior and is able to predict this well too. Busso also implemented his model for studying solder joints in IC packages in a finite element setting (ADINA) and obtained fatigue life predictions based on accumulated plastic strain.

2.4. The Krempl Model (1999)

Proposed by Yao and Krempl (1985)
Revised by Tachibana and Krempl (1995, 97, 98)
Applied to solder deformation by Maciucescu et al. (1999)

Background

Of the four models discussed here, the Krempl model is perhaps the most complex and furthest away in principle from the three mentioned above. This is essentially due to the fact that the VBO model assumes a viscoplastic flow potential Φ and does not evaluate the plastic strain from dislocation and thermodynamics considerations as the previous models did. Also the model, in an attempt to relate a wide range of physical reasoning to observed phenomena, requires a total of 3 state variables: the overstress x (which is the difference between the actual stress σ and the back stress B), the kinematic stress and the isotropic stress. One of the other key differences in the Krempl model is the a priori assumption that all material constants vary with time.

Development of Constitutive Relations

The stress rate is represented by:

$$\dot{\sigma} = \frac{\mu' \dot{\theta}}{\mu}\sigma + 2\mu(\dot{\varepsilon} - \dot{\varepsilon}^p)$$

Where μ is the shear modulus and a primed term refers to the derivative with respect to temperature.

As mentioned before, for the purposes of modeling viscoplastic response, the VBO requires introduction of an overstress *x,* which is defined as the difference between the applied stress and the back (or equilibrium) stress as:

$$x = \sigma - B$$

Additionally, a viscoplastic flow potential is introduced as:

$$\Phi = \left(\frac{g}{m+1}\right)\left(\frac{\overline{x}}{\overline{\sigma}}\right)^m \overline{x}$$

Where $\overline{x}$ represents the effective overstress which is defined as

$$\overline{x} = \sqrt{(3/2)tr(x.x)}$$

And *g,* $\overline{\sigma}$ and *m* are material constants. Since the inelastic strain rate can be thought of as the derivative of the viscoplastic flow potential with respect to the stress, an expression for the inelastic strain rate is determined as:

$$\dot{\varepsilon}^p = \frac{3}{2} g\left(\frac{\overline{x}}{\overline{\sigma}}\right)^m \left(\frac{x}{\overline{x}}\right)$$

State Variables and Material Parameters

The evolution laws of the three state variables are given below:

- Back Stress *B*

$$\dot{B} = \overline{\Psi}'\dot{\theta}\sigma + \overline{\Psi}\dot{\sigma} + E\overline{\Psi}'\dot{p}\left(\frac{x}{\overline{x}} - \frac{B-f}{A}\right)$$

where $\dot{p} = \sqrt{(2/3)tr(\dot{\varepsilon}^p.\dot{\varepsilon}^p)}$, $0 < \Psi < 1$ is a dimensionless shape constant that controls transition from quasi-elastic to fully established inelastic flow and *E* is the modulus of elasticity.

- Kinematic Stress f (Hardening Term)

$$\dot{f} = \frac{2}{3} E_t \dot{\varepsilon}^p = E_t \dot{p} \frac{x}{\bar{x}}$$

- Isotropic Stress (Softening Term)

$$\dot{A} = -\beta \left(\frac{A - A_2}{A_2} \right) p$$

Ignoring work hardening effects eliminates the need for f (more accurately, sets it to zero).

The Krempl model (VBO) originally used these three state variables and a total of 17 material parameters. For the model describing solder deformation, a simplified VBO model with 9 material constants was used. Based on the variation of these material constants with respect to temperature, a fourth order polynomial curve fit was performed to describe their temperature dependence. This lends the model additional robustness when it comes to predicting behavior at various temperatures, even those not included in preliminary testing.

Validation

Material parameters in the Krempl model have been derived from uniaxial tensile testing at different strain rates and at different temperatures. It is validated against experimental results for cyclic softening behavior and as expected (since it has an explicit softening term) predicts this behavior very well. This was tested for a variety of strain amplitudes. The Krempl (VBO) model also predicts the Bauschinger effect well.

3. Discussion

As one would expect, a quantitative comparison of the four selected models is not meaningful. A discussion of the models can only be done in light of the specific requirements and limitations of the user of these models. These are typically related to the testing equipment, the Finite Element Analysis expertise etc. Perhaps more importantly, the behavior of the specific material (or in this case solder alloy specimen) that is being studied can decide which model to use. A suggested procedure is listed in section 2 of this project and a comparison of all models in a tabular format precedes that.

Criticism of the use of these models is often based on the fact that they are all based on a procedure that involves several assumptions, curve fitting and a poor physical basis for the equations. Unfortunately, since the nature of these materials under the given situations is so complex, there appear to be no alternatives to state variable and material parameter based viscoplasticity models.

The application of these modeling results that are mostly gathered from bulk solder specimens to actual solder joints is not apparent. Only Busso’s model seemed to have been applied to solder joints in a finite element setting with good results. For smaller size solders, where microstructure and pad metallurgy may play significant roles, it may prove to be more fruitful to apply these models to specimens that are of the size and nature of actual solder joints.

Finally, in terms of simplicity and range of behavior predicted, it seems the Anand and Busso models have an edge over the other two models. While the Hart/Wilcox model has not enjoyed the same degree of success as the other models, the Krempl model while being very effective in predicting behavior, is somewhat complex. The Anand model has a strong advantage in the fact that it is already implemented in finite element code. The Busso model on the other hand is able to predict important phenomena reasonably well including some that cannot be predicted with the Anand model.

4. References

References are listed depending on which model they represent. Important papers connected to the development of the model are also listed in addition to papers that discuss the model itself.

The Hart Model

- Hart, E.W., "*Constitutive relations for the nonelastic deformation of metals",* Journal of Engineering Materials and Technology, pp 193-202, 1976
- Korhonen, M. A., Hannula, S. P., and Li, C. Y., *"State Variable Theories Based on Hart's Formulation,"* in Unified Constitutive Equations for Creep and Plasticity, Elsevier, pp. 89–137, 1987
- Wilcox, J. R., Subrahmanyan R., Li Che-Yu, "*Thermal Stress Cycles and Inelastic Deformation in Solder Joints", 1990*

The Anand Model

- Adams, P.J., 1986, *"Thermal fatigue of solder joints in micro-electronic devices"*, Thesis, MIT
- Anand, L., *``Constitutive Equations for the Rate-Dependent Deformation of Metals at Elevated Temperatures''*, ASME Journal of Engineering Materials and Technology, 104, 12--17, 1982.
- Brown, S. B., Kim, K. H., and Anand, L., *``An Internal Variable Constitutive Model for Hot Working of Metals''*, International Journal of Plasticity, 5, pp. 95-130, 1989.
- Weber, G. G., Lush, A. M., Zavaliangos, A., and Anand, L., *`` An Objective Time-Integration Procedure For Isotropic Rate-Independent And Rate-Dependent Elastic-Plastic Constitutive Equations''*, International Journal of Plasticity, 6, pp. 701 -- 744, 1990.
- Wilde, J. Becker, K. Thoben, M. Blum, W. Jupitz, T. Guozhong Wang Cheng, Z.N. , *"Rate dependent constitutive relations based on Anand model for 92.5Pb5Sn2.5Ag solder",* Advanced Packaging, IEEE Transactions on, 2000, Vol. 23, pp 408-414
- G. Z. Wang, Z. Cheng, K. Becker and J. Wilde, "*Applying Anand model to represent the viscoplastic deformation behavior of solder alloys",* Trans ASME J. Electronic Packaging, Vol. 123, No. 3, 2001, pp. 247-253

The Busso Model

- Busso, E. P., Kitano, M. and Kumazawa, T., *"A Visco-Plastic Constitutive Model for 60/40 Tin-Lead Solder Used in IC Package Joints "*. ASME Journal Eng. Materials and Technology, V. 114, (1992), 331-337.
- Busso, E. P., Kitano, M. and Kumazawa, T., *"Modelling Complex Inelastic Deformation Processes in IC Packages' Solder Joints "*. Journal of Electronic Packaging, V. 116, (1994), 6-15.

The Krempl Model

- Majors, PS; Krempl, E., "The isotropic viscoplasticity theory based on overstress applied to the modeling of modified 9 Cr-1 Mo steel at 538°C", Mat. Sci. and Engg., A186 (1994) pp. 23-34
- Krempl, E. "Viscoplastic Models for High Temperature Applications," International Journal of Solids and Structures, 37, 279-291, 2000.
- Tachibana Y., Krempl, E., *"Modeling of high homologous temperature deformation behavior using the Viscoplasticity theory based on Overstress (VBO): Part I- Creep and Tensile Behavior"*, ASME Journal of Engineering Materials and Technology, Vol 117, pp 456-461, 1995
- Tachibana Y., Krempl, E*., "Modeling of high homologous temperature deformation behavior using the Viscoplasticity theory based on Overstress (VBO): Part II- Characteristics of VBO Model"*, ASME Journal of Engineering Materials and Technology, Vol 119, pp 1-6, 1997
- Tachibana Y., Krempl, E., *"Modeling of high homologous temperature deformation behavior using the Viscoplasticity theory based on Overstress (VBO): Part III- A Simplified Model",* ASME Journal of Engineering Materials and Technology, Vol 120, pp 193-196, 1998

- Maciucescu, L., Sham, T.-L., Krempl E., *"Modeling the deformation behavior of a Sn-Pb Solder alloy using the Simplified Viscoplasticity theory based on Overstress",* ASME Journal of Electronic Packaging, vol 121, pp 92-98, 1999

Other Models

- Sasaki, K., Ohguchi K-I, Ishikawa, H., *"Viscoplastic deformation of 40Pb/60Sn Solder Alloys-Experiments and Constitutive modeling"*, ASME Journal of Electronic Packaging, Vol. 123, pp 379-387, 2001
- McDowell, D.L., Miller, M.P., Brooks, D.C., *"Unified creep-plasticity theory for solder alloys"*, ASTM STP 1153, S.A. Schroeder and M.R. Mitchell, eds, ASTM, PA, pp 42-59*
- Pao, Y-H, Badgley, S., Jih, E., Govila, R., Browning, J., "Constitutive behavior and low cycle fatigue of 97Sn-3Cu Solder joints", ASME Journal of Electronic Packaging, Vol. 115, pp 147-152, 1993

Other Reading

- Lubliner, J., Plasticity Theory, MacMillan, 1st Edition, 1990

* This paper was not obtained for the purposes of the study but discussions from other papers that discuss it have been used here